LETTER *Special Section on Papers Selected from ITC-CSCC 2003*

# Online SNR and Fading Parameter Estimation for Parallel Combinatorial SS Systems in Nakagami Fading Channels*

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**SUMMARY** This letter discusses the performance of online SNR estimation including fading parameter estimation for parallel combinatorial SS (PC/SS) systems. The PC/SS systems are partial-code-parallel multicode SS systems, which have high-rate data transmission capability. Nakagami-*m* distribution is assumed as fading channel model to cover a wide range of fading conditions. The SNR and fading parameter estimation considered in this letter is based on only a statistical ratio of correlator outputs at the receiver. Numerical results show that SNR estimation performance with fading parameter estimation is close to the one in the case of perfect fading parameter information, if the number of transmitting PN codes is less than a half of assigned PN codes.

***key words:*** *spread spectrum, multicode systems, online SNR estimation, Nakagami fading*

## 1. Introduction

Direct sequence spread spectrum (DS/SS) technique has been successfully implemented in CDMA mobile radio systems [1], wireless LAN [2], and so on. Multicode DS/SS technique is an intuitive solution to speed up the data rate to cover various for multimedia applications. In multicode DS/SS technique, all-code-parallel multicode technique [3] and partial-code-parallel multicode technique referred to parallel combinatorial SS (PC/SS) technique [4] have been investigated.

Forward error correction (FEC) coding is known as effective solution to mitigate the influence of multipath fading in wireless channel. Use of channel state information such as signal-to-noise ratio (SNR) enhances the error correction capability. Online or blind SNR estimation is an attractive topic to avoid insertion of additional training symbols. Summers and Wilson have proposed a simple online SNR estimation scheme and applied it to turbo coded BPSK systems [5]. Ramesh et al. have investigated this technique in Nakagami fading channels [6]. Takizawa et al. applied this online SNR estimation algorithm into PC/SS systems in Nakagami fading channels [7].

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The Nakagami *m*-distribution used as a distribution of fading envelope in [6] and [7] covers the widest range of multipath fading distributions via the parameter *m*. This parameter m shows a ratio of the specular component to the diffused component in Nakagami fading channels. Although this fading parameter is usually unknown at the receiver in practical systems, it is assumed that this information is obtained perfectly at the receiver in [6] and [7]. No attempt to apply both SNR and fading parameter estimation techniques into multicode DS/SS systems has been found so far.

In this letter, performance of online SNR estimation including fading parameter estimation is evaluated in PC/SS systems. The proposed SNR estimator is based on a statistical ratio of the received signals at the PC/SS receiver. Fading parameter estimation method is based on that described by Ali Abdi et al. in [6]. In this letter, both SNR and fading parameter are estimated from a statistical ratio of the correlator outputs at the receiver. The performance of the SNR estimation is evaluated, and compared with the SNR estimation performance with perfect fading parameter information.

The organization of this letter is as follows. In the next section, a PC/SS system model is described. The online SNR and fading parameter estimation for PC/SS systems in Nakagami fading channels is derived in Sect. 3. Performance evaluation and discussion are given in Sect. 4. Finally, we draw some conclusions in Sect. 5.

## 2. System Model

A baseband system model for PC/SS transmitter and receiver is shown in Fig. 1. At the transmitter, a set of *K*-bit binary information data $\mathbf{u}$ is converted into a codeword $\mathbf{x}$ with length *M* given by

$$\mathbf{x} = \{x_1, \cdots, x_j, \cdots, x_M\}, \quad x_j \in \{+1, 0, -1\} \tag{1}$$

The Hamming weight of the codeword $\mathbf{x}$ is $r$. The transmit-

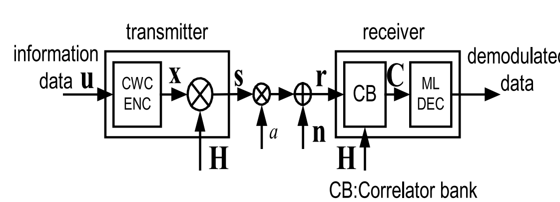


**Fig. 1** PC/SS system model.

ting signal vector **s** is represented as

$$\mathbf{s} = \sqrt{E_c}\mathbf{x} \cdot \mathbf{H} \tag{2}$$

where **H** is an $M \times l$ matrix whose individual rows correspond to the pre-assigned orthogonal PN codes with length $l$. $E_c$ denotes the energy per transmitting PN code. It is related to the energy per information bit $E_b$ by $E_c = E_b \cdot K/r$. The amount of the information data per PN period in PC/SS system is given by

$$K = r + \left\lfloor \log_2 \binom{M}{r} \right\rfloor \text{ [bit/baud]} \tag{3}$$

where $\lfloor p \rfloor$ denotes the maximum integer that is less than $p$. $K$ information bits are divided into $r$ bits and $(K - r)$ bits. The former part corresponds to the polarities at nonzero elements in the codeword. The latter part is converted to the position of $r$ nonzero elements in the codeword.

At the receiver, the received signal vector **r** corrupted by the fading is given by

$$\mathbf{r} = a \cdot \mathbf{s} + \mathbf{n} \tag{4}$$

where **n** is an AWGN sample vector with two-sided power spectral density of $N_0/2$ [W/Hz]. In this letter, fading amplitude $a$ is assumed to be constant during a PN period that is equal to a symbol duration. We also assume that the probability density function (PDF) of $a$ follows Nakagami $m$-distribution given by

$$p(a) = \frac{2m^m a^{2m-1}}{\Gamma(m)} \exp(-ma^2) \tag{5}$$

where $\Gamma(.)$ is the Gamma function. Nakagami parameter $m$ indicates the fraction of specular component in fading channel. The second moment of the fading amplitude $E(a^2)$ is normalized to unity. The fading amplitude $a$ is independent of the noise vector $n$. In this case, the outputs at the bank of $M$ correlators are

$$\mathbf{C} = \mathbf{r} \cdot \mathbf{H}^t / l \tag{6}$$

$$\mathbf{C} = \{C_1, \cdots, C_j, \cdots, C_M\} \tag{7}$$

The PC/SS detector estimates the PN codes corresponding to the correlator outputs that have the first to the $r$th largest magnitude as the transmitted PN codes. A part of information data corresponding to the combination of transmitted PN codes is demodulated based on these transmitted PN code estimates. The other part of information data is demodulated by phase detection of each estimated PN code, which is the same manner as in PSK systems.

## 3. Online SNR and Fading Parameter Estimation Algorithm

Estimation diagram of SNR and Nakagami parameter $m$ is shown in Fig. 2. In PC/SS systems, $r$ PN codes are transmitted per symbol of interest. At the output of the bank of correlators, polarities of individual transmitted PN codes are unknown. If $r < M$, a set of transmitted PN codes usually changes symbol by symbol. A set of correlator outputs in Eq. (7) contains $r$ outputs including signal and noise, $C_s$ and the other $M - r$ outputs including only noise, $C_n$.

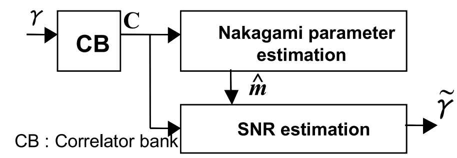


**Fig. 2** Estimation diagram.

$$C_s = \pm a\sqrt{E_c} + n_n \tag{8}$$

$$C_n = n_n \tag{9}$$

where $n_n$ is a Gaussian random variable having zero mean and variance $\sigma^2$.

By using the correlator outputs in Eq. (7), SNR per transmitted PN code given by

$$\gamma = \left(\frac{E_c}{N_0}\right) \cdot E(a^2) = \frac{E_c}{N_0} = \frac{E_c}{2\sigma^2} \tag{10}$$

is estimated. Detail algorithm is described in [5]. The estimated SNR $\tilde{\gamma}$ is given by

$$\tilde{\gamma} = c_4\hat{Z}^4 + c_3\hat{Z}^3 + c_2\hat{Z}^2 + c_1\hat{Z}^1 + c_0 \tag{11}$$

where the coefficients in Eq. (11) will be determined to minimize the square of error. $\hat{Z}$ is given by

$$\hat{Z} = \frac{1}{N_s}\sum_{i=1}^{N_s}\sum_{j=1}^{M} C^2{}_{i,j} \Bigg/ \left[\frac{1}{N_s}\sum_{i=1}^{N_s}\sum_{j=1}^{M} |C_{i,j}|\right]^2 \tag{12}$$

where $C_{i,j}$ is the output of the $j$th correlator at the $i$th symbol. $N_s$ is the number of the symbols per block. If $N_s$ approaches infinity, $Z$ becomes

$$Z = E\left[\sum_{j=1}^{M}(C^2{}_j)\right] \Bigg/ E\left[\sum_{j=1}^{M}(|C_j|)\right]^2 \tag{13}$$

Since $M$ and $r$ are already given, $Z$ is a function of SNR $\gamma$ and Nakagami parameter $m$ [5].

Substituting Eq. (12) to Eq. (11), the SNR estimate $\tilde{\gamma}$ is obtained, if we have perfect knowledge of the Nakagami parameter $m$ [5]. However, in practice, Nakagami parameter $m$ is unknown at the receiver. Moreover, it could change in symbol by symbol in practical wireless channel. In this letter, $m$ is estimated by using a method given in [6]. The estimated Nakagami parameter $\hat{m}$ is given by

$$\hat{m} = \frac{{\mu_2}^2}{\mu_4 - {\mu_2}^2} \tag{14}$$

where $\mu_k$ is the sum of the $k$th moment of correlator outputs that have first to the $r$th largest magnitude, which is given by

$$\mu_k = \frac{1}{N_s}\sum_{j=1}^{N_s}\left\{\frac{1}{r}\sum_{j\in\Omega_i} C_{i,j}\right\} \qquad (15)$$

where $\Omega_i$ is a set of indices that have the first to the $r$th largest magnitude of $C_{i,j}$ at the ith symbol. $N_s$ is the number of samples to compute the moment of received signals, which is set to be the same as $N_s$ in Eq. (12) in this letter.

## 4. Numerical Results and Discussions

The accuracy of the SNR estimates $\tilde{\gamma}$ is evaluated by the expected value $E[\tilde{\gamma}]$. In PC/SS systems, SNR per transmitted PN code depends on $K$ and $r$, if SNR per information bit is equal. Thus, we adopt SNR per information bit as the performance measure for comparison. The estimated SNR per information bit $\tilde{\gamma}_b$ is obtained by simple calculation using the SNR estimate per transmitted PN code, which is given by

$$\tilde{\gamma}_b = \frac{r}{K}\cdot\tilde{\gamma} \qquad (16)$$

In this letter, we set the PN code length $l$ is equal to the number of assigned PN codes $M$.

Figure 3 illustrates average estimated SNR versus the number of transmitted PN codes $r$. $N_s$ is set to 500 symbols. SNR per information bit is set to 10 dB. Each result is determined by 5000 trials. No channel coding is employed. The average estimated SNR in case of having perfect Nakagami parameter information is also plotted as dashed line. The average estimated SNR in both cases look close to real SNR when $r$ is less than four.

The average estimation performance would be influenced by some estimates that have large difference from the real value. Next, the estimation performance is evaluated by the cumulative distribution function (CDF) of the estimation error in SNR. Figure 4 illustrates the CDF of the estimation error in SNR. For reference, the one with perfect Nakagami parameter information is also plotted. Curves showing good performance lies near top and left axes, indicating a high probability of estimating SNR with small errors. Nakagami parameter is set to one and four, respectively. Again, each curve is determined by 5000 trails. We compare the case of $r = 3$ with $r = 8$ because both cases have the same information bit rate. If $r$ is equal to eight, it becomes all-code-parallel multicode systems. In the case of $m = 1$, the CDF curve with Nakagami parameter estimation is very close to the one with perfect Nakagami parameter information when $r$ is equal to three. On contrary, the CDF curve with Nakagami parameter estimation is far from the one with perfect information of Nakagami parameter when $r$ is eight.

Let us look how these CDF curves change in the case of $m = 4$, which is shown in Fig. 4(b). In the case of $r = 3$, the CDF curve with Nakagami parameter estimation is a little bit worse, but still close to the one with perfect Nakagami parameter information. Since large $m$ means having more specular component in fading channel, SNR estimation performance in all-code-parallel case ($r = 8$) becomes better than it is when $m$ is unity, if perfect Nakagami parameter information could be obtained. However, we have to obtain

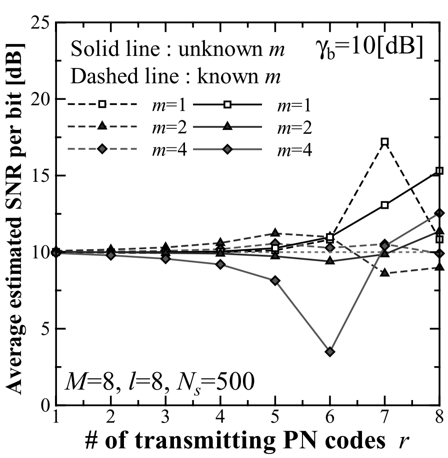


**Fig. 3** SNR estimation performance versus the number of transmitted PN codes $r$.

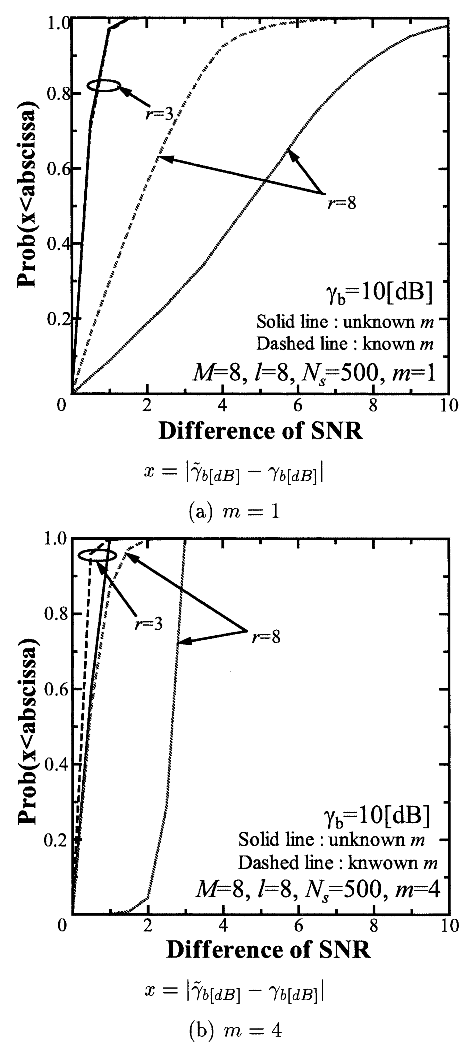


**Fig. 4** Cumulative distribution function (CDF) of SNR estimation error.

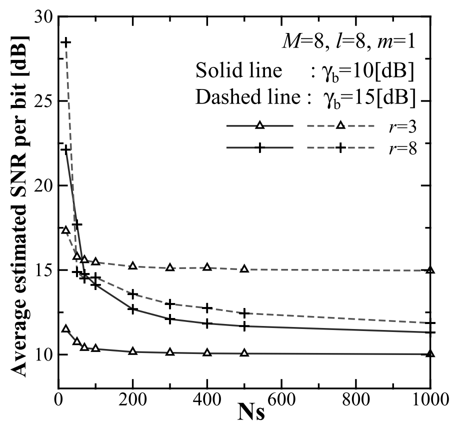


**Fig. 5** Estimated SNR per bit versus the number of symbols observed. Nakagami parameter $m$ is one.

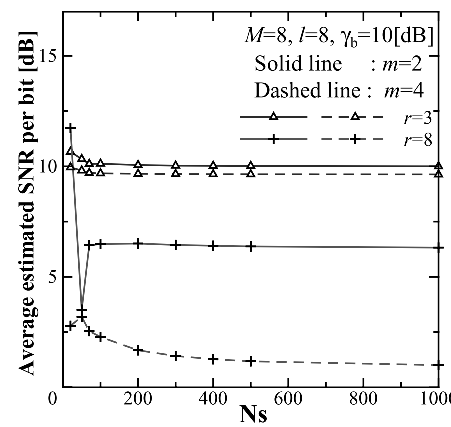


**Fig. 7** Estimated SNR per bit versus the number of symbols observed. Nakagami parameter $m$ is two and four.

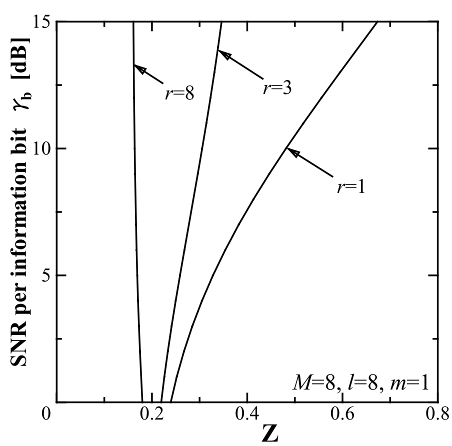


**Fig. 6** SNR per information bit and statistical ratio $Z$ in Eq. (13). $N_s$ goes to infinity.

Nakagami parameter through correlator output. This makes the SNR estimation performance in the case of all code parallel multicode much worse than that in the case of $r = 3$, as shown in Fig. 4(b).

Next, we compare the SNR estimation performance versus the number of symbols observed $N_s$ for different SNR and Nakagami parameter. Figure 5 illustrates the average estimated SNR versus block size $N_s$. In this figure, SNR per bit $\gamma_b$ is set to 10 dB and 15 dB. Nakagami parameter is set to one. In the case of $r = 3$, estimated SNR almost converges at a certain SNR if $N_s$ is more than 200 symbols. In contrast, convergence of the estimated SNR is slower in the case of $r = 8$, which is equivalent to all-code-parallel multicode system. In the case of $\gamma_b = 15$ [dB], the estimated SNR is almost equal to true SNR as well as the case of $\gamma_b = 10$ [dB] when $r$ is three. When $r$ is eight, estimated SNR is much larger than real SNR if $N_s$ is less than 100 symbols. If $N_s$ becomes larger, it converges at a certain SNR as well as it does when $r$ is three. However, the estimated SNR is different from real SNR, especially in the case of $\gamma_b = 15$ [dB]. This estimation error is supposed to be caused by the relation between the statistical ration of the correlator output $Z$ and SNR $\gamma$ shown in Eq. (13). Figure 6 displays SNR per information bit $\gamma_b$ versus the statistical ratio $Z$ according to Eq. (13). In this figure, it implies that $N_s$ goes to infinity. In the case of $r = 8$, the SNR curve versus $Z$ becomes slightly steeper with increasing SNR. It means that SNR estimates will be more sensitive to the fluctuations of $Z$, as SNR increases. This fact will bring more estimation error in higher SNR. In the case of $r = 3$, the SNR curve looks some more gradual than in the case of $r = 8$. Impact of the fluctuations of $Z$ becomes less in this case, even in high SNR.

Figure 7 illustrates the average estimated SNR versus block size when Nakagami parameter $m$ is set to two and four. Real SNR per bit $\gamma_b$ is set to 10 [dB]. If three PN codes have been transmitted, the estimated SNR becomes close to real SNR when $N_s$ exceeds 100 symbols. When $m$ is two, average SNR per bit is almost equal to the real one. When $m$ is four, average estimated SNR becomes a little bit lower than the real SNR. Let us take a look the SNR estimation performance in all-code-parallel multicode system that is the case of $r = 8$. Average estimated SNR also converges at a certain value when $N_s$ becomes more than 200 symbols. However, it is much different from the real SNR. Although those two multicode systems have the same information bit rate, much better SNR estimation performance is obtained by the PC/SS modulation.

## 5. Conclusions

Estimation performance of SNR including Nakagami parameter estimation in PC/SS systems has been investigated. Both SNR and Nakagami parameter are estimated only from output of a bank of correlators. Average of estimated SNR is very close to true SNR when the number of transmitted PN codes $r$ is small comparing to the number of assigned PN codes $M$. It is also demonstrated that the PC/SS system is superior to conventional all-code-parallel multicode DS/SS system on SNR estimation performance, when the both systems have the same information bit rate.

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