LETTER Special Section on Wide Band Systems

# Error Performance of DS-CDMA over Multipath Channel Using Selective Rake Receiver

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**SUMMARY** Error performance of DS-CDMA is discussed over independent Rayleigh faded multipath channel employing selective Rake (SRake) receiver. Simple-to-evaluate and accurate error probabilities are given following Holtzman's simplified improved Gaussian approximation (SIGA). Comparing with SIGA, the validity of standard Gaussian approximation (SGA) is then verified. It is shown that SGA is accurate for SRake until some number of combined paths beyond which it becomes optimistic. It is also shown that as compared to single user performance, the SRake performance is relatively less degraded by multiple access interference (MAI) while the number of combined paths is small.

***key words:*** *DS-CDMA, multipath channel, selective Rake receiver, standard and improved Gaussian approximations*

## 1. Introduction

The Error analysis of DS-CDMA has always been a topic of great interest [1]–[8]. Two classic papers on this topic can be found in [1], [2], which present the error analysis of DS-CDMA in a noiseless channel considering no fading and no multipath. In such a situation, [1] showed that the standard Gaussian approximation (SGA) of multiple access interference (MAI), though simple, is optimistic for small to medium number of users. To obtain accurate results, [1] proposed a better method known as the improved Gaussian approximation (IGA). However, the computational complexity of the IGA is quite intensive that increases with the number of total users. Later, Holtzman in [2] presented a simple and accurate method for error analysis known as simplified improved Gaussian approximation (SIGA).

From practical considerations, the error performances of DS-CDMA over multipath channels are of prime importance. However, the error analysis of DS-CDMA over multipath channels has always been challenging [3]–[8]. Hence, the works considering fading multipath channel have mainly used the SGA only [3]–[5], though, the justification of using the SGA for multipath channel and diversity combining has not been investigated much. Also, the use of the simple SIGA method for multipath channel hasn't received much attention either.

Rake type receivers are usually used for communications over multipaths. A selective Rake (SRake) is one that combines some of the best paths out of total resolved paths [9]. Though there have been many previous papers discussing the performance of DS-CDMA over frequency selective fading channels (see [3]–[7] and the references therein), there has been no previous work considering SRake receiver. Additionally, though the performance of SRake receiver has been presented in [9] in a single user environment, its performance in a multiuser environment hasn't been investigated yet.

In this letter, we present a simple and accurate method of error analysis for DS-CDMA over independent Rayleigh faded multipath channel considering SRake receiver, based on Holtzman's SIGA. We consider two types of channels, one with flat power delay profile (PDP) and the other with exponentially decaying PDP. Though the SIGA method of [2] was basically for additive white Gaussian noise (AWGN) channel, here we extend it for multipath. We also extend the results of [9], which discuss the performance of SRake in a single user case, for a multiple access environment. The combination of the two provides us a way for a very simple and accurate method of error analysis for DS-CDMA over multipath channels employing SRake receiver. We also present results based on SGA and investigate the validity of SGA as compared to the more accurate SIGA. We investigate the impact of the PDP of the channel and the number of selective combined paths of SRake on multiple access performance of the system and on the validity of SGA. Simulation results are also presented to support the theory.

## 2. System Model

### 2.1 Signals Model

The asynchronous binary DS-CDMA system under consideration has $K$ simultaneous users. Assuming perfect power control, the transmitted signal of the $k$-th user can be given by [3],

$$s_k(t-\tau_k) = \sqrt{2P}a_k(t-\tau_k)b_k(t-\tau_k)\cos(\omega_c t+\phi_k) \quad (1)$$

where $P$ is the received signal power, $b_k(t)$ is the data sequence, $a_k(t)$ is the random spreading chip sequences and $\omega_c$ is the carrier frequency. Assume $k \in [1, 2, \ldots, K]$. $\tau_k$ and $\phi_k$ are random delay and phase respectively relative to the desired signal of 1st user ($\tau_1 = 0, \phi_1 = 0$). $\tau_k$ and $\phi_k$

Manuscript received February 27, 2005.
Manuscript revised June 1, 2005.
Final manuscript received July 13, 2005.

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DOI: 10.1093/ietfec/e88–a.11.3118

are independent and uniformly distributed over $[0, T_b]$ and $[0, 2\pi]$ respectively, where $T_b$ is bit duration. $b_k(t)$ is a random process having outcomes uniform on $\{+1, -1\}$. $a_k(t)$ over each bit duration is defined as

$$a_k(t) = \sum_{i=0}^{N-1} a_{k,i} p(t - iT_c) \tag{2}$$

where $p(t)$ is chip waveform, $T_c$ is chip duration and assuming a rectangular pulse shape $p(t) = 1, 0 \le t < T_c$; $p(t) = 0$, otherwise. The energy of $p(t)$ is normalized to $T_c$. $a_{k,i}$ is a random process uniform on $\{+1, -1\}$. Additionally, $a_{k,i}$ is periodic with period $N = T_b/T_c$ and $a_k(t)$ and $b_k(t)$ are independent. The processing gain ($PG$) of the system is $N$.

The random sequence of the $k$-th user is considered to have a time asynchronism of $\tau_k$ where $\tau_k = \Delta_k + \gamma_k T_c$. Here, $\gamma_k$ is an integer within 0 to $N-1$ and $\Delta_k$ is uniformly distributed over $[0, T_c]$.

### 2.2 Channel Model

We consider an independent Rayleigh faded multipath channel. The channel is characterized by a tapped delay line model with length $L$ and delays $(l-1)T_c$, $l = 1, 2, \ldots, L$ [3]. The maximum delay spread of the channel is $T_d = LT_c$. The received signal at the end of the channel is given by

$$\begin{aligned} y(t) &= \sum_{k=1}^{K} h_k(t) * s_k(t) + n(t) \\ &= \sum_{k=1}^{K} \sum_{l=1}^{L} \alpha_{k,l} e^{j\theta_{k,l}(t)} s_{k,l}(t) + n(t) \end{aligned} \tag{3}$$

where $h_k(t)$ is the impulse response of the channel, $\alpha_{k,l}(t)$, $\theta_{k,l}(t)$ are the time varying fading amplitude and phase respectively ($j = \sqrt{-1}$) and $s_{k,l}(t)$ is the signal component of the $l$-th path of the $k$-th user. Additionally, $n(t)$ is the AWGN component with a two-sided variance of $\sigma^2_{AWGN} = N_o/2$.

*Power Delay Profile(PDP):* A flat PDP considering equal average energy over all multipath components is common in literature. A flat PDP can be a good assumption for relatively low rate and long distance (i.e. as cellular) communications. However, presently very high rate short distance (i.e. mainly indoor) communications are evolving. Considering an exponentially decaying PDP is necessary for this type of channels [10]. So, we consider both flat and decaying PDPs. Equal energies are considered below the both types of the PDPs to facilitate a fair comparison.

We assume that all the users experience similar type of fading in average and so we drop the subscript $k$ in $E[\alpha^2_{k,l}]$ [5], where $E[.]$ represents the mean value. The mean signal to interference and noise ratio (SINR) per path can be given by

$$\gamma_l^{(\mu)} = \Gamma^{(\mu)} E[\alpha_l^2], \qquad l = 1, 2, \ldots, L \tag{4}$$

where $\Gamma^{(\mu)}$ is the average total SINR per bit and

$$E[\alpha_l^2] = \begin{cases} \frac{1}{L}, \\ \qquad \text{for flat PDP} \\ \left[\frac{1-\exp(-C/L)}{1-\exp(-C)}\right] e^{-C\left(\frac{l-1}{L}\right)}, \\ \qquad \text{for decaying PDP, } C > 0 \end{cases} \tag{5}$$
$$l = 1, 2, \ldots, L$$

The average total SINR per bit $\Gamma^{(\mu)}$ is given by

$$\Gamma^{(\mu)} = \frac{E_b}{2\left(\sigma^2_{AWGN} + E[\sigma^2_{MAI}] + E[\sigma^2_S]\right)} \tag{6}$$

where $E_b(= PT_b)$ is the energy per bit and $E[\sigma^2_{MAI}]$, $E[\sigma^2_S]$ are the means of the variances of multiple access interference (MAI) and self-interference (due to inter chip interference (ICI) and inter symbol interference (ISI)) respectively. Here note that the superscript $\mu$ in (4) and (6) means that the respective parameters are evaluated by using the mean value of MAI variance, $\mu = E[\sigma^2_{MAI}]$.

### 2.3 Receiver Model

We consider correlation type Rake receiver [9]. The receiver is assumed to be able to synchronize with the first path and is assumed to know the amplitude and phase of each path. In other words, a maximal ratio combining (MRC) is considered. The Rake receiver places its correlator at delays $(l-1)T_c$, $l = 1, 2, \ldots, L$. The selective Rake (SRake) receiver combines $L_c$ best paths out of total resolved paths $L$ ($L_c \le L$) [9].

## 3. Performance Analysis

Though we will employ SRake receiver with $L_c \le L$, we need to model the variances of self-interference and MAI over all resolved paths $L$ to obtain the average total SINR per bit as given in (6). Later we need to obtain the average SINR per path according to (4) to use in the error probability equation [9], [10].

### 3.1 Self-Interference Modeling

The variance of the self-interference for an arbitrary resolved path $i$, can be given by

$$\begin{aligned} \left(\sigma_S^i\right)^2 &= \frac{E_b}{N} \sum_{l=1, l\neq i}^{L} E[\alpha_l^2] \cos^2(\theta_{1,i} - \theta_{1,l}) \\ &\approx \frac{E_b(1 - E[\alpha_i^2])}{2N} \end{aligned} \tag{7}$$

where $\theta_{1,i} - \theta_{1,l}$ is the phase difference between the $l$-th and $i$-th path of the desired user $k = 1$. Here, note that the mean value $E[\cos^2(\theta_{1,i} - \theta_{1,l})] = 1/2$ can be used without degrading the performance [3].

Averaging over all resolved paths, $i = 1, 2, \ldots, L$, we get for both types of PDPs

$$E\left[\sigma_S^2\right] \approx \frac{1}{2N}\left(1 - \frac{1}{L}\right) E_b$$

$$= \frac{(L-1)E_b}{2NL} = \frac{T_d(L-1)E_b}{2T_b L^2} \tag{8}$$

Note that (8) shows the mean value of self-interference under SGA. The SGA of self-interference is considered to be reasonably accurate [5].

### 3.2 Multiple Access Interference (MAI) Modeling

Because we consider $k = 1$ to be the desired user, the variance of the MAI can be given by [1]

$$\sigma^2_{MAI} = \sum_{k=2}^{K} Z_k \tag{9}$$

where

$$Z_k = \frac{E_b}{N^2} \sum_{l=1}^{L} \alpha_l^2 [2(x_{k,l}^2 - x_{k,l})(2B+1) + N] \times \cos^2 \beta_{k,l} \tag{10}$$

where $x_{k,l} = \Delta_k/T_c$ is a random variable uniformly distributed over [0, 1], $B$ is a quantity that represents the number of adjacent chips having opposite polarity per bit [1] and $\beta_{k,l} = \phi_k + \theta_{k,l} - \theta_{1,l}$. Note that $E[B] = (N-1)/2$ and $E[B^2] = N(N-1)/4$ [1], [2]. Also note that $E[x_{k,l}] = 1/2$, $E[x^2_{k,l}] = 1/3$, $E[\cos^2 \beta_{k,l}] = 1/2$ and $E[\cos^4 \beta_{k,l}] = 3/8$. Considering the MAI from different users to be independent, the mean of $\sigma^2_{MAI}$ under SGA can be given by

$$\mu = E[\sigma^2_{MAI}] = \frac{E_b(K-1)}{3N} \sum_{l=1}^{L} E[\alpha_l^2] = \frac{E_b T_d (K-1)}{3T_b L} \sum_{l=1}^{L} E[\alpha_l^2] \tag{11}$$

However, it is well known in literature that the SGA of MAI is not accurate for AWGN channel [1]. A more accurate method known as SIGA [2], [11] has previously been used in AWGN. Here we apply the SIGA in multipath. The SIGA requires the variance in addition to the mean of $\sigma^2_{MAI}$ to be known. The variance under the assumption that the fadings over different paths are independent can be given by

$$\upsilon = \mathrm{var}[\sigma^2_{MAI}] = (K-1)\left[E[Z_k^2] - E[Z_k]^2\right] \approx \frac{23E_b^2(K-1)}{360N^2}\left(\sum_{l=1}^{L} E[\alpha_l^2]\right)^2 \tag{12}$$

### 3.3 Bit Error Probabilities

The bit error probability (BEP) for SRake under SGA can be given by [9]

$$P(\mu) = \frac{1}{\pi}\int_0^{\pi/2} \prod_{l=1}^{L_c}\left[\frac{\sin^2\theta}{\gamma_l^{(\mu)} + \sin^2\theta}\right] \times \prod_{l=L_c+1}^{L}\left[\frac{\sin^2\theta}{\gamma_l^{(\mu)}\frac{L_c}{l} + \sin^2\theta}\right] d\theta \tag{13}$$

The BEP under SIGA can be given by [2]

$$P^{SIGA} = \frac{2}{3}P(\mu) + \frac{1}{6}P(\mu + \sqrt{3\upsilon}) + \frac{1}{6}P(\mu - \sqrt{3\upsilon}) \tag{14}$$

where $P(\mu \pm \sqrt{3\upsilon})$ are obtained from (13) by replacing $\gamma_l^{(\mu)}$ by $\gamma_l^{(\mu\pm\sqrt{3\upsilon})} = \Gamma^{(\mu\pm\sqrt{3\upsilon})}E[\alpha_l^2]$ where $\Gamma^{(\mu\pm\sqrt{3\upsilon})}$ is given by

$$\Gamma^{(\mu\pm\sqrt{3\upsilon})} = \frac{E_b}{2\left(\sigma^2_{AWGN} + E[\sigma^2_S] + E[\sigma^2_{MAI}] \pm \sqrt{3\mathrm{var}[\sigma^2_{MAI}]}\right)} \tag{15}$$

## 4. Numerical Examples

In this section, the results presented in the previous sections are illustrated using specific examples. Figure 1 shows the BEP vs. total users $K$ and Fig. 2 shows the BEP vs. selective combined paths $L_c$ for SRake receiver over channels having flat and exponentially decaying ($C = 10$) PDPs. We set $N = 63$, $L = 100$ and $E_b/N_o = 20$ dB. Bit rates are kept fixed at $R_b = L/(63T_d)$ with arbitrary $T_d$. Figure 1 shows the results of SIGA only. It is seen that increasing $L_c$ generally brings performance improvement. However, the larger the value of $L_c$, the more is the performance variations due to change in $K$. In other words, as compared to single user performance, increasing $K$ imposes comparatively less performance degradation for an SRake while it combines smaller number of $L_c$. As a consequence, increasing $L_c$ doesn't bring much performance improvement if the number of simultaneous users $K$ is large. Additionally, for certain fixed number of users $K$, performance variations due to change in $L_c = 1, 2, \ldots, L$ is much wider in a channel with flat PDP.

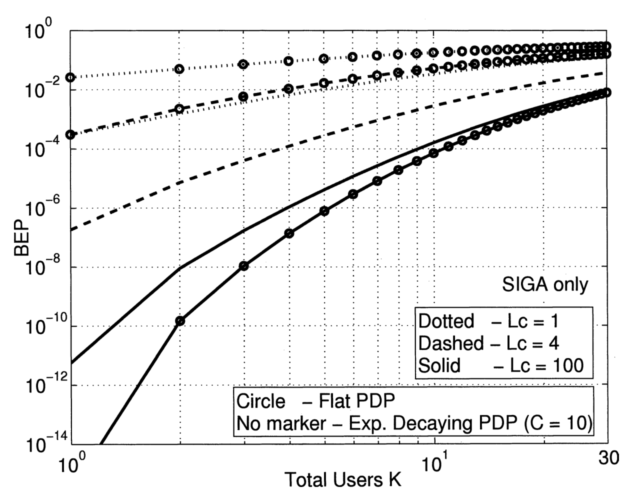


**Fig. 1** BEP vs. total users $K$ for SRake receivers in channels having flat and exponentially decaying ($C = 10$) PDPs. $N = 63$, $E_b/N_o = 20$ dB and $L = 100$. Results are shown for $L_c = 1$, 4 and 100 considering SIGA. $R_b = 1/T_b = L/(NT_d)$, $T_d$ being an arbitrary constant.

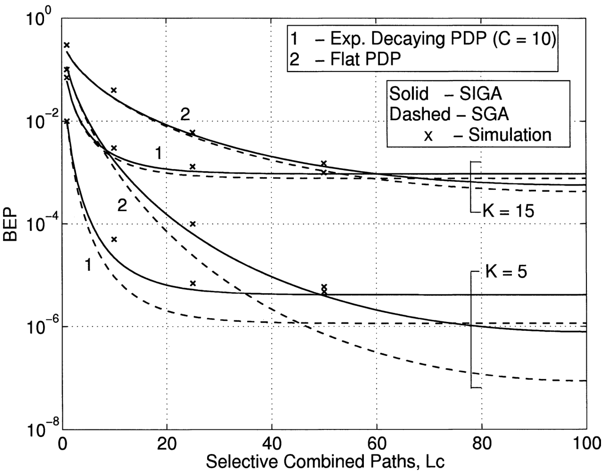


**Fig. 2** BEP vs. number of combined paths $L_c$ of SRake receiver in channels having flat and exponentially decaying ($C = 10$) PDPs. $N = 63$, $E_b/N_o = 20$ dB and $L = 100$. Results are shown for $K = 5$ and 15 from SGA, SIGA and simulations. $R_b = 1/T_b = L/(NT_d)$, $T_d$ being an arbitrary constant.

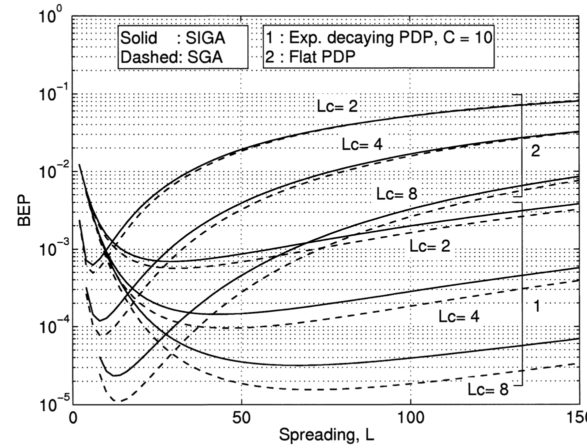


**Fig. 3** BEP vs. spreading $L$ for SRake receiver in channels having flat and exponentially decaying ($C = 10$) PDPs. $N = 63$, $E_b/N_o = 20$ dB and $L_c = 2, 4, 8$. Results are shown for $K = 5$ considering SGA and SIGA. $R_b = 1/T_b = L/(NT_d)$, $T_d$ being an arbitrary constant.

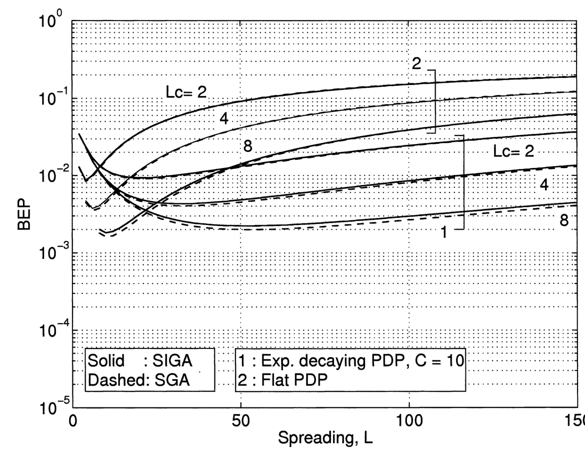


**Fig. 4** BEP vs. spreading $L$ for SRake receiver in channels having flat and exponentially decaying ($C = 10$) PDPs. $N = 63$, $E_b/N_o = 20$ dB and $L_c = 2, 4, 8$. Results are shown for $K = 15$ considering SGA and SIGA. $R_b = 1/T_b = L/(NT_d)$, $T_d$ being an arbitrary constant.

In Fig. 2, we present the BEP performance vs. $L_c$. From the performances over two types of PDPs, SRake is seen to perform better in decaying PDP for small to medium $L_c$ and in flat PDP afterwards. In Fig. 2, we also compare the validity of SGA and SIGA for SRake receiver depending upon the number of combined paths, simultaneous users and shape of the PDP. The results from SIGA closely match with those from simulation and can be considered to be reliable approximations. The SGA can be considered more or less accurate in both types of the PDPs for almost all $L_c = 1, 2, \ldots, L$ while $K$ is large. However, while $K$ is small, SGA is accurate only for small $L_c$, afterwards, which becomes quite optimistic. Here note that while there are many multipath components, the density of MAI in each path is approximately Gaussian according to the central limit theorem [12]. Hence, if we capture only one path ($L_c = 1$), the SGA for MAI should be very accurate even for small $K$. However, for $L_c > 1$, the respective path energies are MRC combined, which distorts the density. As a consequence, as $L_c$ increases, the density moves away from the Gaussian density and the SGA becomes inaccurate. However, with increasing $K$, the CLT again can be used and the SGA becomes accurate.

Finally the BEP vs. spreading $L$ for SRakes with $L_c = 2$, 4 and 8 are shown in Figs. 3 and 4, for $K = 5$ and 15 respectively. We set $N = 63$ and $E_b/N_o = 20$ dB as before. These figures provide several important implications for multiple access performance by employing SRake receiver. If the spreading is small, better performance is obtained in flat PDP, whereas the performance degrades drastically after reaching a well-defined optimum $L$ if the spreading is increased. In contrast, for medium to large spreading, better performance is seen in exponentially decaying PDP. Though there is an optimum $L$ in this case too, this is not that well defined and overspreading doesn't bring drastic performance degradation. As a whole, if reasonably high spreading is maintained, MAI is less effective for SRake if the PDP is decaying than being flat.

Figures 3, 4 also show the validity of SGA for SRake at different spreading. While $K = 15$, the SGA is accurate in all cases. For $K = 5$, the SGA is nearly accurate for all spreading for both flat and decaying PDP while $L_c = 2$. For $L_c = 4$ and 8, the SGA is optimistic for flat PDP while the spreading is near the optimum value. For small and large spreading, the SGA can be considered reasonably accurate in this case. However, if the PDP is decaying, the SGA is accurate only at small spreading, being optimistic afterwards.

## 5. Conclusions

A study presenting error performance of DS-CDMA over Rayleigh faded multipath channel has been presented while SRake receiver is employed. It has been shown that the

SIGA previously used for AWGN channel can be successfully used in multipath channel as well. One of the advantages of the method is that the results can be evaluated very easily finding the effects of various system, channel and receiver parameters. It has been shown that the SGA, though is the simplest way of MAI modeling, is accurate in some circumstances. However, because the SGA becomes optimistic in most of the cases of practical interest, the SIGA that is accurate in all cases without increasing the computational complexity should be the better choice for performance analysis of DS-CDMA over multipath channels.

## Acknowledgment

This work was supported in part by the Grant-in-Aid for scientific research (No. 16560328) of JSPS, International Communication Foundation (ICF) and Union Tool Scholarship Foundation of Japan. Graduate studies of Mr. Rahman are supported by the Government of Japan under Monbukagakusho scholarship program.

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