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Analysis of Mean Waiting Time for Delivery of a Message in Mobile Multi-Hop Networks

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SUMMARY Analysis of waiting time to deliver a message M from a source S to a destination D is deeply related to connectivity analysis, which is an important issue in fundamental studies of mobile multi-hop networks. In [1], we compared the mean waiting times of two methods to deliver M with the mean value of the minimum waiting time. The mean minimum waiting time was obtained by computer simulation because theoretical analysis of this mean is not easy, although another two methods were analyzed theoretically. In this paper, we propose an approximate method to theoretically analyze the mean minimum waiting time in a one-dimensional street network, and show that this method gives a good approximation of the mean minimum waiting time. Also, we consider shadowing and change of directions of mobile nodes at intersections as negative factors arising in two-dimensional street networks. We extend the above method to compute the mean minimum waiting time considering these factors, and discuss how the mean minimum waiting time is affected by these factors.

key words: *mobile multi-hop network, connectivity, mobility, epidemic routing*

1. Introduction

In mobile multi-hop networks [2], [3], each node has capabilities of direct communication and relaying. These capabilities enable mobile nodes to construct a multi-hop path between mobile nodes. Mobile nodes can exchange information if there is a multi-hop path between them; however, mobile nodes sometimes encounter a situation where there is no multi-hop path between them because positions of mobile nodes are generally random and the transmitting range is finite. Therefore, connectivity analysis is important for understanding behavior of mobile multi-hop networks [4].

As a measure of connectivity, the probability that there is at least one path between a source S and a destination D is often used. As another measure for mobile multi-hop networks, [1], [5] use the mean of the waiting time from the time when a request for transmission of a message M destined for D arrives at S to the time when D receives M. Assume that M arrives at S at $t = \tau_1$ and that M is received by D at τ_2. The waiting time $\tau_2 - \tau_1$ depends on how M is delivered from S to D. Also, from the viewpoint of applications, there are applications that accept a long waiting time, such as data collection through a sensor network, where S, D, and M correspond to a sensor node, a data center, and data observed by S, respectively. Here, we assume that sensors and relay nodes in the sensor network can be mobile nodes. In mobile multi-hop networks for such applications, the mean waiting time can be an appropriate measure of connectivity, and it depends on the method to send M to D. Then, in [1], the following three methods were compared.

Method 1: If there is at least a multi-hop path between S and D at τ_1, then S sends M to D immediately. Otherwise, S keeps searching a multi-hop path to D by a certain routing protocol until the routing protocol finds a multi-hop path between S and D expecting that a multi-hop path will appear between S and D because mobility of relay nodes changes topology with time. Finally, S sends M to D through this multi-hop path. In [1], the routing protocol was assumed to be an ideal one that can always notice appearance of a connected multi-hop path between S and D. This method reflects the basic behavior of ordinary ad-hoc routing protocols.

Method 2: Epidemic routing [6] delivers M to D by making mobile nodes move M closer to D, and can be realized in various manners like the proposed algorithm in [6]. In [1], we compared the simplest epidemic routing, which does not use multi-hop transmission, with Method 1 to explore how mobility of nodes affects the mean waiting time. The simplest epidemic routing is called Method 2. In Method 2, S directly sends M to all mobile nodes in its communication range. It is expected that some of these nodes approach D as time passes and may finally enter the communication range of D. If this actually occurs, M can be delivered to D directly from mobile nodes having M. In actual situations, we need some protocol to realize Method 2. To do this, for example, S has to periodically broadcast M to mobile nodes in its communication range, and mobile nodes having M have to periodically check whether they are in the communication range of D by exchanging control signals. Although there may be various ways to realize Method 2, here we assume an ideal situation where S can always notice that there are mobile nodes in its communication range and mobile nodes having M can always notice that they enter the communication range of D. Method 2 does not use multi-hop transmission and utilizes only capability of mobility to move M closer to D. Then, we can intuitively see that the waiting time becomes long if Method 2 is used;

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however, Method 2 can sometimes deliver M to D faster than Method 1 for a small density of nodes as shown in [1].

Method 3: This method is a combination of Methods 1 and 2. If there is at least a multi-hop path between S and D at τ_1, then S sends M to D immediately like Method 1. Otherwise, S sends M to all mobile nodes that have multi-hop paths from S, and mobile nodes that received M repeatedly distribute M to other nodes in the same manner. For example, such distribution of M is done by flooding. Finally, M is delivered to D. Here, we assume that scheduling of flooding is ideally executed and neglect delay caused by collision of packets for simplifying analysis and clarifying how the change of topology with time affects the waiting time. Hence, Method 3 minimizes the waiting time in this case.

Let $T_{w,1}$, $T_{w,2}$ and $T_{w,3}$ be $\tau_2 - \tau_1$ for Methods 1, 2 and 3, respectively, and let $E(\cdot)$ be the mean value of $\cdot$. In [1], $E(T_{w,1})$, $E(T_{w,2})$ and $E(T_{w,3})$ were analyzed and characterized relatively in a one-dimensional mobile multi-hop network consisting of a fixed source S, a fixed destination D, and pedestrians walking at a constant velocity v along a street. Note that S, D, and pedestrians correspond to a sensor node, a data center, and mobile relay nodes, respectively, if the network is used as a sensor network. We analyzed these mean waiting times assuming that positions of nodes are randomly decided and using probabilistic properties of relative positions of nodes.

As shown in [1], Methods 2 and 3 outperform Method 1 for a small density of nodes and Method 1 is no longer effective in such a situation because $E(T_{w,1})$ rapidly increases as the density of nodes decreases. Therefore, we pay attention to situations where the density of nodes is small and the mean waiting time is large. Based on this policy, we neglect short delays caused in a queue of packets or in transmissions of packets through direct links or multi-hop paths for simplifying analysis. Also, as mentioned, we assume an ideal situation where S, D, and mobile nodes can always notice existence of nodes in their communication ranges and appearance of multi-hop paths. From these assumptions, $T_{w,1}$, $T_{w,2}$ and $T_{w,3}$ in [1] are considered as lower bounds of the waiting times of Methods 1, 2 and 3 in actual situations, respectively.

Furthermore, comparison between $E(T_{w,1})$, $E(T_{w,2})$ and $E(T_{w,3})$ gives some insights to know how differently mobility of nodes affects the mean waiting time as mentioned in [1]. In Method 1, mobility helps nodes to construct a connected multi-hop path by changing the topology of the network. This capability of mobility is called Capability 1 in [1]. In Method 2, mobility helps nodes to move M closer to D. This capability of mobility is called Capability 2 in [1]. Method 3 maximally utilizes these capabilities of mobility to minimize the waiting time. We showed that $E(T_{w,2}) > E(T_{w,1}) \approx E(T_{w,3})$ if the density of nodes is large, and that $E(T_{w,1}) > E(T_{w,2}) \approx E(T_{w,3})$ if the density of nodes is sufficiently small. This means that Capability 1 works sufficiently well without any assistance from Capability 2 if the density of nodes is large, and that the reverse is true if the density of nodes is sufficiently small. From these results, we can well understand how $E(T_{w,1})$, $E(T_{w,2})$ and $E(T_{w,3})$ behave differently due to different effects of mobility.

In [1], $E(T_{w,1})$ and $E(T_{w,2})$ were theoretically analyzed. However, $E(T_{w,3})$ was obtained by computer simulation although a theoretical lower bound was given because theoretical analysis is not easy, as mentioned in [1]. In [7], an approximate method to analyze the mean waiting time of Method 3 in a one-dimensional vehicle network was recently proposed.

Although a one-dimensional street network on a line was assumed in [1], street networks usually include corners and intersections. As a result, shadowing occurs at corners and mobile nodes can change their directions at intersections. If shadowing occurs at a corner between S and D, Method 1 cannot construct any multi-hop paths between them. If a mobile node having M does not move toward D due to a change of direction at an intersection, Method 2 cannot deliver M to D. It is expected that these negative factors also affect Method 3 and increase the mean minimum waiting time; however, $E(T_{w,3})$ was not theoretically analyzed in these situations.

With this as background, in this paper, we propose an approximate method to compute $E(T_{w,3})$ theoretically. We analyze $E(T_{w,3})$ in three models, called Models 1, 2 and 3. Model 1 is almost the same model as that used in [1]. In Model 1, mobile nodes are pedestrians moving along a line. In Model 2, we assume that there are some corners along a street and shadowing occurs around the corners. In Model 3, we assume that there are intersections along a street and mobile nodes change their directions at intersections. We show that the approximate methods well describe $E(T_{w,3})$ in these models and show how shadowing and the change of direction affect $E(T_{w,3})$. Although Model 1 and the approximate method derived for Model 1 are similar to the network model and the approximate method in [7], the equations for Model 1 derived in this paper are different from those in [7], and [7] does not consider effects of shadowing and the change of direction of mobile nodes on the waiting time.

2. Mean Waiting Time in Model 1

2.1 Definitions and Assumptions

First, we propose an approximate method to compute $E(T_{w,3})$ theoretically in Model 1. The following are definitions and assumptions made in Model 1. The assumptions are essentially the same as those in [1] except for A_1.

A1: Suppose that all nodes including a source S and a destination D are on a line. In [1], we considered transmission of a message M from S to D assuming that S and D were fixed nodes. To simplify analysis, in this paper, we assume that a request of transmission of M arrives at a mobile node moving toward D at $t = \tau_1$. Denote this node by S. Assume that D is fixed and that other nodes are mobile nodes. As mentioned, we consider that S is a mobile sensor node that moves with a pedestrian and D is a fixed data center. Let $x(n, t)$ be the coordinate of a node n at time t on the

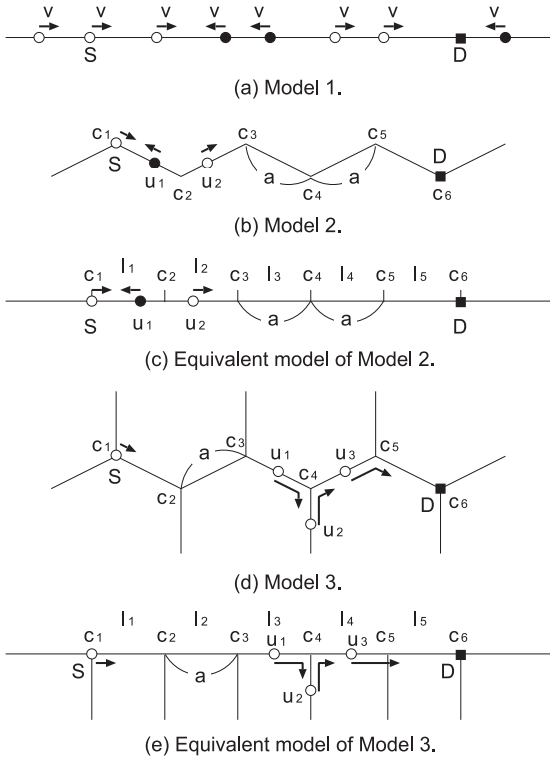


Fig. 1 Models.

line. Let L be the distance from S to D at τ_1. Suppose that $x(S, \tau_1) = 0$, and that $x(D, t) = L$ for all t because D is stationary. Define again that τ_2 is the moment when D receives M.

A2: Assume that a mobile node is a pedestrian walking along the line at a constant velocity v independently of other nodes without changing direction. As represented in Fig. 1(a), mobile nodes moving left (in the direction from D to S) are represented by black nodes and those moving right (in the direction from S to D) are represented by white nodes. As mentioned, we consider that these mobile nodes comprise a mobile multi-hop path along which M is transmitted or move M closer to D.

A3: Let N_b and N_w be the sets of mobile nodes moving left and those moving right, respectively. Denote a mobile node of N_b by b_i, where i is an integer such that $x(b_i, t) < x(b_{i+1}, t)$ for all i and all t. Denote a mobile node of N_w by w_j, where j is an integer such that $x(w_j, t) < x(w_{j+1}, t)$ for all j and all t. Note that the above definition of w_j is slightly different from that in [1].

A4: Assume that the distributions of nodes of N_b and N_w obey Poisson distributions of intensity $\frac{\lambda}{2}$ at the initial moment $t = 0$, respectively. Note that the same distributions are observed at time $t > 0$ because mobile nodes move independently from other nodes [8].

A5: Assume that there exists a wireless link between two nodes if the distance between them is not longer than r, where r is a positive constant and $r < L$. Otherwise, there is no link between them.

A6: Let $N_M(t)$ be the set of nodes having M at time t. Let $N'_M(t)$ be the set of nodes that do not have M at time t. Then, $S \in N_M(\tau_1)$. Let n_h^t be the node whose coordinate is the largest in $N_M(t)$. Let $n_{h,w}^t$ be the node of N_w whose coordinate is the largest in $N_M(t)$. Let $X(t)$ and $X_w(t)$ be the coordinates of n_h^t and $n_{h,w}^t$ at t, respectively.

A7: Suppose that $u_1 \in N_M(\tau_a)$ and $\{u_2, u_3, ..., u_k\} \subset N'_M(\tau_a)$, where $k \geq 2$, and that $x(u_1, \tau_a) < x(u_2, \tau_a) < \cdots < x(u_k, \tau_a)$. Topology of the network changes with time under the above assumption. Suppose that u_1 becomes connected to $\{u_2, u_3, ..., u_k\}$ due to the change of topology at $\tau_b = \tau_a + \Delta\tau$, where $\Delta\tau$ is a small constant, that is, $x(u_2, \tau_b) - x(u_1, \tau_b) \leq r$, $x(u_3, \tau_b) - x(u_2, \tau_b) \leq r$, ..., $x(u_k, \tau_b) - x(u_{k-1}, \tau_b) \leq r$. Assume that M is forwarded from u_1 to u_2, ..., u_k simultaneously at τ_b without delay.

A8: If an interval has no node and the length of this interval is greater than r, we call this interval a gap. If an interval has no node of N_b and the length of this interval is greater than r, we call this interval a gap of N_b. A gap of N_w is defined in the same manner.

2.2 Related Work

In [7], an approximate method to compute $E(T_{w,3})$ in a vehicle network similar to Model 1 was recently proposed. This method computes $E(T_{w,3})$ by dividing the distance between S and D by the average message transfer velocity. The average message transfer velocity was computed from $t_{DTN}^{(i)}$ and S_{m-hop}, defined in [7]. As can be seen from the following subsections, this concept is similar to our analysis in Model 1, and $t_{DTN}^{(i)}$ and S_{m-hop} correspond to T_{walk} and the mean of $X_{fwd,w}$, which are defined in the following subsections, respectively. In [7], $t_{DTN}^{(i)}$ and S_{m-hop} are derived by dividing a street into vehicle slots with length l, which is the length of each vehicle, and using the probability that there exists at least one vehicle in a slot, and it is assumed that $t_{DTN}^{(i)}$ obeys an exponential distribution. The validity of this analysis is shown by comparing numerical results of this analysis with simulation results. In [7], it is assumed that the velocity of mobile node is a random variable. This assumption is more complicated than that in this paper; however, only a line network was considered in [7] while this paper considers effects of shadowing and inflow and outflow of mobiles at intersections in Models 2 and 3.

Although the outline of our approximate method to compute $E(T_{w,3})$ is similar to that of [7], we derive the mean value of $X_{fwd,w}$ differently from [7] and derive the density function of T_{walk} from properties of clumps as explained below while these properties are not used in [7]. As a result, in this paper, the density function of T_{walk} is obtained as a function different from an exponential distribution, and other probabilities and mean values not used in [7] are derived. Also, the final form of $E(T_{w,3})$ is different from that in [7]. Furthermore, as can be understood later, our analysis in Model 1 can be simply extended to those in Models 2 and 3 because the final form of $E(T_{w,3})$ in Model 1 is suitable to extension to analyses in Models 2 and 3.

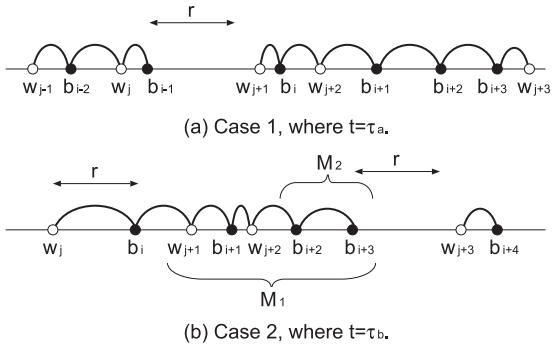


Fig. 2 Examples of forwarding.

2.3 Mean Progress of M in One Forwarding

In Model 1, we can move M closer to D by forwarding M through a multi-hop path. Even if forwarding is impossible, M approaches D as mobile nodes of N_w having M walk toward D. We call a time interval in which forwarding is impossible a "walking period." After a walking period, forwarding is done and the next walking period begins. We consider how much M is made closer to D by one forwarding to compute $E(T_{w,3})$. To do this, we consider the progress of M assuming that the distance to D is very long. Although this progress can be measured by $X(t)$, we consider $X_w(t)$ as the progress of M for simplifying analysis. In the following, we first explain the reason we consider $X_w(t)$ as the progress of M, and second explain how much M is made closer to D by one forwarding.

Figure 2(a) shows a situation at $t = \tau_a$ in a walking period where $\{w_{j-1}, b_{i-2}, w_j, b_{i-1}\} \subset N_M(\tau_a)$ and $\{b_i, ..., b_{i+3}, w_{j+1}, ..., w_{j+3}\} \subset N'_M(\tau_a)$. Because there is a gap between b_{i-1} and w_{j+1}, M cannot be forwarded to any nodes of $N'_M(\tau_a)$. To forward M to nodes of $N'_M(\tau_a)$ after τ_a, we have to wait until the distance between w_j and b_i becomes equal to r. In general, if a walking period ends at t, then $n_h^{t-\Delta t}$ must be in N_w and there must be a node of N_b at $x(n_h^{t-\Delta t}, t) + r$ at t, where Δt is a small positive constant. Therefore, nodes of N_b cannot be sources to forward M to mobile nodes which do not have M.

Figure 2(b) shows a situation where the above walking period ends. Suppose that $t = \tau_b$ and that $n_h^{\tau_b-\Delta t} = w_j$. Then, the distance between w_j and b_i is equal to r as mentioned. M is forwarded from w_j to b_i, w_{j+1}, b_{i+1}, w_{j+2}, b_{i+2} and b_{i+3}. Then, $n_h^{\tau_b} = b_{i+3}$ and $n_{h,w}^{\tau_b} = w_{j+2}$. Because a node of N_b cannot be a source of the next forwarding after τ_b as mentioned, the source node of the next forwarding is w_{j+2}. Consider $X(t) - X_w(t)$, which is the distance between n_h^t and $n_{h,w}^t$ at t. Define that ΔX_{fwd} is the mean of $X(t) - X_w(t)$ given that a walking period ends at t. After τ_b, $X(t) - X_w(t)$ decreases and finally becomes equal to 0 because n_h^t and $n_{h,w}^t$ approach if $n_h^t \in N_b$. Note again that $n_h^t \in N_b$ cannot forward M to nodes $N'_M(\tau_b)$ until n_h^t and $n_{h,w}^t$ come to the same position. Consequently, we can see that the nodes in the right of $n_{h,w}^t$ cannot essentially contribute to the progress of M. Therefore, we consider $X_w(t)$ as the progress of M instead of $X(t)$ for simplifying analysis of $E(T_{w,3})$.

Consider the situation in Fig. 2(b) again. Using the notations in Fig. 2(b), we consider the mean progress of M by one forwarding. Let M_1 be the number of nodes that are on the right of b_i and have at least a multi-hop path to b_i as shown in Fig. 2(b). Denote these M_1 nodes by u_1, u_2, ..., u_{M_1}, such that $x(u_1, \tau_b) < x(u_2, \tau_b) < \cdots < x(u_{M_1}, \tau_b)$. Let u_0 be b_i. Suppose that $u_{M_1-M_2+1} \in N_b$, $u_{M_1-M_2+2} \in N_b$,..., $u_{M_1-1} \in N_b$ and $u_{M_1} \in N_b$. Let X_{fwd} be the distance between w_j to u_{M_1}. Let $X_{fwd,w}$ be the distance between w_j to $u_{M_1-M_2}$ given that $M_1 > M_2$, namely there is at least a node of N_w in u_1, ..., u_{M_1}. Then, we mainly pay attention to $E(X_{fwd,w})$ as the mean progress of M in one forwarding because it corresponds to the increase of $X_w(t)$. As shown in Appendix A,

$$E(X_{fwd,w}) = \frac{e^{\lambda r}}{\lambda} - \frac{r}{e^{\lambda r} - 1}. \tag{1}$$

For reference, we have

$$\Delta X_{fwd} = E(X_{fwd} | M_1 > M_2) - E(X_{fwd,w}) \le r \tag{2}$$

as shown in Appendix A. Namely, ΔX_{fwd} is at most r, and this property supports the above assumption from another point of view that the progress of M realized by nodes of N_b does not essentially contribute to the progress of M.

As mentioned, in the approximate method in [7], S_{m-hop} corresponds to $E(X_{fwd,w})$. As can be seen from the above equations and the equations in [7], the derivation and final form of $E(X_{fwd,w})$ are different from those of S_{m-hop}.

2.4 Density Function of the Length of a Walking Period

In this subsection, we consider the length of a walking period that begins at τ_c. Denote this length by T_{walk}. As mentioned, T_{walk} corresponds to $t_{DTN}^{(i)}$ in [7]. There is a big difference between [7] and this paper in the derivation of the density function of T_{walk}. In the analysis in [7], $t_{DTN}^{(i)}$ is assumed to obey an exponential distribution. As explained in the following, however, T_{walk} depends on the distance between adjacent nodes of N_w and the lengths of connected components consisting of nodes of N_b, which are called clumps in the following. Due to this dependence, it is not easy to prove that T_{walk} obeys an exponential distribution. In the following, we derive the density function of T_{walk}, which is not expressed as an exponential distribution, although some approximations are used.

First, we explain the concept of a clump, which is deeply related to T_{walk}. Consider nodes on the right of a node u_k at t. Denote these nodes by u_{k+1}, u_{k+2}, ... and suppose that $x(u_k, t) < x(u_{k+1}, t) < x(u_{k+2}, t) < ...$. If $x(u_{k+1}, t) - x(u_k, t) \le \alpha$, $x(u_{k+2}, t) - x(u_{k+1}, t) \le \alpha$, ..., $x(u_{k+\ell}, t) - x(u_{k+\ell-1}, t) \le \alpha$ and $x(u_{k+\ell+1}, t) - x(u_{k+\ell}, t) > \alpha$, where α is a constant, then the interval $[x(u_k, t), x(u_{k+\ell}, t)+\alpha]$ is called a clump, which begins from u_k. For a clump consisting of nodes distributed according to a Poisson distribution of intensity β, the density function of the length of

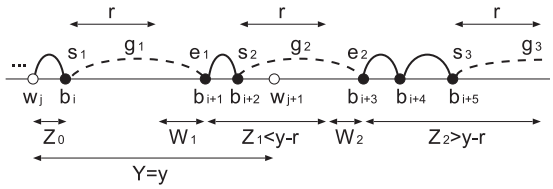


Fig. 3 Variables to define T_{walk}.

a clump has been given in [9] and the mean of the clump length is $\frac{e^{\alpha\beta}-1}{\beta}$. Also, in [9], it has been mentioned that the exponential distribution with the mean $\frac{e^{\alpha\beta}-1}{\beta}$ can be used as an approximation to the distribution of the length of a clump for a large β. In the following, we compute the density of T_{walk} using these properties.

Let C be an event where there is a gap between w_j and w_{j+1} at τ_c. We show an example of the situation at τ_c in Fig. 3, and explain T_{walk} using some variables as shown in this figure. Suppose that w_j corresponds to $n^{\tau_c}_{h,w}$. Let Y be a random variable identical to the distance between w_j and w_{j+1}. Suppose that $Y = y$ at τ_c, where y is a positive constant. Here, $Y > r$ because of the gap as mentioned. Denote by g_k kth gap of N_b on the right of w_j at τ_c, where $k = 1, 2, ...$ Let s_k and e_k be coordinates of black nodes at both ends of g_k, where $s_k < e_k$. Then, g_k corresponds to an interval (s_k, e_k), and the black nodes at e_{k-1} and s_k have a connected path consisting of black nodes between them for all k. Let Z_k and W_k be $s_{k+1} - e_k + r$ and $e_k - s_k - r$, respectively, where $k = 1, 2, ...$. Let Z_0 be $s_1 - x(w_j, \tau_c)$. Let Z_{sum} and W_{sum} be $\sum_{k=1}^{m-1} Z_k$ and $\sum_{k=1}^{m} W_k$, respectively, if $Z_1 < y - r$, $Z_2 < y - r$, ..., $Z_{m-1} < y - r$ and $Z_m \geq y - r$. Then, we have

$$T_{walk} = \frac{Z_0 + Z_{sum} + W_{sum}}{2v} \tag{3}$$

because if $Z_1 < y - r$, $Z_2 < y - r$, ..., $Z_{m-1} < y - r$ and $Z_m \geq y - r$, then the black nodes between e_m and s_{m+1} can connect w_j and w_{j+1} when the distance between the black node that is at e_m at time τ_c and w_j becomes equal to r.

By letting α and β be r and $\frac{\lambda}{2}$, respectively, we can see that Z_k for $k \geq 1$ corresponds to the length of a clump consisting of nodes of N_b from the definitions of Z_k and a clump, where $\alpha = r$, $\beta = \frac{\lambda}{2}$, and the mean of Z_k is $\frac{e^{\frac{\lambda r}{2}}-1}{\frac{\lambda}{2}}$. For simplicity, we assume that Z_k approximately obeys an exponential distribution with the mean $\frac{e^{\frac{\lambda r}{2}}-1}{\frac{\lambda}{2}}$ as mentioned. Based on this assumption, by letting α and β be $y - r$ and $\Lambda = \frac{\frac{\lambda}{2}}{e^{\frac{\lambda r}{2}}-1}$, we can see that $Z_{sum}+y-r$ again means the length of a clump with the mean $\frac{e^{\Lambda(y-r)}-1}{\Lambda}$ if $Z_1 < y - r$, $Z_2 < y - r$, ..., $Z_{m-1} < y - r$ and $Z_m \geq y - r$. We can again assume that $Z_{sum} + y - r$ obeys an exponential distribution of intensity $\frac{\Lambda}{e^{\Lambda(y-r)}-1}$. Because we need the density of Z_{sum}, for simplicity, we roughly compute the density of Z_{sum} as an exponential distribution of intensity Λ_Z, where $\Lambda_Z = \{\frac{e^{\Lambda(y-r)}-1}{\Lambda} - (y-r)\}^{-1}$.

Also, W_{sum} is the sum of exponential random variables. Suppose that M_3 is a random variable such that $Z_1 < y - r$, $Z_2 < y - r$, ..., $Z_{M_3-1} < y - r$ and $Z_{M_3} \geq y - r$. Then, the density function of W_{sum}, denoted by $f_{W_{sum}}(w)$ is as follows:

$$\begin{aligned} & f_{W_{sum}}(w) \\ &= \sum_{m_3=1}^{\infty} f_{W_{sum}}(w|M_3 = m_3)P(M_3 = m_3) \\ &= \Lambda_W e^{-\Lambda_W w} \end{aligned} \tag{4}$$

where $f_{W_{sum}}(w|M_3 = m_3) = \frac{(\frac{\lambda w}{2})^{m_3}}{(m_3-1)!w} e^{-\frac{\lambda}{2}w}$ because W_k obeys an exponential distribution of $\frac{\lambda}{2}$, $P(M_3 = m_3) = (1 - e^{-\Lambda(y-r)})^{m_3-1} e^{-\Lambda(y-r)}$ and $\Lambda_W = \frac{\lambda}{2} e^{-\Lambda(y-r)}$.

Here, Z_0 is identical to the $X(\tau_c) - X_w(\tau_c)$, and the mean of this length is identical to ΔX_{fwd} and is at most r as mentioned. For simplicity, therefore, we approximate Z_0 as $E(Z_0) = \Delta X_{fwd}$. Then, from this approximation and the above equations, we have

$$\begin{aligned} & f_{T_{walk}}(t|Y = y) \\ &= \int_0^{2vt-E(Z_0)} 2v f_{Z_{sum}}(2vt - E(Z_0) - w) f_{W_{sum}}(w) dw \\ &= \frac{2v\Lambda_W\Lambda_Z\{e^{-\Lambda_Z(2vt-E(Z_0))} - e^{-\Lambda_W(2vt-E(Z_0))}\}}{\Lambda_W - \Lambda_Z} \end{aligned} \tag{5}$$

if $t \geq \frac{E(Z_0)}{2v}$ and is equal to 0 otherwise. Let $f_Y(y)$ be the density function of Y. Then,

$$f_{T_{walk}}(t) = \int_{-\infty}^{\infty} f_{T_{walk}}(t|Y = y) f_Y(y|C) dy. \tag{6}$$

For unconditioning of Eq. (5), we use $f_Y(y|C)$ given in Appendix B. Although unfortunately we cannot obtain the closed form of $f_{T_{walk}}(t)$, we numerically compute $f_{T_{walk}}(t)$.

2.5 Velocity of Movement of M and Mean Waiting Time

To compute $E(T_{w,3})$, we use V_M, which is the mean progress of M carried by nodes of N_w per unit time. Define that B is an event where S cannot forward M to any nodes of N_w before the distance from S to D becomes equal to r. Denote V_M by $V_{M,B}$ if B occurs, and by $V_{M,\overline{B}}$ otherwise. If B occurs, then S does not have multi-hop paths to any nodes of N_w at τ_1 and T_{walk} is longer than $\frac{L-r}{v}$. Then

$$P(B) = P(C)P\left(T_{walk} > \frac{L - r}{v}\right), \tag{7}$$

where $P(C)$ is given in Appendix B and $P\left(T_{walk} > \frac{L-r}{v}\right)$ can be computed from Eq. (6). If B occurs, then M is carried by S without multi-hop transmission to nodes of N_w. Hence,

$$V_{M,B} = v. \tag{8}$$

In general, M is moved closer to D by walking of $n^t_{h,w}$ and forwarding from $n^t_{h,w}$ alternately. If B does not occur, we pay attention to this property and compute $V_{M,\overline{B}}$ as the ratio of the mean progress of M in one cycle of walking and forwarding to the mean length of a walking period, and approximately compute $V_{M,\overline{B}}$ as follows:

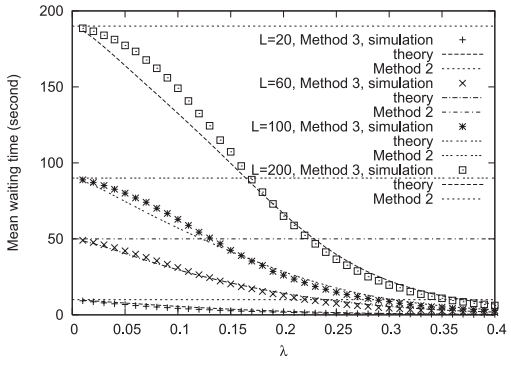


Fig. 4 Numerical results of $E(T_{w,3})$ in Model 1.

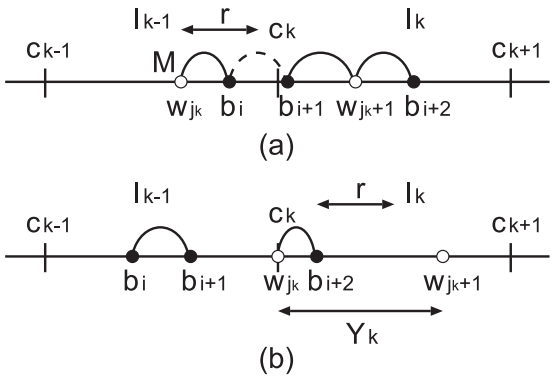


Fig. 5 Relation between w_{j_k} and w_{j_k+1} in Model 2.

$$V_{M,\overline{B}} = \frac{vE\left(T_{walk}|T_{walk} \le \frac{L-r}{v}\right) + E(X_{fwd,w})}{E\left(T_{walk}|T_{walk} \le \frac{L-r}{v}\right)} \tag{9}$$

because $T_{walk} \le \frac{L-r}{v}$ if B does not occur. Using V_M, we roughly compute $E(T_{w,3})$ as

$$E(T_{w,3}) = P(B)\frac{L-r}{V_{M,B}} + \{1-P(B)\}\frac{L-r}{V_{M,\overline{B}}}. \tag{10}$$

The reason we divide $L-r$ by $V_{M,B}$ or $V_{M,\overline{B}}$ is that D can receive M if the distance between D and M is r. Note that although [7] uses a concept similar to Eq. (10), [7] does not consider two events B and $\overline{B}$ separately. This classification will be utilized in the analyses of $E(T_{w,3})$ in Models 2 and 3.

We show the numerical results of $E(T_{w,3})$ obtained by the above method in Fig. 4 together with the simulation results. To obtain the simulation results, we use the same method as used in [1]. Here, $r = 10$ m and $v = 1$ m/sec. Although we use some approximations, the numerical results agree well with the simulation results. From these results, we can confirm the validity of the approximate method. We can also confirm that $E(T_{w,3})$ decreases as λ increases and that $E(T_{w,3})$ increases as L increases.

For reference, we also showed $E(T_{w,2})$, which is the mean of the maximum waiting time of epidemic routing while $E(T_{w,3})$ is the mean of the minimum waiting time of epidemic routing. To compute $E(T_{w,2})$, we assume that S delivers M to D by itself. Then, $E(T_{w,2})$ is $\frac{L-r}{v}$. We can confirm that Method 3 can move M faster than Method 2. Their difference becomes smaller as λ decreases, and finally Method 3 is almost identical to Method 2 for a small λ. As shown in this result, by comparing the mean waiting times of Methods 2 and 3, we can understand how forwarding decreases the mean waiting time for a given λ. We can see that the difference increases as L increases, and forwarding demonstrates its ability for a large L.

3. Mean Waiting Time in Model 2

The approximate method provided in the preceding section enables us to theoretically compute $E(T_{w,3})$ in a one-dimensional street network. As a next step, we consider the same problem in two-dimensional street networks. First, we consider effects of shadowing at corners along a street. Suppose that a street is surrounded by buildings and that there are some corners on the street. Then, a mobile node may not be able to communicate with a mobile node beyond the corner. Here, we assume that mobile nodes having a corner between them cannot directly communicate with each other even if the distance between them is not longer than r. We try to compute $E(T_{w,3})$ with this assumption together with the assumptions in Sect. 2. We call the model considered here Model 2. Here, we consider $n_c + 1$ corners. We assume that S is at a corner c_1 at τ_1 and D is at a corner c_{n_c+1}. Suppose that corners c_2, c_3, ..., c_{n_c} exist between S and D as shown in Fig. 1(b). In this example, nodes u_1 and u_2 cannot directly communicate with each other due to shadowing at c_2. Denote that I_k be the interval between c_k and c_{k+1}. Suppose that the length of each interval is a constant a, where $a \ge r$. Other than the above assumptions and definitions, we use the assumptions and definitions made in Model 1. To do this, we assume the street in Model 2 as a straight line in which there are $n_c + 1$ corners at a constant interval a and shadowing occurs at each corner as shown in Fig. 1(c). From this assumption, we can define N_b, N_w and the coordinates of nodes in the same manner as in Model 1.

In Model 2, M cannot be forwarded from a node on I_k to another node on I_{k+1} because of shadowing. Then, M is delivered to a new interval always by walking of a node of N_w having M. We try to roughly compute $E(T_{w,3})$ in Model 2 assuming that flow of M in I_k is almost identical to that in Model 1, where $L = a$. Suppose that w_{j_k} is in I_{k-1} and has M and that w_{j_k+1} is not in I_{k-1} and does not have M as shown in Fig. 5(a). In this figure, w_{j_k+1} cannot receive M because b_i and b_{i+1} are not connected owing to shadowing at c_k. Assume that w_{j_k} arrives at c_k at τ_a as shown in Fig. 5(b). Let Y_k be the distance between w_{j_k} and w_{j_k+1}. Let C_k be an event where there is no path between w_{j_k} and w_{j_k+1} at τ_a. For example, C_k occurs in Fig. 5(b). Note that C_k includes an event where there is a corner between w_{j_k} and w_{j_k+1}.

In Model 1, we implicitly suppose a situation where cycles of walking and forwarding are repeated. In Model 2, however, we sometimes meet a situation where a walking period never ends. Such a situation occurs if Y_k is longer than a. Considering such a situation separately, we define

that $T_{walk,k}$ is the length from τ_a to the end of a walking period that begins at τ_a given that C_k occurs, and $Y_k \leq a$. Define that B_k is an event where w_{j_k} cannot forward M to w_{j_k+1} before w_{j_k} leaves I_k. Then, B_k is the union of the following events:

- $Y_k > a$,
- $Y_k \leq a$, C_k and $T_{walk,k} > \frac{a-r}{v}$.

Here, we make an assumption that is not needed in Model 1. In Model 2, if $Y_k > a$, then a walking period never ends after M enters I_k as mentioned, and B_k, B_{k+1}, ... must occur. From this fact, we assume that $P(B_{k+1}|B_k) = 1$ to simplify analysis. Then, for $k = 2, 3, ..., n_c - 1$,

$$P(B_k) = P(B_{k-1}) + P(\overline{B_{k-1}})\Bigg\{P(Y_k > a) + P(Y_k \leq a)P(C_k|Y_k \leq a)P\left(T_{walk,k} > \frac{a-r}{v}\right)\Bigg\}. \quad (11)$$

$P(B_1)$ and $P(B_{n_c})$ are slightly different from this equation because S exists at c_1 initially and there is no effect of shadowing in I_{n_c}. Then,

$$P(B_1) = P(Y_1 > a) + P(Y_1 \leq a)P(C_1|Y_1 \leq a) \times P\left(T_{walk,1} > \frac{a-r}{v}\right), \quad (12)$$

$$P(B_{n_c}) = P(B_{n_c-1}) + P(\overline{B_{n_c-1}})P(C_{n_c}) \times P\left(T_{walk,n_c} > \frac{a-r}{v}\right). \quad (13)$$

To compute $P(Y_k > a)$ and $P\left(T_{walk,k} > \frac{a-r}{v}\right)$, we give the density functions of Y_k and $T_{walk,k}$ in Appendix C. Also, we give $P(C_k|Y_k \leq a)$ in Appendix C. By using equations in Appendix C, we can compute $P(B_k)$ for $k = 1, 2, ..., n_c$.

Denote V_M by V_{M,B_k} if B_k occurs, and by $V_{M,\overline{B_k}}$ otherwise. Then, we roughly compute V_{M,B_k} and $V_{M,\overline{B_k}}$ in the same manner as in Model 1 as follows:

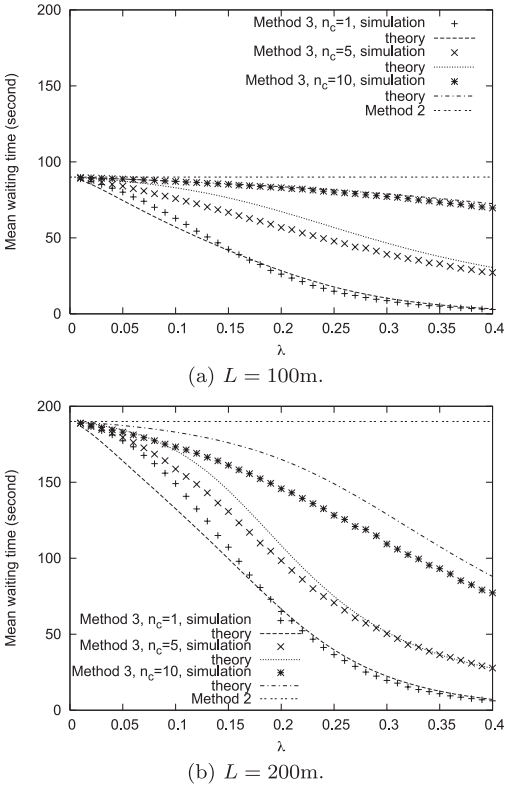
$$V_{M,B_k} = v, \quad (14)$$

$$V_{M,\overline{B_k}} = \frac{vE(T_{walk,k}|T_{walk,k} \leq \frac{a-r}{v}) + E(X_{fwd,w})}{E(T_{walk,k}|T_{walk,k} \leq \frac{a-r}{v})}. \quad (15)$$

Here, we compute $E(X_{fwd,w})$ by Eq. (1) to simplify computation of $V_{M,\overline{B_k}}$. To compute $V_{M,\overline{B_k}}$, we use the density function of $T_{walk,k}$ represented in Appendix C.

We define that $T_{w,3,k}$ is the length of an interval from the time when M enters I_k to the time when M enters I_{k+1} for $k = 2, ..., n_c - 1$. We define that $T_{w,3,1}$ is the length of an interval from τ_1 to the time when M enters I_2 and that $T_{w,3,n_c}$ is the length of an interval from the time when M enters I_{n_c} to the time when D receives M. We compute $T_{w,3,k}$ as follows: For $k = 1, ..., n_c - 1$,

$$E(T_{w,3,k}) = P(\overline{B_k})\left\{\frac{a - E(Y_{k+1,1})}{V_{M,\overline{B_k}}} + \frac{E(Y_{k+1,1})}{V_{M,B_k}}\right\} + P(B_k)\frac{a}{V_{M,B_k}}. \quad (16)$$



(a) $L = 100$m.

(b) $L = 200$m.

Fig. 6 Numerical results of $E(T_{w,3})$ in Model 2 for $n_c = 1$, 5 and 10.

$$E(T_{w,3,n_c}) = P(\overline{B_{n_c}})\frac{a-r}{V_{M,\overline{B_{n_c}}}} + P(B_{n_c})\frac{a-r}{V_{M,B_{n_c}}}. \quad (17)$$

For $k = 1, ..., n_c - 1$, $n^t_{h,w}$ must walk in the last part of I_k due to shadowing; therefore, we assume in Eq. (16) that the mean length of this "walking" part is $E(Y_{k+1,1})$, which is given in Appendix C.

Finally, we can compute $E(T_{w,3})$ as

$$E(T_{w,3}) = \sum_{k=1}^{n_c} E(T_{w,3,k}). \quad (18)$$

Numerical results of $E(T_{w,3})$ in Model 2 for $n_c = 1$, 5 and 10 are represented in Fig. 6 together with simulation results. We assume that $L = 100$ m and $L = 200$ m in Figs. 6(a) and (b), respectively. Note that the results for $n_c = 1$ mean that those when shadowing never occurs. For reference, we also showed $E(T_{w,2})$, which is $\frac{L-r}{v}$.

From these results, we can confirm that the numerical results well describe characteristics of $E(T_{w,3})$ although there is a difference between the numerical and simulation results in some cases because of approximations used in the analysis. In the analysis, we assumed that $P(B_{k+1}|B_k) = 1$. This approximation means that B_{k+1} strongly depends on B_k. We consider that the tendency of the numerical results of $E(T_{w,3})$ well describes that of the simulation results due to this assumption of dependence; however, the exact value of $P(B_{k+1}|B_k)$ is not equal to 1. Therefore, the above approximation of $P(B_{k+1}|B_k)$ is one factor that sometimes causes a difference between the numerical results and the simulation

results. The more accurate analysis of $P(B_{k+1}|B_k)$ is considered as a future problem. These figures show that the mean waiting time rapidly increases as shadowing occurs more frequently, and it increases as λ decreases. We can see that Method 3 can move M faster than Method 2 even if shadowing occurs. As explained in the analysis of $E(T_{w,3})$ in Model 2, the velocity of M decreases and becomes equal to the velocity of walking if a large gap appears between nodes. Due to such a negative effect of shadowing, $E(T_{w,3})$ approaches $E(T_{w,2})$ as λ decreases.

Although Method 1 cannot send M from S to D if shadowing occurs between S and D as mentioned, Method 3 can deliver M to D even if shadowing occurs. This feature is an advantage of Method 3 other than its quickness while Method 3 is faster than Method 1 in nature.

4. Mean Waiting Time in Model 3

In Model 2, we considered effects of shadowing at corners on the message delivery. At corners of streets, there is another factor that may increase the mean waiting time. If there is an intersection at a corner, a mobile node sometimes changes its direction. The change of direction of a mobile node with M may increase the waiting time if this node does not move toward D. We consider such a situation, called Model 3, and extend the analysis in Model 2 to consider negative effects of the change of direction at intersections on the mean waiting time.

For Model 3, we consider the network depicted in Fig. 1(d). In this network, there are intersections, each of which is connected to three streets. We assume that shadowing occurs between each pair of streets. For example, in Fig. 1(d), shadowing occurs between u_1 and u_2, between u_2 and u_3, and between u_3 and u_1. To simplify explanation, we transform the network in Fig. 1(d) to the network in Fig. 1(e).

Consider the network in Fig. 1(e). Assume that a mobile node never turns back and turns each corner after selecting its direction from among the two directions. We assume that flows of mobile nodes are as represented in Fig. 7 and Table 1. We assume the density of nodes moving along a route from γ_1 to γ_2 is $\frac{\lambda(1-P_{turn})}{2}$, where $0 \le P_{turn} \le 1$, as shown in this table. In the same manner, we show the density of nodes moving along other routes in Table 1. From this assumption, the density of nodes moving from γ_1 to γ_4 is $\frac{\lambda}{2}$ and that moving from γ_4 to γ_1 is also $\frac{\lambda}{2}$. Therefore, if we observe the flow of mobile nodes moving along I_1, I_2, ..., I_{n_c}, this flow is almost the same as that in Model 2 except for the fact that some nodes leave or enter I_1, I_2, ..., I_{n_c} in Model 3. Hence, by comparing Models 2 and 3, we can observe effects of the change of direction of nodes. In the following, we consider $E(T_{w,3})$ in Model 3 using the same definitions of variables by considering a sequence of I_1, I_2, ..., I_{n_c} in Model 3 as the street in Model 2.

If $Y_k > a$ in Model 2, then $V_M = v$ because the topology of the network consisting of nodes of N_w does not change. On the other hand, in Model 3, a mobile node turns at each corner as mentioned. Then, topology of the network changes with time more dynamically than those in Model 2. Even if $Y_k > a$ in Model 3, Y_{k+1} can be smaller than or equal to a. This is a difference between Models 2 and 3. Therefore, although we assume that $P(B_{k+1}|B_k) = 1$ in Model 2, we eliminate this assumption in Model 3 and suppose that

$$P(B_k) = P(Y_k > a) + P(Y_k \le a) \times P(C_k|Y_k \le a)P\left(T_{walk,k} > \frac{a-r}{v}\right) \quad (19)$$

for $k = 1, 2, ..., n_c - 1$, and

$$P(B_{n_c}) = P(C_{n_c})P\left(T_{walk,n_c} > \frac{a-r}{v}\right). \quad (20)$$

We compute the probabilities in the right terms of these equations in the same manner as in Model 2.

Also, a mobile node having M does not always move toward the direction of D. In this case, M must be carried along a redundant path. For example, in Fig. 7(b), u_1, which has M and moves toward D, changes its direction at c_k. In this example, u_2 receives M from u_1 and moves toward D, and u_3, which has M and follows u_1, also moves toward D. Then, u_2 and u_3 can play the same role as u_1. However, the length of an interval of time when u_1 arrives at c_k to the time when u_2 or u_3 arrives at c_k must be added to the waiting time in this case. This detour of M is another difference between Models 2 and 3.

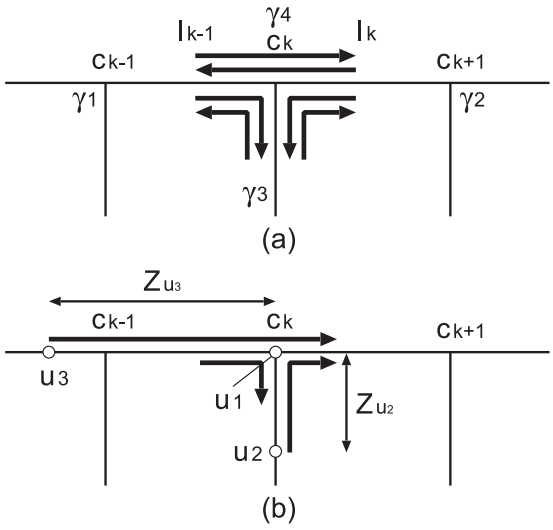


Fig. 7 Flows of mobiles in Model 3.

Table 1 Density of nodes of each flow.

Direction	Density of nodes
$\gamma_1 \to \gamma_2$	$\frac{\lambda(1-P_{turn})}{2}$
$\gamma_1 \to \gamma_3$	$\frac{\lambda P_{turn}}{2}$
$\gamma_2 \to \gamma_1$	$\frac{\lambda(1-P_{turn})}{2}$
$\gamma_2 \to \gamma_3$	$\frac{\lambda P_{turn}}{2}$
$\gamma_3 \to \gamma_1$	$\frac{\lambda P_{turn}}{2}$
$\gamma_3 \to \gamma_2$	$\frac{\lambda P_{turn}}{2}$

To take into account these effects, we consider Fig. 7 again. Suppose that u_1, which has M, is at the corner c_k and that there is no node that has M in $I_k, I_{k+1}, \ldots$. Suppose that u_2 moves toward c_{k+1} through c_k after passing u_1 and that u_3, which follows u_1, moves toward c_{k+1} through c_k. Then, u_2 must receive M from u_1. For simplicity, we assume that u_3 also receives M before arriving at c_k from nodes that received M although u_3 may not able to receive M. Then, u_2 and u_3 can move M in I_k by walking or forwarding instead of u_1. Hence, we add the mean length of an interval from the time when u_1 arrives at c_k to the time when u_2 or u_3 arrives at c_k to Eqs. (16) and (17) after substituting Eqs. (19) and (20) into these equations, and use the result of this addition as $E(T_{w,3,k})$ in Model 3. Let Z_{u_2} and Z_{u_3} be the distances from u_1 to u_2 and u_3 when u_1 is at c_k, respectively. Z_{u_2} is an exponential random variable with intensity $\frac{\lambda P_{turn}}{2}$, and Z_{u_3} is an exponential random variable with intensity $\frac{\lambda(1-P_{turn})}{2}$ as can be seen from Table 1. Then, we approximately compute $E(T_{w,3})$ in Model 3 as

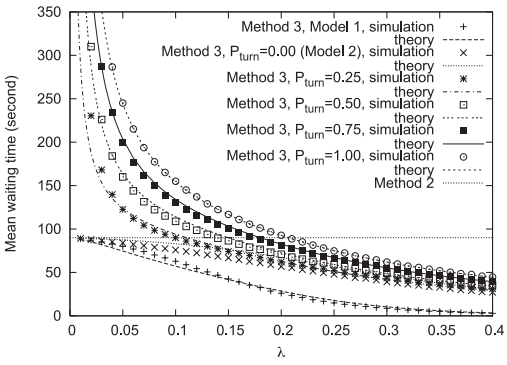
$$E(T_{w,3}) = E(T_{w,3,1}) + \sum_{k=2}^{n_c} \left\{ E(T_{w,3,k}) + P_{turn} \frac{E(\min(Z_{u_2}, Z_{u_3}))}{v} \right\}, \quad (21)$$

where $E(T_{w,3,k})$ and $E(T_{w,3,n_c})$ are computed by Eqs. (16) and (17), respectively, after substituting $P(B_k)$ for Model 3 into these equations and $E(\min(Z_{u_2}, Z_{u_3})) = \frac{2}{\lambda}$.

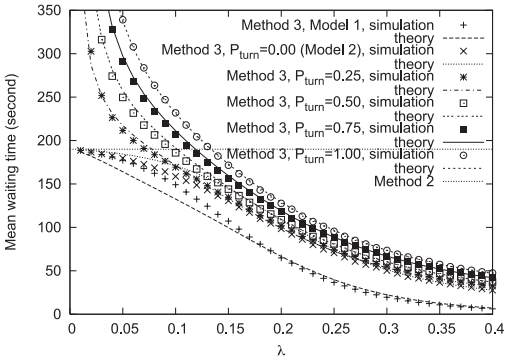
We show the numerical results of $E(T_{w,3})$ in Model 3 for $P_{turn} = 0$, 0.25, 0.5, 0.75 and 1 in Fig. 8 together with simulation results. We assume that $L = 100$ m and $L = 200$ m in Figs. 8(a) and (b), respectively, and that $n_c = 5$. From the results, we can see that the numerical results agree with the simulation results. If we compare Fig. 6 and Fig. 8, we can see that, in Model 3, the numerical results agree with the simulation results better than in Model 2. One reason for this result is as follows: $E(T_{w,3,k})$ deeply depends on $E(T_{w,3,k-1})$ in Model 2 as explained to compute $P(B_k)$ in Model 2, and difficulty in analysis of this dependence causes the difference between numerical results and simulation results in Model 2. On the other hand, in Model 3, the distributions of nodes in I_{k-1} and I_k are less dependent than those in Model 2 because of inflow and outflow of nodes at c_k. As a result, a simple approximation analysis in Model 3 provides better results than in Model 2.

From Fig. 8, $E(T_{w,3})$ increases as λ decreases. $E(T_{w,3})$ increases as P_{turn} increases for the following reasons. Here, a situation where $P_{turn} = 0$ means that every node moves along a line from c_1 to c_{n_c+1} without changing its direction; namely this situation is identical to Model 2. A situation where $P_{turn} = 1$ means that there is no node moving along a line from c_1 to c_{n_c+1} without changing its direction; namely this situation causes detours of M at intersections most frequently. Then, we can observe the above characteristic.

In Fig. 8, we plot $E(T_{w,2})$ given that S successfully delivers M to D by itself although Method 2 cannot always deliver M to D in Model 3 because S does not always arrive at D because of change of direction. Namely, $E(T_{w,2}) = \frac{L-r}{v}$.



(a) $L = 100$m.



(b) $L = 200$m.

Fig. 8 Numerical results of $E(T_{w,3})$ in Model 3 for $P_{turn} = 0$, 0.25, 0.5, 0.75 and 1.0, where $n_c = 5$.

In this case, the difference between $E(T_{w,3})$ for $P_{turn} > 0$ and that for $P_{turn} = 0$ corresponds the increase of $E(T_{w,3})$ caused by the detours of M. Due to the increase of $E(T_{w,3})$ caused by the detours, $E(T_{w,3})$ is sometimes greater than $\frac{L-r}{v}$. As mentioned, however, Method 3 can deliver M to D with the mean waiting time depicted in Fig. 8 as opposed to Method 2, which cannot always deliver M to D. This property is also an advantage of Method 3.

5. Discussions on Results in Models 2 and 3

In the analysis of $E(T_{w,3})$ in Model 2, we assumed that B_{k+1} strongly depends on B_k for $k = 1, 2, ..., n_c - 1$. Furthermore, we approximately set $P(B_{k+1}|B_k)$ to be 1 as an extreme approximation. However, the numerical results well describe the tendency of $E(T_{w,3})$. Hence, we can confirm the validity of our analysis. Then, this result indicates that in Model 2, forwarding rarely occurs in intervals I_{k+1}, I_{k+2}, ... once forwarding does not occur in I_k because of the dependence between B_k and B_{k+1}. Therefore, we can see that the dependence between B_k and B_{k+1} is an important factor that should be considered in the analysis of $E(T_{w,3})$ in Model 2.

In contrast, in Model 3, we eliminated the above assumption on the dependence between B_k and B_{k+1}, and assumed that B_{k+1} is independent from B_k for $k = 1, 2, ..., n_c - 1$ because it is expected that inflow and outflow of mobile nodes weaken the dependence. In addition to this assump-

tion, in Model 3, we added $P_{turn}\frac{E(\min(Z_{u_2},Z_{u_3}))}{v}$ to $E(T_{w,3,k})$ for $k = 2, 3, ..., n_c$ as a negative factor of inflow and outflow of mobile nodes. From comparison between numerical results of the analysis and simulation results, we can see that our analysis well approximates $E(T_{w,3})$ in Model 3. This result indicates that we have two important factors in Model 3. One is that we can consider that B_k and B_{k+1} are independent for $k = 1, 2, ..., n_c - 1$. The other is that $P_{turn}\frac{E(\min(Z_{u_2},Z_{u_3}))}{v}$ increases the mean waiting time. Also, we can analyze $E(T_{w,3})$ in Model 3 more simply than in Model 2 because we can compute $E(T_{w,3,k})$ independently.

The above results give some insights to know what factors affect $E(T_{w,3})$ in Models 2 and 3. Furthermore, the next step of this research will be understanding behavior of $E(T_{w,3})$ in more complicated networks like lattice street networks. To analyze $E(T_{w,3})$ in the complicated networks, we need to consider more complicated problems; however, it is expected that network structures in Models 2 and 3 will appear as parts of such complicated networks, and the above facts that indicate what affects $E(T_{w,3})$ in Models 2 and 3 will play a fundamental role to understand behavior of $E(T_{w,3})$ in the complicated networks, and the equations derived in this paper will be applied to the analysis of $E(T_{w,3})$ in the complicated networks.

6. Conclusions

In this paper, we considered delivery of a message M from a source S to a destination D, and analyzed the mean of the minimum waiting time needed for delivery of M. First, we analyzed the mean waiting time in a one-dimensional network consisting of mobile nodes moving along a line. We proposed an approximate method to compute the mean minimum waiting time by using the velocity of flow of M along the line, and showed that this method gave a good approximation to the mean of the minimum waiting time. Second, we modified the approximate method to compute the mean waiting time in a case where shadowing occurs at corners. Using the modified method, we showed how much the mean waiting time is increased by shadowing. Third, we considered another case where mobile nodes change their directions at the corners in addition to shadowing, and approximately analyzed the mean of the minimum waiting time in this case. From the results of this analysis, we showed how much the mean waiting time is increased by the change of direction at the corners. Also, from the results of theoretical analyses, we showed what factors affect the mean waiting time in the last two cases. Extensions of the approximate methods to other models are considered as future works.

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Appendix A: X_{fwd}

The mean of $x(u_{k+1},\tau_b) - x(u_k,\tau_b)$ is equal to $\frac{1}{\lambda} - \frac{r}{e^{\lambda r}-1}$ for $k = 0, 1, ..., M_1 - 1$, because $x(u_{k+1},\tau_b) - x(u_k,\tau_b)$ is exponentially distributed with intensity λ given that it is not longer than r. Then

$$\begin{aligned} &E(X_{fwd,w}) \\ &= \sum_{m_2=0}^{\infty}\sum_{m_1=m_2+1}^{\infty} P(M_1 = m_1, M_2 = m_2 | M_1 > M_2) \\ &\quad \times \left\{ r + (m_1 - m_2)\left(\frac{1}{\lambda} - \frac{r}{e^{\lambda r}-1}\right)\right\} \\ &= \frac{e^{\lambda r}}{\lambda} - \frac{r}{e^{\lambda r}-1}, \end{aligned} \tag{A·1}$$

where $P(M_1 = m_1, M_2 = m_2 | M_1 > M_2)$ can be computed from

$$P(M_1 = m_1) = (1 - e^{-\lambda r})^{m_1} e^{-\lambda r}, \tag{A·2}$$

$$P(M_2 = m_2 | M_1 = m_1) = \begin{cases} \frac{1}{2^{m_2+1}}, & (m_2 \le m_1 - 1), \\ \frac{1}{2^{m_1}}, & (m_2 = m_1), \end{cases} \tag{A·3}$$

$$\begin{aligned} &P(M_1 > M_2) \\ &= \sum_{m_2=0}^{\infty}\sum_{m_1=m_2+1}^{\infty} P(M_1 = m_1, M_2 = m_2) \\ &= \frac{e^{\lambda r} - 1}{e^{\lambda r} + 1}. \end{aligned} \tag{A·4}$$

From the above equations,

$$\Delta X_{fwd} = \frac{e^{\lambda r} - 1 - \lambda r}{\lambda(e^{\lambda r} + 1)}. \tag{A·5}$$

An inequality $\Delta X_{fwd} \leq r$ can be rewritten as

$$h(\lambda) = \lambda r e^{\lambda r} + 2\lambda r - e^{\lambda r} + 1 \geq 0. \quad (\text{A}\cdot 6)$$

This holds for $\lambda \geq 0$ because $h(0) = 0$ and $\frac{dh(\lambda)}{d\lambda} = \lambda r^2 e^{\lambda r} + 2r \geq 0$ if $\lambda \geq 0$. Therefore, $\Delta X_{fwd} \leq r$ for $\lambda \geq 0$.

Appendix B: Density Function of Y in Model 1

For $y \leq r$, we have $f_Y(y|C) = 0$ obviously. From Bayes' theorem [10],

$$f_Y(y|C) = \frac{P(C|Y = y) f_Y(y)}{\int_0^\infty P(C|Y = y) f_Y(y) dy}. \quad (\text{A}\cdot 7)$$

In Model 1, $f_Y(y) = \frac{\lambda}{2} e^{-\frac{\lambda}{2} y}$ because Y is an exponential random variable with intensity $\frac{\lambda}{2}$. Also,

$$P(C|Y = y) = 1 - P_c\left(y, \frac{\lambda}{2}, r\right), \quad (\text{A}\cdot 8)$$

where $P_c\left(y, \frac{\lambda}{2}, r\right)$ is the probability that w_j and w_{j+1} are connected by a multi-hop path consisting of nodes of N_b given that $Y = y$, and can be computed by Eq. (A·1) in [1]. From these equations, we can compute $f_Y(y|C)$ for $y \geq r$. Also, we can compute $P(C)$ as

$$P(C) = \int_0^\infty P(C|Y = y) f_Y(y) dy. \quad (\text{A}\cdot 9)$$

Appendix C: Density Function of $T_{walk,k}$

For $k = 1, 2, ..., n_c - 1$,

$$f_{T_{walk,k}}(t) = \int_{-\infty}^{\infty} f_{T_{walk}}(t|Y_k = y) f_{Y_k}(y|Y_k \leq a, C_k) dy. \quad (\text{A}\cdot 10)$$

$$f_{T_{walk,n_c}}(t) = \int_{-\infty}^{\infty} f_{T_{walk}}(t|Y_{n_c} = y) f_{Y_{n_c}}(y|C_{n_c}) dy. \quad (\text{A}\cdot 11)$$

For $y \leq r$ or $y > a$, we have $f_{Y_k}(y|Y_k \leq a, C_k) = 0$ obviously. By Bayes' theorem and some calculations,

$$f_{Y_k}(y|Y_k \leq a, C_k) = \frac{P(C_k|Y_k = y) f_{Y_k}(y)}{\int_0^a P(C_k|Y_k = y) f_{Y_k}(y) dy} \quad (\text{A}\cdot 12)$$

for $k = 1, 2, ..., n_c - 1$. As $f_{Y_{n_c}}(y|C_{n_c})$, we can use Eq. (A·7). Also, we can compute $P(C_k|Y_k = y)$ by Eq. (A·8) for $k = 1, 2, ..., n_c$. Also, for $k = 1, 2, ..., n_c - 1$,

$$P(C_k|Y_k \leq a) = \int_0^a P(C_k|Y_k = y) f_{Y_k}(y|Y_k \leq a) dy. \quad (\text{A}\cdot 13)$$

$$P(C_{n_c}) = \int_0^\infty P(C_{n_c}|Y_{n_c} = y) f_{Y_{n_c}}(y) dy. \quad (\text{A}\cdot 14)$$

$f_{Y_1}(y)$ is identical to $f_Y(y)$ for Model 1; however, $f_{Y_k}(y)$ is slightly different from $f_Y(y)$ for $k = 2, 3, ..., n_c$. Consider Fig. A·1, which shows a typical situation in Model 2. In this figure, w_{j_k} is $n^t_{h,w}$ and has to walk until arriving at c_k because there is no node of N_w between w_{j_k} and c_k. Then, the distance between w_{j_k} and c_k affects Y_k in Model 2. Let $Y_{k,1}$ and $Y_{k,2}$ be the distance between w_{j_k} and c_k and that between c_k and w_{j_k+1} in the situation of the above figure, respectively. Then, Y_k is $Y_{k,1} + Y_{k,2}$. We assume that $Y_{k,1}$ and $Y_{k,2}$ are exponential random variables with intensity $\frac{\lambda}{2}$. Then, the density function of $Y_{k,1}$ is $f_{Y_{k,1}}(y) = \frac{\frac{\lambda}{2} e^{-\frac{\lambda}{2} y}}{1 - e^{-\frac{\lambda}{2} a}}$ because $Y_{k,1}$ must be smaller than or equal to a because w_{j_k} is in I_{k-1}. Also, the density function of $Y_{k,2}$ is $f_{Y_{k,2}}(y) = \frac{\lambda}{2} e^{-\frac{\lambda}{2} y}$. Then,

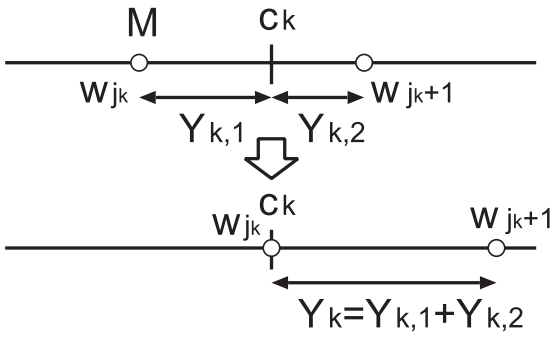

Fig. A·1 Relation between w_{j_k} and w_{j_k+1} in Model 2.

$$f_{Y_k}(y) = f_{Y_{k,1}+Y_{k,2}}(y) = \frac{\lambda^2 y e^{\frac{\lambda}{2}(a-y)}}{4\left(e^{\frac{\lambda}{2}a} - 1\right)}. \quad (\text{A}\cdot 15)$$

Note that $E(Y_{k,1}) = \frac{2}{\lambda} - \frac{a}{e^{\frac{\lambda a}{2}} - 1}$ for $2 \leq k \leq n_c$.

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