LETTER *Special Section on Multi-dimensional Mobile Information Networks*

Evaluation of Selective Rake Receiver in Direct Sequence Ultra Wideband Communications

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SUMMARY Performance of selective Rake (SRake) receiver is evaluated for direct sequence ultra wideband (DS-UWB) communications considering an independent Rayleigh channel having exponentially decaying power delay profile (PDP). BEP performances are shown. The results obtained are compared with similar results in a channel having flat PDP. Assumption of a flat PDP is found to predict the optimum spreading bandwidth to be lower and sub-optimum operating performance beyond optimum spreading bandwidth to be severely worse than that is achievable in a channel having exponentially decaying PDP by employing an SRake receiver having fixed number of combined paths. Optimum spreading bandwidth for SRake in a channel having exponentially decaying PDP is shown to be much larger than the one in a channel having flat PDP; that is specifically a good-news for UWB communications. Effects of partial band interference are also investigated. Interference is found to be less effective in exponentially decaying PDP.

key words: *ultra wideband, selective Rake receiver, spreading bandwidth, power delay profile, partial band interference*

1. Introduction

Recently considerable interests have grown on ultra wideband (UWB) communication systems [1]. One of the most promising features of UWB communications is the improved multipath resolvability [2]. Rake types of receivers are usually employed for communications over multipath channels [3]. If we consider an all Rake (ARake) receiver that combines all resolved multipaths by maximal ratio combining (MRC) [3], [4], the bit error probability (BEP) performance for any particular signal-to-noise ratio (SNR) will be increasingly better with increased spreading bandwidth. But as a concern of practical considerations, implementation of ARake receiver may become impractical because of the increased complexity, as the system bandwidth gets large [2]. As a result, a performance vs. complexity trade off may be often needed [2]. If we consider ARake receiver to be in the one extreme with its optimum performance and highest complexity, selection combining (SC) that combines only one path having the highest SNR can be considered to be in the other extreme because of it's least complexity and considerable performance degradation as compared with ARake [4]. A selective Rake (SRake) receiver is one that combines L_c number of best paths out of L total paths ($L_c < L$) by MRC [5], [6]. An SRake receiver, a compromise between ARake and SC, can be a good candidate for application in UWB communications.

Previous works have discussed the performance of SRake receivers in direct sequence spread spectrum (DS-SS) communications assuming flat power delay profile (PDP) [5]–[7]. In this letter, we evaluate the performance of SRake receivers in direct sequence ultra wideband (DS-UWB) communications considering exponentially decaying PDP and compare the results obtained with those in a flat PDP. We also present how different PDP's affect the optimum spreading bandwidth for as SRake receiver having fixed number of combined paths. Another contribution of this letter is we consider the effects of partial band interference. We develop the expressions for average SNR for each path at non-interfering and interfering conditions. Then we use these expressions in symbol error probability (SEP) expression for SRake receiver. We also investigate the effects of interference having different bandwidths and placed at different locations of the communication band.

The organization of the letter is as follows. In the next section we present system model. In Sect. 3 we present the expression of SEP for SRake applicable for both flat and exponentially decaying PDP. In Sect. 4 we consider effects of interference. We present numerical examples in Sect. 5 and finally we conclude in Sect. 6.

2. System Model

2.1 Signals and Channel Model

We represent the equivalent low-pass (ELP) transmitted signal by $s(t)$ and ELP received signal by $y(t)$. So we can write

$$y(t) = \int_{-\infty}^{+\infty} h(t,\tau)s(t-\tau)d\tau \tag{1}$$

where $h(t,\tau)$ is the ELP impulse response of the channel with t and τ denoting the time and delay variables. A block diagram of the system using ELP representation of the signals is shown in Fig. 1. The ELP received signal can be modeled as,

$$r(t) = y(t) + n_w(t) + n_J(t) \tag{2}$$

where $n_w(t)$ represents white Gaussian noise and $n_J(t)$ represents interference. The white Gaussian noise has a one-sided power spectral density (psd) of N_o and the interference psd depends on the average power applied J and the

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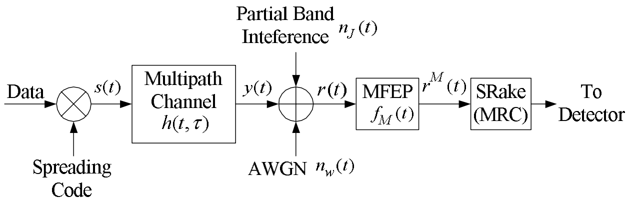

Fig. 1 Block diagram of the system using equivalent low pass (ELP) representation of signals.

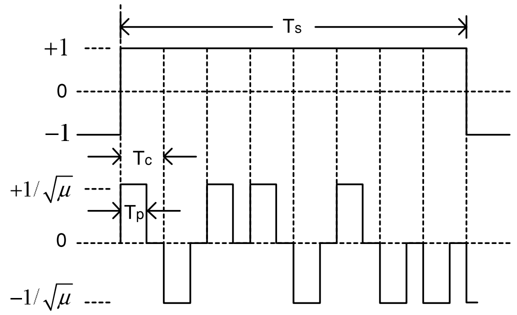

Fig. 2 ELP representation of DS-UWB signal having chip duty-factor $\mu = T_p/T_c$.

bandwidth covered W_J by the interference.

2.2 Receiver Front-End

We represent the Rake receiver by its equivalent matched filter version representation [6] and employ a matched front-end processor (MFEP) followed by a tapped delay line and a maximal ratio combiner (MRC) [5], [6]. Resolution of resolvability of multipath fading achieved by MFEP is equal to communication pulse-width T_p that is approximately equal to the inverse of the spreading bandwidth B. We consider a MFEP with ELP impulse response

$$f_M(t) = \begin{cases} s^*(T_s - t), & 0 < t < T_s \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where T_s represents symbol period and * is for complex conjugate. So, the output of the MFEP can be written as

$$r^M(t) = \int_0^t r(t-\zeta) f_M(\zeta) d\zeta \tag{4}$$

2.3 The DS-UWB System

A conventional DS-SS communication system uses chip duty-factor of 1. For certain data rate, spreading is provided in the system by increasing the chip rate. Unlike DS-SS, we consider a DS-UWB system where spreading is provided by decreasing chip duty-factor starting from a value of 1 keeping data rate and chip rate constant (Fig. 2). As a result, in general, the DS-UWB system transmits a pulse of duration T_p per chip of duration T_c where $T_p \leq T_c$. We define chip duty-factor, $\mu = T_p/T_c (0 < \mu \leq 1)$. However, we assume to increase the peak power inversely proportional to μ to keep the average power constant. Processing gain of the proposed DS-UWB system can be given by

$$PG_{DS-UWB} = \frac{T_s}{T_p} = \frac{T_s}{T_c}\frac{T_c}{T_p} = \frac{PN\ code\ \text{length}}{\mu} \tag{5}$$

where T_s is symbol period and PN means pseudorandom. Here we note that the spreading bandwidth of the DS-UWB system is $B \approx 1/T_p = 1/(\mu T_c)$. We consider a channel with constant maximum delay of T_d with respect to the first arriving path. At any spreading bandwidth B, the total number of resolvable paths is given by $L = T_d/T_p$. So, we can express spreading bandwidth as $B = L/T_d$, from which we see, for a constant T_d, total number of resolvable paths L is proportional to spreading bandwidth B. The chip duty-factor $\mu = T_p/T_c = 1/(BT_c) = T_d/(LT_c)$. With constant data rate and chip rate, both B and L are inversely proportional to μ. Figure 2 shows an example PN sequence for the DS-UWB system and Fig. 3 shows the relationship among chip duty-factor, PN code length and processing gain based on Fig. 2.

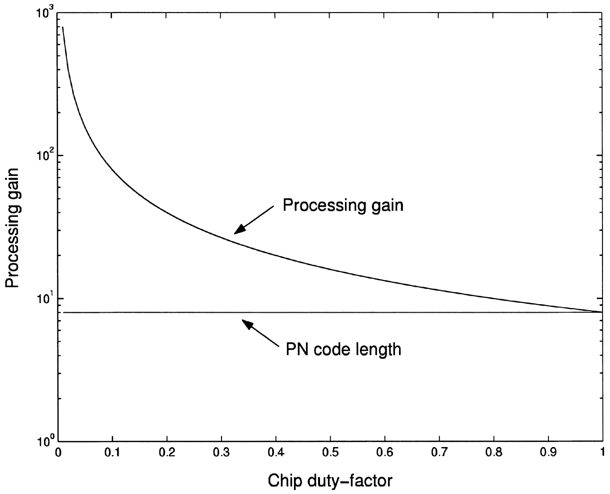

Fig. 3 Relationship among chip duty-factor, *PN* code length and processing gain in Fig. 2.

3. Performance in AWGN

3.1 Flat and Exponentially Decaying PDP

The ELP received signal in (4) for AWGN and single user can be written as

$$r^M(t) = \sum_{l=1}^{L} \alpha_l \exp^{j\theta_l} s_l(t) + n_w^M(t) \tag{6}$$

where α_l and θ_l are the amplitude and phase of the l-th path respectively, $s_l(t)$ is the signal component received in the l-th path and $n_w^M(t)$ is the AWGN component. Here α_l is considered to be Rayleigh distributed. Considering a receiver having knowledge of the phase and amplitude of each path and considering a flat PDP, the average received SNR for each path under unit energy constraint can be given (at the end of MFEP) by

$$\gamma_{o,l} = \beta\Gamma\mathbf{E}\{\alpha_l^2\} = \beta\frac{\Gamma}{L},\ l = 1, 2, 3, \ldots, L \tag{7}$$

where Γ is SNR (E_s/N_o), E_s being the symbol energy, $\mathbf{E}\{.\}$ represents mean value and $\mathbf{E}\{\alpha_l^2\} = 1/L$. We assume the multipath amplitudes of each symbol to be independent and identically distributed (i.i.d). However, actually, adjacent multipath amplitudes of the same symbol will be weakly correlated that we neglect here [6]. In (7) β is a parameter that depends on the shape of the pulses used. Values of β were reported to be 0.6666, 0.5866 and 0.4811 for rectangular, half-sine and raised cosine pulses respectively [6].

In actual case, especially for UWB, the PDP of a real channel will not be flat. This motivates us to incorporate an exponentially decaying PDP. The multipath PDP can be considered to depend only on specific channel parameters [3]. Motivated by this, we consider a channel having PDP independent of bandwidth. In a channel having exponentially decaying PDP, we model the average SNR of each path under unit energy constraint to be [9]

$$\gamma_{o,l} = \beta\Gamma\left[\frac{1 - \exp(-C/L)}{1 - \exp(-C)}\right] e^{-C\left(\frac{l-1}{L}\right)}, \qquad C > 0,\ l = 1, 2, 3, \ldots, L \tag{8}$$

where C is a positive quantity. Reasonable values of C can be, for example, 10 [9]. Here note that for different values of C, the total energy below the PDP given by (8) will be same for certain value of L which is also equal to the energy below the flat PDP given by (7) for the same L.

3.2 Symbol Error Probability (SEP)

The SEP for SRake was previously derived in [6, Eq. (33)] for a DS-SS system considering a channel having flat PDP. M-ary phase shift keying (MPSK) was considered. The results of [6] are also applicable to our DS-UWB system if flat PDP is considered. However, to apply similar results for exponentially decaying PDP, we need to extend the results of [6] to a general form that will be applicable for various types of PDPs. Using the average SNR for each path shown in the previous sub-section and following the procedures of [6], the SEP of SRake receiver in a channel having arbitrary PDP, can be shown to be

$$SEP_{SRake} = \frac{1}{\pi}\int_0^{\Theta}\prod_{l=1}^{L_c}\left[\frac{\sin^2\theta}{\delta_{MPSK}\gamma_{o,l} + \sin^2\theta}\right] \times \prod_{l=L_c+1}^{L}\left[\frac{\sin^2\theta}{\delta_{MPSK}\gamma_{o,l}\dfrac{L_c}{l} + \sin^2\theta}\right] d\theta \tag{9}$$

where L_c represents the number of combined paths having best SNRs out of L total number of resolvable multipaths, $\delta_{MPSK} = \sin^2(\pi/M)$ and $\Theta = \pi(M-1)/M$. For BPSK they become $\delta_{MPSK} = 1$ and $\Theta = \pi/2$. For an exponentially decaying PDP, we consider $\gamma_{o,l}, l = 1, 2, 3, \ldots, L$ to be the average SNR for each path arranged in descending order.

4. Performance in Interference

We consider a partial band interference with bandwidth W_J and average power J [11, Chapter 6]. We denote the communication system bandwidth by $W(= 2/T_p)$. The fraction of the communication bandwidth that is interfered is denoted by $\rho = W_J/W$. Interference one-sided psd over the interfered bandwidth is given by $N_J' = J/W_J$ and over the full communication system bandwidth is given by $N_J = J/W$ (Fig. 4). It should be noted here that we model the partial band interference as Gaussian noise [11].

The interference one-sided psd at the end of despreader output at receiver intermediate frequency f_{IF} can be given by [11]

$$S(f_{IF}) = \frac{2N_J'}{W}\int_{f_J - W_J/2}^{f_J + W_J/2} \mathrm{sinc}^2\{(f_o - \lambda) \times 2/W\} d\lambda \tag{10}$$

where f_o is communication system center frequency and f_J is interference center frequency. Now (10) can be rewritten as

$$S(f_{IF}) = \frac{N_J T_d}{\rho L}\int_{f_J - \rho L/T_d}^{f_J + \rho L/T_d} \mathrm{sinc}^2\{(f_o - \lambda) \times T_d/L\} d\lambda \tag{11}$$

With this partial band interference in action, Γ changes to

$$\Gamma_J = \frac{E_s}{N_o + S(f_{IF})} = \frac{1}{\left(\dfrac{1}{E_s/N_o} + \dfrac{\dfrac{T_d}{\rho L}\int_{f_J - \rho L/T_d}^{f_J + \rho L/T_d} \mathrm{sinc}^2\{(f_o - \lambda) T_d/L\} d\lambda}{(P/J)(W/R_s)}\right)} \tag{12}$$

As a special case, if narrow band interference is in action ($\rho \ll 1$) that equals its center frequency with that of the communication system, (12) can be simplified. Considering the communication system having an average signal power of P and symbol rate of R_s ($E_s = P/R_s$), (12) can be rewritten as [11]

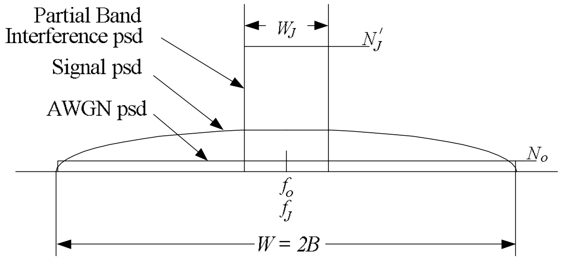

Fig. 4 Power spectral density (psd) of signal, additive white Gaussian noise (AWGN) and partial band interference.

$$\Gamma_J = \frac{1}{\left(\dfrac{1}{E_s/N_o} + \dfrac{2}{(P/J)(W/R_s)}\right)} = \frac{1}{\left(\dfrac{1}{E_s/N_o} + \dfrac{1}{(P/J)(LT_s/T_d)}\right)} \tag{13}$$

where T_s is symbol period and L is total number of resolved multipaths. To evaluate the performance of SRake in interference, Γ in (7) and (8) can be replaced by Γ_J from (12) or (13) and finally (9) can be used.

5. Numerical Examples

In this section, the results presented in the previous sections are illustrated by specific examples. We consider a DS-UWB communication system with sufficiently long *PN* code length. We keep both T_d and T_c constant. We assume our system to be free from inter chip interference (ICI). To increase the spreading bandwidth (and hence L) more, we decrease the duty factor μ gradually starting from 1. We can see, $L = T_d/(\mu T_c)$. BPSK modulation and rectangular pulse shape have been used. Because of using BPSK modulation, bit and symbol will express same meaning.

We plot the BEP performance of SRake ($L_c = 4$) in channels having flat PDP and exponentially decaying PDP with $C = 10$ in Fig. 5. From Fig. 5 it is seen, until some value of L, SRake performance in flat PDP is better than that in exponentially decaying PDP. However, if L is increased further, SRake performance becomes better in exponentially decaying PDP. In Fig. 6 we present the optimum spreading bandwidth for SRake ($L_c = 4$) that provides minimum achievable BEP for channels having flat PDP and exponentially decaying PDPs with $C = 1, 2, 3, \ldots, 10$. The points to note are the increase of optimum spreading bandwidth and the decrease in the slope of the plots after each one has reached its minima with increase in C. The plot for flat PDP tends to suggest that in UWB, we will need to work with a sub-optimum BEP. But in actual case, because the PDP of real channel will be decaying with time, we may be able to work optimally still using a much larger bandwidth. Another important implication of the results shown in Fig. 6 is, the larger the value of C, the more the BEP performance degradation due to overspreading is alleviated. So in our proposed DS-UWB system with exponentially decaying PDP with $C = 10$, overspreading costs much less BEP degradation as compared to flat PDP.

If no interference is present, the optimum spreading for flat PDP is $L^* = 17$ and for exponentially decaying PDP with $C = 10$, is $L^* = 90$ (Fig. 6). In an interfering case, as shown in Fig. 7, an optimally designed SRake ($L_c = 4$) is found to operate sub-optimally in the presence of interference, because the optimum spreading bandwidth moves away with increase in the interfering power. However, in exponentially decaying PDP, which should be the actual case of DS-UWB, especially at short distance, interference is found to be less effective. Figures 7 and 8 have been plotted using values T_d = 200 ns and R_b = 100 kbps.

In presence of interference, firstly, we consider the effects of interference employing $f_J = f_o$. If the interference is a narrowband (NB) one (*i.e.* $\rho \ll 1$), increase in J/P means an increase in noise psd N'_J. Here we consider P a constant. Partial band interference on the other hand may have different bandwidths. We consider such a case with constant psd N'_J. As a result, in this case J is proportional to ρ. However, in actual case of UWB communications, interference may not be placed at the center frequency. This motivates us to investigate, secondly, what happens if f_J is not equal to f_o. In this case we consider NB interference only. Results for $f_o - f_J = 0$, $W/6$ and $W/3$ are shown in Fig. 8 for flat and exponentially decaying ($C = 10$) PDPs at respective optimum

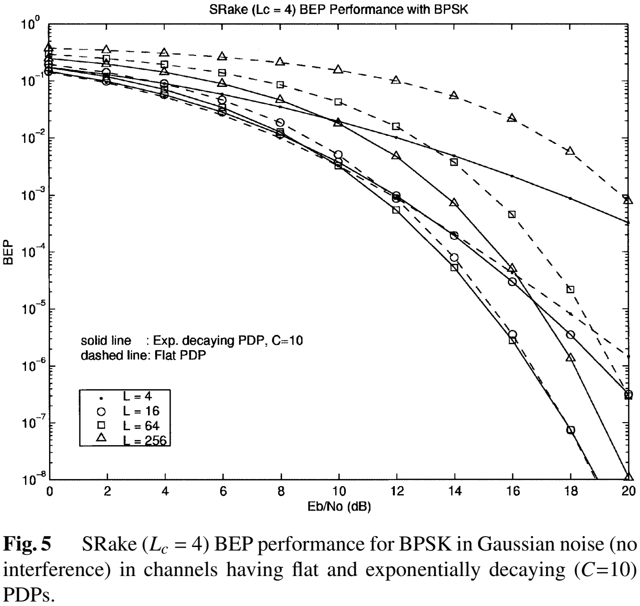

Fig. 5 SRake ($L_c = 4$) BEP performance for BPSK in Gaussian noise (no interference) in channels having flat and exponentially decaying (C=10) PDPs.

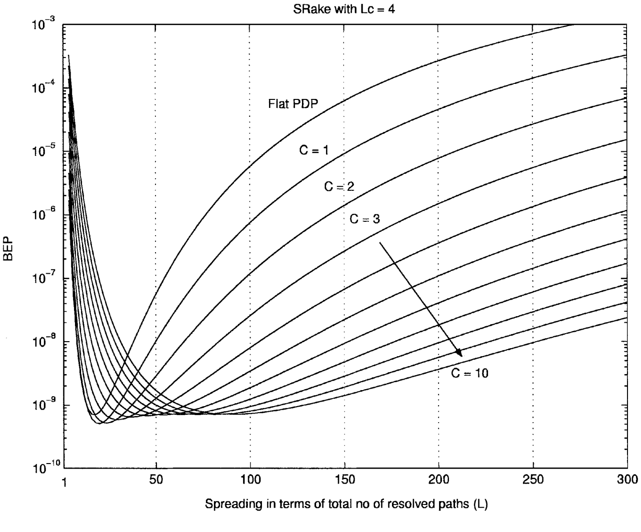

Fig. 6 SRake ($L_c = 4$) BEP performance for BPSK in Gaussian noise (no interference) in channels having flat PDP and exponentially decaying PDPs with $C = 1, 2, 3, \ldots, 10$. E_b/N_o is kept fixed at 20 dB.

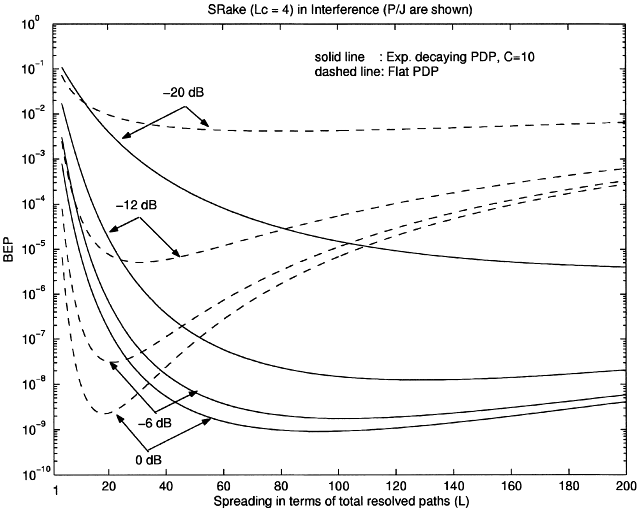

Fig. 7 SRake (L_c = 4) BEP performance with BPSK for interference ($\rho \ll 1, f_o = f_J$) in channels having flat and exponentially decaying (C = 10) PDPs. Curves are shown for P/J = 0 dB, −6 dB, −12 dB and −20 dB in successively higher positions. E_b/N_o is kept fixed at 20 dB.

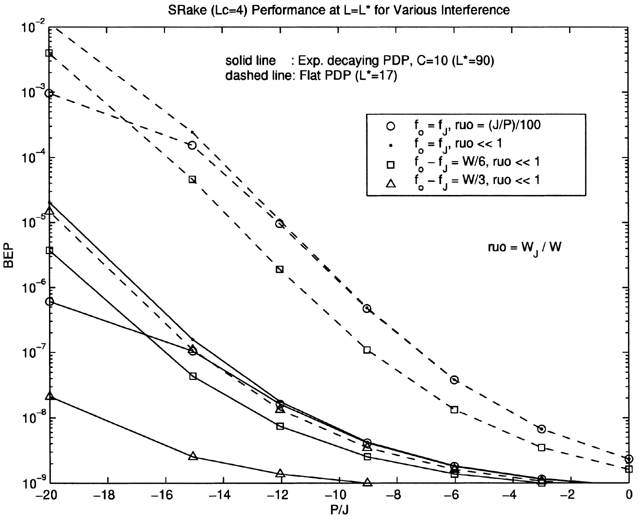

Fig. 8 SRake (L_c = 4) BEP vs P/J variations for various interferences in flat and exponentially decaying (C = 10) PDPs at optimum spreading bandwidths of the respective systems. E_b/N_o is kept fixed at 20 dB, BPSK.

spreading bandwidths of the systems. NB interference ($\rho \ll 1$) placed at the center frequency is found to be the most effective in both types of PDPs. However, interference is seen to be less effective in exponentially decaying PDP.

6. Conclusions

Performance of SRake receiver has been evaluated for DS-UWB system in non-interfering and interfering environments considering a channel having exponentially decaying PDP. The results obtained have been compared with similar results in a channel having flat PDP. DS-UWB system that usually uses huge spreading bandwidth and employs SRake receiver, has been shown to perform better in exponentially decaying PDP than in flat PDP in both non-interfering and interfering environments.

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