LETTER

A Design Method of Odd-Channel Linear-Phase Paraunitary Filter Banks with a Lattice Structure

Shogo MURAMATSU† *and* Hitoshi KIYA†, *Members*

SUMMARY In this letter, a design method of linear-phase paraunitary filter banks is proposed for an odd number of channels. In the proposed method, a non-linear unconstrained optimization process is assumed to be applied to a lattice structure which makes the starting guess of design parameters simple. In order to avoid insignificant local minimum solutions, a recursive initialization procedure is proposed. The significance of our proposed method is verified by some design examples.
key words: *multirate filter banks, paraunitary system, linear-phase filter, subband image coding, non-linear optimization*

1. Introduction

The linear-phase (LP) and paraunitary (PU) properties of filter banks are particularly significant for subband coding of images [1]–[3]. Thus, several linear-phase paraunitary filter banks (LPPUFBs) have been studied so far [3]–[8]. In the article [4], a special case of such systems, which is known as the lapped orthogonal transforms (LOT), was shown. Then, the more general systems were established [5], [6]. For even number of channels, such systems have been well developed, especially with lattice structures [3], [7], [8]. Those structures enable us to design LPPUFBs with a non-linear unconstrained optimization process. However, non-linear optimization processes are sensitive to their starting guess and has no guarantee to yield the global minimum solution. Thus, in order to avoid at least an insignificant local minimum solution, a recursive initialization design procedure was proposed [8].

Recently, some approaches to construct odd-channel LPPUFBs have been developed. Soman et al. showed the existence of odd-channel LPPUFBs and provided the lattice structure [6]. Then, Nagai et al. improved the lattice structure to cover larger class of LPPUFBs than Soman's system [9], [10]. In the article [9], in order to avoid the use of non-linear optimization, the design problem is reduced to solving a set of linear equations iteratively. In compensation for this approach, object functions are restricted and some practical ones, such as coding gain, are excluded.

In this letter, we consider applying a non-linear optimization process to a lattice structure of odd-channel LPPUFBs. In order to avoid at least insignificant local minimum solutions, we provide a lattice structure which makes the starting guess of design parameters simple, and propose a recursive initialization design procedure. The procedure starts from the design for a non-overlapping system and evolutionarily increases the design problem. This work can be regarded as a modification of that for even-channel system [8]. This letter is based on the article [16], and our method is independently devised with the article [9].

All through this letter, the notations $\boldsymbol{\Gamma}_M$, $\boldsymbol{I}_M$, and $\boldsymbol{J}_M$ denote the $M \times M$ diagonal matrix which has $+1$ and -1 elements alternatively on the diagonal, the identity matrix and the counter identity matrix [1], [11]. In addition, $\boldsymbol{O}$ and $\boldsymbol{o}$ are the null matrix and vector, respectively, and the superscript 'T' on a matrix or a vector represents the transposition.

2. Review of LPPUFBs

In this section, we review LPPUFBs. Figure 1 shows a parallel structure of M-channel maximally decimated filter banks [1], where $H_k(z)$ and $F_k(z)$ are the analysis and synthesis filters, respectively. When the reconstructed output sequence $\hat{x}(n)$ is identical to the input $x(n)$ except for the delay and scaling, the analysis-synthesis system is called *perfect reconstruction (PR) filter banks*. Let $\boldsymbol{E}(z)$ and $\boldsymbol{R}(z)$ be the $M \times M$ type-I polyphase matrix of an analysis bank and type-II polyphase matrix of a synthesis bank, respectively [1].

If $\boldsymbol{E}(z)$ and $\boldsymbol{R}(z)$ satisfy the condition that $\boldsymbol{R}(z)\boldsymbol{E}(z) = cz^{-N}\boldsymbol{I}_M$ for some integer N and some non-zero constant c, then the system has PR property [1]. In addition, if $\boldsymbol{E}(z)$ holds the condition that

$$\tilde{\boldsymbol{E}}(z)\boldsymbol{E}(z) = \boldsymbol{I}_M, \tag{1}$$

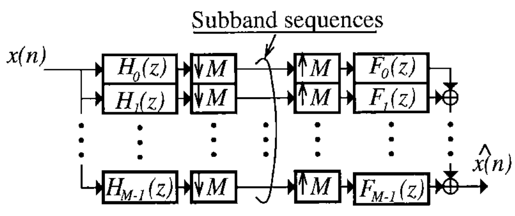


Fig. 1 M-channel maximally decimated filter banks. The box including $\downarrow M$ and $\uparrow M$ denote the down- and up-sampler with the factor M, respectively.

Manuscript received December 19, 1997.
†The authors are with the Department of Electrical Engineering, Graduate School of Engineering, Tokyo Metropolitan University, Hachioji-shi, 192–0364 Japan.

then it is said to be *paraunitary (PU)*, where $\tilde{\boldsymbol{E}}(z)$ is the paraconjugation of $\boldsymbol{E}(z)$ [1]. The condition as in Eq. (1) is sufficient to construct PR filter banks, since the PR property is guaranteed by choosing the synthesis polyphase matrix as $\boldsymbol{R}(z) = cz^{-N}\tilde{\boldsymbol{E}}(z)$.

Next, let us show the LP property of filter banks. We assume that the elements of the polyphase matrix $\boldsymbol{E}(z)$ is real, causal and FIR of order N. On this assumption, the corresponding analysis filters $H_k(z)$ are also real, causal and FIR, and the order results in $K = (N+1)M - 1$. If $\boldsymbol{E}(z)$ holds

$$z^{-N}\boldsymbol{\Gamma}_M\boldsymbol{E}(z^{-1})\boldsymbol{J}_M = \boldsymbol{E}(z), \quad (2)$$

then each analysis filter $H_k(z)$ for even k is symmetric and one for odd k is antisymmetric

In this letter, we consider constructing LPPUFBs for odd M, which satisfy both Eqs. (1) and (2). For the sake of convenience, the order N of a polyphase matrix is sometimes referred to as *the overlapping factor.*

3. Lattice Structure

For an odd-channel LPPUFB of overlapping factor $N = 2L$, it can be verified that the product form

$$\boldsymbol{E}(z) = \boldsymbol{P}^T\left\{\prod_{\ell=1}^{L}\boldsymbol{R}_{\mathrm{E}\ell}\boldsymbol{Q}_{\mathrm{E}}(z)\boldsymbol{R}_{\mathrm{O}\ell}\boldsymbol{Q}_{\mathrm{O}}(z)\right\}\boldsymbol{R}_{\mathrm{E}0}\boldsymbol{C}\boldsymbol{J}_M \quad (3)$$

provides a lattice structure, where

$$\boldsymbol{C} = \frac{1}{\sqrt{2}}\begin{bmatrix}\boldsymbol{\Phi}_{\mathrm{S}} & \boldsymbol{O}\\ \boldsymbol{O} & \boldsymbol{\Phi}_{\mathrm{A}}\end{bmatrix}\begin{bmatrix}\boldsymbol{I}_{\frac{M-1}{2}} & \boldsymbol{o} & \boldsymbol{J}_{\frac{M-1}{2}}\\ \boldsymbol{o}^T & \sqrt{2} & \boldsymbol{o}^T\\ \boldsymbol{J}_{\frac{M-1}{2}} & \boldsymbol{o} & -\boldsymbol{I}_{\frac{M-1}{2}}\end{bmatrix}, \quad (4)$$

where $\boldsymbol{\Phi}_{\mathrm{S}}$ and $\boldsymbol{\Phi}_A$ denote arbitrary $(M+1)/2\times(M+1)/2$ and $(M-1)/2 \times (M-1)/2$ orthonormal matrices, respectively. In addition,

$$\boldsymbol{R}_{\mathrm{E}\ell} = \begin{bmatrix}\boldsymbol{W}_{\mathrm{E}\ell} & \boldsymbol{O}\\ \boldsymbol{O} & \boldsymbol{U}_{\mathrm{E}\ell}\end{bmatrix}, \quad (5)$$

$$\boldsymbol{R}_{\mathrm{O}\ell} = \begin{bmatrix}\boldsymbol{W}_{\mathrm{O}\ell} & \boldsymbol{o} & \boldsymbol{O}\\ \boldsymbol{o}^T & 1 & \boldsymbol{o}^T\\ \boldsymbol{O} & \boldsymbol{o} & \boldsymbol{U}_{\mathrm{O}\ell}\end{bmatrix}, \quad (6)$$

where $\boldsymbol{W}_{\mathrm{E}\ell}$ is an $(M+1)/2 \times (M+1)/2$ orthonormal matrix, and all of $\boldsymbol{W}_{\mathrm{O}\ell}$, $\boldsymbol{U}_{\mathrm{E}\ell}$ and $\boldsymbol{U}_{\mathrm{O}\ell}$ are $(M-1)/2 \times (M-1)/2$ orthonormal matrices. Furthermore, $\boldsymbol{Q}_{\mathrm{E}}(z) = \boldsymbol{B}\boldsymbol{\Lambda}_{\mathrm{E}}(z)\boldsymbol{B}$ and $\boldsymbol{Q}_{\mathrm{O}}(z) = \boldsymbol{B}\boldsymbol{\Lambda}_{\mathrm{O}}(z)\boldsymbol{B}$, where

$$\boldsymbol{\Lambda}_{\mathrm{E}}(z) = \begin{bmatrix}\boldsymbol{I}_{\frac{M+1}{2}} & \boldsymbol{O}\\ \boldsymbol{O} & z^{-1}\boldsymbol{I}_{\frac{M-1}{2}}\end{bmatrix}, \quad (7)$$

$$\boldsymbol{\Lambda}_{\mathrm{O}}(z) = \begin{bmatrix}\boldsymbol{I}_{\frac{M-1}{2}} & \boldsymbol{O}\\ \boldsymbol{O} & z^{-1}\boldsymbol{I}_{\frac{M+1}{2}}\end{bmatrix}, \quad (8)$$

$$\boldsymbol{B} = \frac{1}{\sqrt{2}}\begin{bmatrix}\boldsymbol{I}_{\frac{M-1}{2}} & \boldsymbol{o} & \boldsymbol{I}_{\frac{M-1}{2}}\\ \boldsymbol{o}^T & \sqrt{2} & \boldsymbol{o}^T\\ \boldsymbol{I}_{\frac{M-1}{2}} & \boldsymbol{o} & -\boldsymbol{I}_{\frac{M-1}{2}}\end{bmatrix}. \quad (9)$$

Figure 2 shows the lattice structure, which consists of $(M+1)/2$ symmetric and $(M-1)/2$ anti-symmetric filters. The counterpart synthesis bank holding PR property is simply obtained as the paraconjugation of analysis bank because of the PU property [1].

The product form in Eq. (3) is obtained in the similar way to the approach for even M shown in the article [8], where it should be noted that the overlapping factor N is even when M is odd [12]. The product form guarantees both of the PU and LP properties, since the the order-increasing process

$$\boldsymbol{E}_{2(\ell+1)}(z) = \boldsymbol{R}_{\mathrm{E},\ell+1}\boldsymbol{Q}_{\mathrm{E}}(z)\boldsymbol{R}_{\mathrm{O},\ell+1}\boldsymbol{Q}_{\mathrm{O}}(z)\boldsymbol{E}_{2\ell}(z) \quad (10)$$

holds both of the PU and LP properties when $\boldsymbol{E}_{2\ell}(z)$ is PU and LP, where $\boldsymbol{E}_m(z)$ denotes a polyphase matrix of LPPUFBs whose overlapping factor is m.

Substituted $\boldsymbol{\Phi}_{\mathrm{S}} = \boldsymbol{I}_{\frac{M+1}{2}}$ and $\boldsymbol{\Phi}_{\mathrm{A}} = -\boldsymbol{J}_{\frac{M-1}{2}}$, Eq. (3) results in the factorization given in the articles [9], [10]. The factorization was shown to be minimal and complete for odd-channel LPPUFBs whose filters all have length $(N+1)M$. Note that any choice of $\boldsymbol{\Phi}_{\mathrm{S}}$ and $\boldsymbol{\Phi}_{\mathrm{A}}$ does not affect the minimality and the completeness. As we will show, proper choice of these matrices makes the starting guess of the design parameters simple, and these matrices contribute only for the starting guess and are fixed during the design phase.

4. Design Procedure

By controlling the matrices $\boldsymbol{W}_{\mathrm{E}\ell}$, $\boldsymbol{W}_{\mathrm{O}\ell}$, $\boldsymbol{U}_{\mathrm{E}\ell}$ and $\boldsymbol{U}_{\mathrm{O}\ell}$,

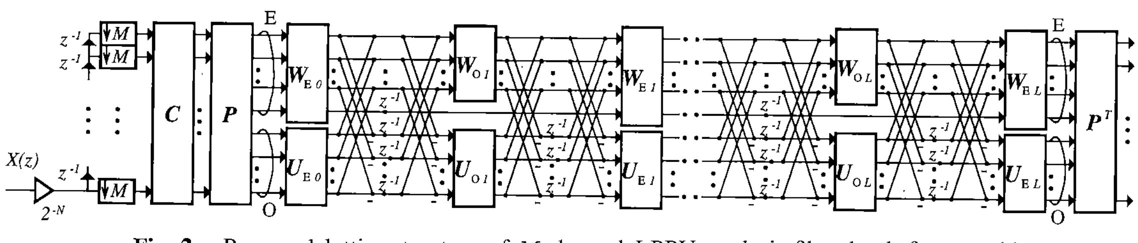


Fig. 2 Proposed lattice structure of M-channel LPPU analysis filter bank for an odd number of channel M.

we can design LPPUFBs for odd M. Since $\boldsymbol{W}_{\mathrm{E}\ell}$ can be characterized by $(M+1)(M-1)/8$ plane rotations, and each of the others can be done by $(M-1)(M-3)/8$ ones [1], a non-linear unconstrained optimization process can be used to design them. Any non-linear optimization, however, has no guarantee to yield the global minimum solution, and the result is sensitive to the starting guess. In this section, let us consider avoiding insignificant local minimum solutions in this approach.

4.1 The Recursive Initialization Approach

The lager overlapping factor is, the more complex the starting guess becomes. One of the feasible approaches to guess the starting point is an evolutionary approach, which starts from lower order problems and uses the results as the starting points for higher order ones. For even number of channels M, such an approach has been developed [8]. It is based on a technique of delay realization with the lattice structure. Similarly, we can provide such an approach for odd M. Note that, however, the lattice structure is slightly different from that of even one. Therefore, some modification is required in terms of the parameter mapping.

In the following, let us show our proposed design procedure with an evolutionary approach and a technique of delay realization for odd M, which will be shown as a lemma. The proposed procedure is as follows, where N is the overlapping factor, that is, the order of the polyphase matrix:

Step 1: Start with proper $\boldsymbol{E}_0(z)$, for example, by putting the M-point type-I DCT (DCT-I) as the matrix $\boldsymbol{C}$ and letting $\boldsymbol{R}_{\mathrm{E}0} = \boldsymbol{I}_M$. Then, set $\ell = 0$ and optimize $\boldsymbol{E}_0(z)$.
Step 2: Initialize $\boldsymbol{E}_{2(\ell+1)}(z)$ by using $\boldsymbol{E}_{2\ell}(z)$ as $\boldsymbol{E}_{2(\ell+1)}(z) = z^{-1}\boldsymbol{E}_{2\ell}(z)$, and increment ℓ as $\ell \leftarrow \ell+1$.
Step 3: Optimize $\boldsymbol{E}_{2\ell}(z)$, and go to Step 2 until the order 2ℓ reaches to N, that is, ℓ reaches to $L = N/2$.

This procedure is applicable to any object function. Furthermore, there is a simple mapping procedure by which the initialization in Step 2 can be achieved in the lattice structure. The procedure is based on the following lemma:

Lemma 1: Let $\boldsymbol{E}_n(z)$ be a matrix of order n provided as in Eq. (3) and $\ell = n/2$. When

$$\boldsymbol{R}_{\mathrm{E}\ell} = \boldsymbol{R}_{\mathrm{O}\ell} = \begin{bmatrix} \boldsymbol{I}_{\frac{M+1}{2}} & \boldsymbol{O} \\ \boldsymbol{O} & -\boldsymbol{I}_{\frac{M-1}{2}} \end{bmatrix}, \tag{11}$$

$\boldsymbol{E}_n(z)$ can be represented as follows:

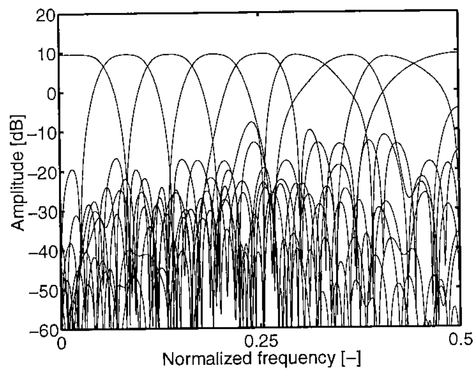
$$\boldsymbol{E}_n(z) = z^{-1}\boldsymbol{E}_{n-2}(z), \tag{12}$$

where $\boldsymbol{E}_{n-2}(z)$ is a polyphase matrix of order $n-2$, which satisfies the LP and PU properties.

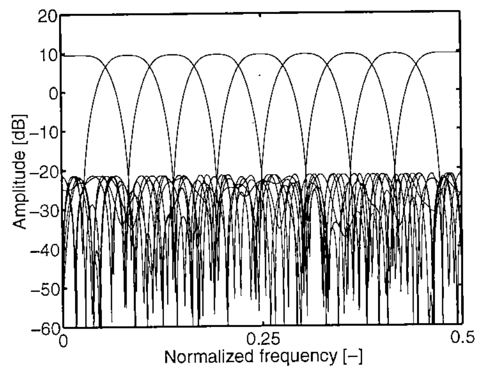
proof: Substituted Eq. (11), $\boldsymbol{R}_{\mathrm{E}\ell}\boldsymbol{Q}_{\mathrm{E}}(z)\boldsymbol{R}_{\mathrm{O}\ell}\boldsymbol{Q}_{\mathrm{O}}(z)$ results in $z^{-1}\boldsymbol{I}_M$. Hence, from Eq. (3), Eq. (12) holds. □

Equation (12) implies that $\boldsymbol{E}_n(z)$ is identical to $\boldsymbol{E}_{n-2}(z)$ except for the delay. Thus, when $\boldsymbol{E}_{n-2}(z)$ has good performance, for example high coding gain and stop-band attenuation, so does $\boldsymbol{E}_n(z)$. From this fact, in order to design $\boldsymbol{E}_n(z)$, well-designed $\boldsymbol{E}_{n-2}(z)$ should be a good candidate for the starting guess, appended the section $\boldsymbol{P}^T\boldsymbol{R}_{\mathrm{E}\ell}\boldsymbol{Q}_{\mathrm{E}}(z)\boldsymbol{R}_{\mathrm{O}\ell}\boldsymbol{Q}_{\mathrm{O}}(z)\boldsymbol{P}$ with the matrices in Eq. (11).

In addition, our proposed procedure at least guarantees that the performance of the resulting system is not worse than that of the lower order system. In this point of view, our proposed structure is preferable since, by simply choosing the matrices $\boldsymbol{\Phi}_S$ and $\boldsymbol{\Phi}_A$ as $(M+1)/2$-point DCT-I and $(M-1)/2$-point type-III DCT (DCT-III), respectively, the matrix $\boldsymbol{C}$ in Eq. (3) can be set as the M-point DCT-I [13], which provides a good starting guess of $\boldsymbol{E}_0(z)$ with $\boldsymbol{R}_{\mathrm{E}0} = \boldsymbol{I}_M$ for most practical object functions. In other words, insignificant local minimum solutions can be avoided.



(a) Filters designed for maximizing coding gain G_{TC} for AR(1) process with $\rho = 0.95$. $G_{\mathrm{TC}} = 9.65$ [dB].



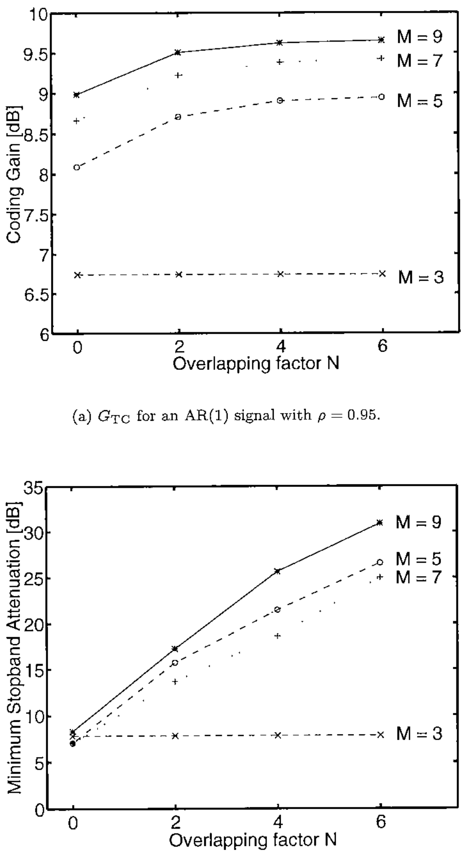
(b) Filters designed for maximizing minimum stop-band attenuation, where each transition-band width is set to $\pi/2M = \pi/18$ [rad]. $A_S = 30.88$ [dB].

Fig. 3 Design examples: amplitude responses of 9 analysis filters, where $M = 9$, $N = 6$ ($L = 3$) and the length of each filter is 63.

4.2 Design Examples

In order to verify the significance of our proposed method, we show some design examples, where $\boldsymbol{C}$ is fixed as the M-point DCT-I matrix. Figure 3 (a) and (b) give the amplitude responses of 9 analysis filters designed for coding gain G_{TC} for AR(1) process with the correlation coefficient $\rho = 0.95$ [14], and those for minimum stop-band attenuation A_{S}, respectively, where $M = 9$, $N = 6 (L = 3)$ and each analysis filter has $M(N+1) = 63$ tap length. For maximizing A_S, transition-band width of each filter is set to $\pi/2M = \pi/18$ [rad]. These examples are obtained by using the routines 'fminu' for (a) and 'minimax' for (b) provided by MATLAB optimization toolbox [15]. The resulting coding gain and minimum stop-band attenuation are $G_{\mathrm{TC}} = 9.65$ [dB] and $A_{\mathrm{S}} = 30.9$ [dB], respectively.

In Fig. 4, the resulting G_{TC} and A_{S} are shown for $M = 3, 5, 7$ and 9. The coding gain G_{TC} is maximized for AR(1) process with $\rho = 0.95$, and the minimum stop-band attenuation A_{S} is maximized with the transition-band-width $\pi/2M$ [rad]. Figure 4 shows that, as the overlapping factor (or the order of polyphase matrix) N increases, both of G_{TC} and A_{S} increase for $M = 5, 7$ and 9. This illustrates that the recursive initialization procedure does not yield worse solution than that of the system which is used as the starting guess. This statement is also true for $M = 3$. However, the performance does not improved even if the order increases. This is because the LPPU condition is crucial for this case.



(a) G_{TC} for an AR(1) signal with $\rho = 0.95$.

(b) A_{S} with transition-band width $\pi/2M$ [rad].

Fig. 4 Resulting coding gain G_{TC} and minimum stop-band attenuation A_{S} versus overlapping factor N.

5. Conclusions

In this letter, a design method of LPPUFBs was proposed for an odd number of channels. In the proposed method, we assumed that an unconstrained non-linear optimization process is applied to a lattice structure which makes the starting guess of design parameters simple. In order to avoid insignificant local minimum solutions, a recursive initialization procedure, which starts from the design for a non-overlapping system and evolutionarily increases the design problem, was proposed. This work can be regarded as a modification of that for even-channel system [8]. By showing some design examples, we verified the significance.

Acknowledgement

This work was presented at EUSIPCO'96 [16] with the support of the telecommunications advancement foundation. We would like to appreciate their support.

References

[1] P.P. Vaidyanathan, "Multirate Systems and Filter Banks," Prentice Hall, Englewood Cliffs, 1993.

[2] M. Vetterli and J. Kovačević, "Wavelets and Subband Coding," Prentice Hall, Englewood Cliffs, 1995.

[3] G. Strang and T.Q. Nguyen, "Wavelets and Filter Banks," Wellesley-Cambridge, Press, 1996.

[4] H.S. Malvar, "Signal Processing with Lapped Transforms," Artech House, 1991.

[5] M. Vetterli and D.L. Gall, "Perfect reconstruction FIR filter banks: Some properties and factorizations," IEEE Trans. Acoust., Speech, & Signal Process., vol.37, pp.1057–1071, July 1989.

[6] A.K. Soman, P.P. Vaidyanathan, and T.Q. Nguyen, "Linear phase paraunitary filter banks: Theory, factorizations and designs," IEEE Trans. Signal Processing, vol.41, pp.3480–3496, Dec. 1993.

[7] R.L. de Queiroz, T.Q. Nguyen, and K.R. Rao, "Generalized linear-phase lapped orthogonal transforms," Proc. IEEE ISCAS, vol.2, pp.277–280, May 1994.

[8] S. Muramatsu and H. Kiya, "A new factorization technique for the generalized linear-phase LOT and its fast implementation," IEICE Trans. Fundamentals, vol.E79-A, pp.1173–1179, Aug. 1996.

[9] T. Nagai, C.W. Kok, M. Ikehara, and T.Q. Nguyen, "Design and lattice structure of FIR paraunitary filter banks with linear phase," IEICE Trans. Fundamentals, vol.E80-

A, pp.712–721, April 1997.
[10] C.W. Kok, T. Nagai, M. Ikehara, and T.Q. Nguyen, "Structures and factorizations of linear phase paraunitary filter banks," Proc. IEEE ISCAS, pp.365–368, 1997.
[11] R.A. Horn and C.R. Johnson, "Matrix Analysis," Cambridge University Press, 1985.
[12] T.D. Tran and T.Q. Nguyen, "On M-channel linear-phase FIR filter banks and application in image compression," IEEE Trans. Signal Processing, vol.45, pp.2175–2187, Sept. 1997.
[13] K.R. Rao and P. Yip, "Discrete Cosine Transform," Academic Press, 1990.
[14] A.K. Soman and P.P. Vaidyanathan, "Coding gain in paraunitary analysis/synthesis systems," IEEE Trans. Signal Processing, vol.41, pp.1924–1835, May 1993.
[15] The Math Works, Inc., "Optimization Toolbox User's Guide," Oct. 1994.
[16] S. Muramatsu and H. Kiya, "A new design method of linear-phase paraunitary filter banks with an odd number of channels," Proc. EUSIPCO'96, vol.1, pp.73–76, Sept. 1996.