

Characterization of Minimum Route MTM in One-Dimensional Multi-Hop Wireless Networks

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SUMMARY In multi-hop wireless networks, since source and destination nodes usually have some candidate paths between them, communication quality depends on the selection of a path from these candidates. For network design, characterizing the best path is important. To do this, in [1], [2], we used expected transmission count (ETX) as a metric of communication quality and showed that the best path for ETX is modeled by a path that consists of links whose lengths are close to each other in static one-dimensional multi-hop networks with a condition that the ETX function of a link is a convex monotonically increasing function. By using the results of this characterization, a minimum route ETX can be approximately computed in a one-dimensional random network. However, other metrics fail to satisfy the above condition, like medium time metric (MTM). In this paper, we use MTM as a metric of communication quality and show that we cannot directly apply the results of [1], [2] to the characterization of the best path for MTM and the computation of minimum route MTM. In this paper, we characterize the path that minimizes route MTM in a different manner from [1], [2] and propose a new approximate method suitable for the computation of minimum route MTM.

key words: multi-hop wireless networks, medium time metric, theoretical analysis

1. Introduction

In multi-hop wireless networks [3], [4], source node S can send a packet to destination node D through a multi-hop path consisting of other nodes. Since there may be several candidates for multi-hop paths that connect S and D, choosing a path with high quality from these candidates is important. Communication quality can be evaluated by various metrics, including per-hop round trip time (RTT) [5], expected transmission count (ETX) [6], medium time metric (MTM) [7], and expected transmission time metric (ETT) [8]. For example, the ETX of a link (link ETX) is defined as the expected number of transmissions required to successfully deliver a packet through the link, and the ETX of a path (route ETX) is defined as the sum of the ETXs of all links in the path. $f_1(z)$, $f_2(z)$, and $f_3(z)$ in Fig. 1 are examples of link ETX functions. Here, $f_1(z)$ is derived by assuming mica2 mote as a wireless node. As explained in [1], [2], the link ETX function can be modeled as a convex monotonically increasing function of the length of a link. MTM, which is used for a

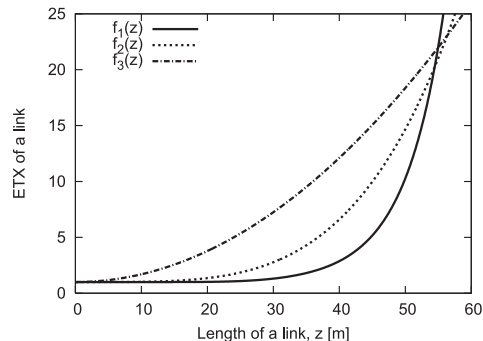


Fig. 1 Examples of ETX functions.

Table 1 Relation between transmission rate, maximum length of a link, and link MTM [7].

Transmission rate [Mbps]	Maximum length of a link [m]	Link MTM [μ sec]
11.0	26.3	2542
5.5	35.1	3673
2.0	44.2	7634
1.0	52.5	13858

different purpose from ETX, is a metric for multi-rate environments while ETX is for single-rate environments. The MTM of a link (link MTM) is defined as the medium time consumed in the link to send a packet through the link, and the MTM of a path (route MTM) is defined as the sum of the MTMs of all links in the path. For example, for IEEE 802.11b, we can use four transmission rates: 1 M, 2 M, 5.5 M, and 11 Mbps. In [7], the authors evaluated the maximum length of a link and link MTM for each rate. Table 1 shows these relations. From this table, we can represent link MTM as a nondecreasing step-like function of the length of a link, assuming that we always use the highest rate among the available rates. $f_4(z)$, $f_5(z)$, and $f_6(z)$ in Fig. 2 are examples of the MTM functions. $f_4(z)$ is derived from the relation in Table 1. $f_5(z)$ and $f_6(z)$ are defined as nondecreasing step-like functions that have characteristics different from $f_4(z)$ as will be explained in Sect. 4.2. In this paper, we call a path that minimizes its ETX or its MTM the best path. We call the ETX of the best path *minimum route ETX* and the MTM of the best path *minimum route MTM*.

For the design of networks, characterizing the best path is important; however, if nodes are randomly distributed, it is difficult to theoretically identify the best path and pre-

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cisely analyze minimum routes ETX and MTM. In [2], we considered a static one-dimensional network where nodes are located at constant intervals and mathematically showed that the best path for ETX consists of links whose lengths are close to each other in the network. Also in [1], we considered a static one-dimensional network where nodes are randomly distributed and showed that route ETX is approximately minimized by making the length of all links in the path close to a certain constant. Based on this result, we theoretically analyzed the mean value of minimum route ETX in a one-dimensional random network in [1]. Furthermore, in [2], we characterized the best path for ETX in two-dimensional lattice and random networks. In this characterization, we utilized the above property indicating that the best path for ETX consists of links whose lengths are close to each other in one-dimensional networks. However, these analyses in [1], [2] assume that a link ETX function is a convex monotonically increasing function, so the same results cannot always be applied to the analysis of minimum route MTM, since a link MTM function is not always a convex monotonically increasing function, as shown in the above example. In fact, the best path for MTM can consist of links whose lengths are not close to each other, as shown in Sect. 3.

With these things as background, in this paper, we characterize the best path for MTM in a static one-dimensional network where nodes are randomly distributed as the first step of characterization of the best path for MTM. For the characterization, we consider two policies for path-selection to approximate the best path. For Policy 1, we determine a reference length and construct a path using links whose lengths are close to the reference length. This policy was used to approximate the best path for ETX in [1]. Policy 2 constructs a path by selecting links whose MTM per length is minimized. We compare the characterization of the best path for MTM using Policy 2 with Policy 1. Through the above comparisons by theoretical considerations and simulation results, we show that Policy 2 can approximate the best path better than Policy 1 while MTM is used. We also show how to theoretically compute the mean value of route MTM for Policy 2. Finally, we show that this mean value well approximates the mean value of minimum route MTM in a one-dimensional random network.

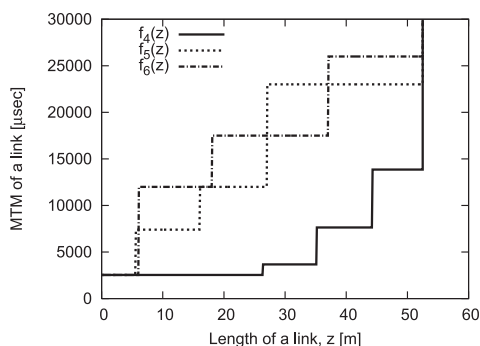


Fig. 2 Examples of MTM functions.

The rest of this paper is organized as follows. In Sect. 2, we explain the definitions and assumptions used in this paper. In Sect. 3, we explain the differences between the constructions of the best paths for ETX and MTM in a one-dimensional network where nodes are located at constant intervals. In Sect. 4, we consider two path-selection policies and model the best path for MTM with them in a one-dimensional random network. In Sect. 5, we theoretically analyze the mean value of minimum route MTM using Policy 2. Sect. 6 concludes this paper.

2. Definitions and Assumptions

In this paper, we characterize the best path for MTM in static one-dimensional multi-hop networks. Let S and D be source and destination nodes, respectively. Let L be the distance between S and D. Suppose that N nodes are distributed between S and D. Denote the $N + 2$ nodes including S and D by v_0, v_1, \dots, v_{N+1} , where $v_0 = S$ and $v_{N+1} = D$. For $i = 0, 1, \dots, N + 1$, let X_i be the coordinate of v_i . Suppose that $X_0 = 0 \leq X_1 \leq \dots \leq X_{N+1} = L$. In this paper, we consider two kinds of networks: a one-dimensional regular and a one-dimensional random. In the regular network, nodes are located at constant intervals with distance a , where a is a positive real number and $\frac{L}{a}$ is an integer. On the other hand, in the random network, nodes are distributed based on a Poisson process with intensity λ . Figures 3 and 4 show examples of these networks. Let d be the maximum transmitting range. Two nodes can communicate with each other if and only if the distance between them is not greater than d . Let $u(z)$ be ETX or MTM of a link with length z . Assume that $u(z)$ is a convex monotonically increasing function where ETX is used as a metric, and also assume that $u(z)$ is a nondecreasing step-like function, where MTM is used as a metric. Let R be the set of all paths between S and D. Let $U(r)$ be route ETX or route MTM of path $r \in R$. Let r_O be the best path. Minimum routes ETX and MTM are represented as $U(r_O)$. Define Optimum Routing as a routing method that minimizes routes ETX or MTM. Also, we define d_0 as the value of z to minimize $\frac{u(z)}{z}$ because d_0 is an important parameter to characterize r_O .

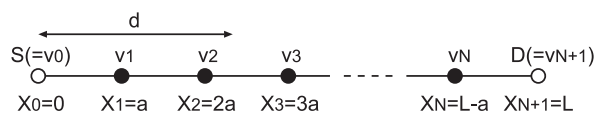


Fig. 3 One-dimensional regular network.

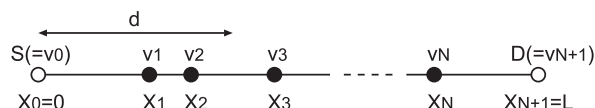


Fig. 4 One-dimensional random network.

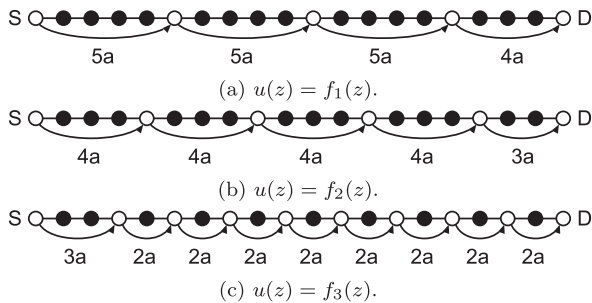


Fig. 5 Examples of best paths for ETX, where $a = 5.25$ m, $d = 10a = 52.5$ m, and $L = 19a = 99.75$ m.

3. The Best Path in a One-Dimensional Regular Network

In this section, we briefly explain the characterization of the best path for ETX in the regular network in [2] and show that this characterization cannot be always applied for MTMs.

In [2], we proved that if $u(z)$ is a convex monotonically increasing function, then there are at most two kinds of link lengths in the best path, and the difference between them is a in a one-dimensional regular network. This means that the best path for ETX consists of links whose lengths are close to each other. Figures 5(a), (b), and (c) show the best path for link ETX functions $f_1(z)$, $f_2(z)$, and $f_3(z)$ in Fig. 1, respectively, where $a = 5.25$ m, $d = 10a = 52.5$ m, and $L = 19a = 99.75$ m. In these figures, we confirmed that the best paths satisfy the above condition; the difference of the link length between any pair of links included in the best path is always less than or equal to a . We also proved in [2] that if the number of hops is given, then only one path satisfies the above condition except for the difference of the order of links. Specifically, if the number of hops is k , then the path consists of $k - \lfloor \frac{L}{a} \rfloor + k \lfloor \frac{L}{ka} \rfloor$ links with lengths $\lfloor \frac{L}{ka} \rfloor a$ and $\frac{L}{a} - k \lfloor \frac{L}{ka} \rfloor$ links with lengths $\lfloor \frac{L}{ka} \rfloor a + a$. Clearly, the number of hops of the best path is not greater than $\frac{L}{a}$; therefore, we can limit the best path candidates to at most $\frac{L}{a}$ paths only under the condition that $u(z)$ is a convex monotonically increasing function. From this property, we can easily characterize the best path for ETX in a one-dimensional regular network. Furthermore, in [1], we showed that an approximate method called Adjustable Routing, which constructs a path in which the length of each link is as close to a reference length as possible, well describes Optimum Routing even in a one-dimensional random network. This fact is supported by the above property in a regular network.

On the other hand, we have the following property:

Property 1. *If $u(z)$ is a nondecreasing step-like function, then for any path r , there exists $u(z)$ such that $r = r_O$.*

The proof is provided in Appendix A. This property means that the difference of the link length between any pair of links in the best path is not always less than or equal to a if

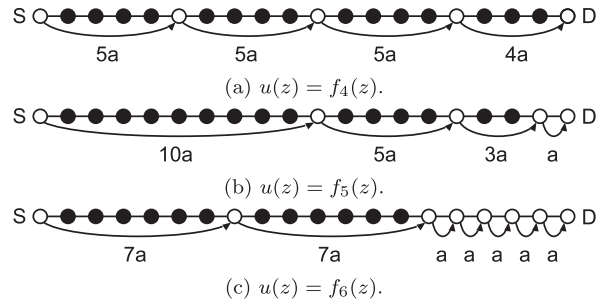


Fig. 6 Examples of best paths for MTM, where $a = 5.25$ m, $d = 10a = 52.5$ m, and $L = 19a = 99.75$ m.

a nondecreasing step-like function is used like $u(z)$. Figures 6(a), (b), and (c) show the best path for link MTM functions $f_4(z)$, $f_5(z)$, and $f_6(z)$ in Fig. 2, respectively, where $a = 5.25$ m, $d = 10a = 52.5$ m, and $L = 19a = 99.75$ m. These paths are algorithmically calculated by the Dijkstra algorithm [9], which solves shortest path problems. Although the difference between the link lengths is not longer than a for $f_4(z)$, it is sometimes greater than a for $f_5(z)$ and $f_6(z)$. For these reasons, Adjustable Routing is not an appropriate method to approximate Optimum Routing if MTM is used, so we need another way to characterize the best path for MTM. Hence, in the following section, we define a new policy called Policy 2 to approximate Optimum Routing while the policy used in Adjustable Routing is called Policy 1, and compare the approximate performances of these policies.

4. Characterization of the Best Path for MTM in a One-Dimensional Random Network

4.1 Policies 1 and 2

Policy 1 is the same as the Adjustable Routing mentioned above. It makes the lengths of all links in the path close to d_s , which is a reference link length. Policy 1 selects a path as follows:

Policy 1: Let r_{P1} be the path selected by Policy 1. Determine constant d_s from interval $[0, d]$. For $i = 0, 1, \dots, N$, let t_i be the number of nodes in interval $(X_i, X_i + d_s]$. Suppose that v_i is S or a relay node selected by this policy. If $t_i \geq 1$, v_{i+t_i} is selected as the next relay node of v_i , and v_{i+1} is selected as the next relay node of v_i if $t_i = 0$.

Policy 2 makes link MTM per link length as small as possible for every link in the path as follows:

Policy 2: Let r_{P2} be the path selected by Policy 2. For $i = 0, 1, \dots, N$, let t'_i be the number of nodes in interval $(X_i, X_i + d]$. Suppose that v_i is S or a relay node selected by this policy. For integer $m' = \operatorname{argmin}_{i+1 \leq m \leq i+t'_i} \frac{u(X_m - X_i)}{X_m - X_i}$, $v_{m'}$ is selected as the next relay node of v_i .

4.2 Properties of Policies 1 and 2

In this subsection, we explain that Policy 2 is expected to approximate Optimum Routing better than Policy 1 if $u(z)$ is a nondecreasing step-like function because MTM is used. For real number $c \geq 1$, we define the set of positive real numbers $I(c) \subseteq [0, d]$ as follows:

$$I(c) = \left\{ z \in [0, d] \mid u(z) \leq c \frac{u(d_0)}{d_0} z \right\}. \quad (1)$$

$I(c)$ monotonically increases with c . If $c_1 \leq c_2$, then $I(c_1) \subseteq I(c_2)$. Also, $I(c)$ is represented as the union of some intervals. Let $\eta(c)$ be the number of intervals in $I(c)$. Figures 7, 8, 9, and 10 show the relations between $I(c)$ and c for $f_1(z)$, $f_4(z)$, $f_5(z)$, and $f_6(z)$, respectively, where $d = 52.5$ m. In these figures, the horizontal axis is c , and the vertical axis means $z \in [0, d]$. The gray region is the set of (c, z) such that $z \in I(c)$, and the white region is the set of (c, z) such that $z \notin I(c)$. For example, if $c = 2$ in Fig. 8, then $I(c)$ consists of the union of the two intervals of z , namely, $[z_1, z_2]$ and $[z_3, z_4]$, and does not include three intervals $[0, z_1)$, (z_2, z_3) , and $(z_4, 52.5]$, where $z_1 = 13.15$ m, $z_2 = 35.1$ m, $z_3 = \frac{26.3 \times 7634}{2542 \times 2}$ m ≈ 39.5 m, and $z_4 = 44.2$ m. We can see that $\eta(c) = 1$ for any c in Fig. 7, and $1 \leq \eta(c) \leq 4$ for any c in Figs. 8, 9, and 10.

We have the following properties for $I(c)$ and $\eta(c)$:

Property 2. Consider k -hop path r . Suppose that the lengths of the links in r are Y_1, Y_2, \dots, Y_k . If $Y_i \in I(c)$ for $i = 1, 2, \dots, k$, then $U(r) \leq cU(r_0)$.

Property 3. If $u(z)$ is a convex monotonically increasing function, then $\eta(c) = 1$ for any $c \geq 1$.

Property 4. Suppose that we set d_s to $\max_{z \in I(c)} z$ in Policy 1.

Suppose candidates for the next link whose lengths are included in $I(c)$. Then for $\eta(c) = 1$, Policy 1 always selects the next link from these candidates, even if there are some candidates for the next link whose lengths are not included in $I(c)$ while Policy 1 may select a link whose length is not included in $I(c)$ for $\eta(c) \geq 2$.

Property 5. Suppose candidates for the next link whose lengths are included in $I(c)$. Then Policy 2 always selects the next link from these candidates, even if there are some candidates for the next link whose lengths are not included in $I(c)$.

The proofs of Properties 2 and 3 are provided in Appendices B and C, respectively, and Properties 4 and 5 can be easily derived from the definitions of Policies 1 and 2, respectively.

From Property 2, we can make $U(r)$ of path r at most $cU(r_0)$ while constructing a path using links whose lengths are included in $I(c)$. Of course, c is desired to be as small as possible so that $U(r)$ is close to $U(r_0)$. As mentioned, however, $|I(c)|$ becomes smaller as c becomes smaller. For a

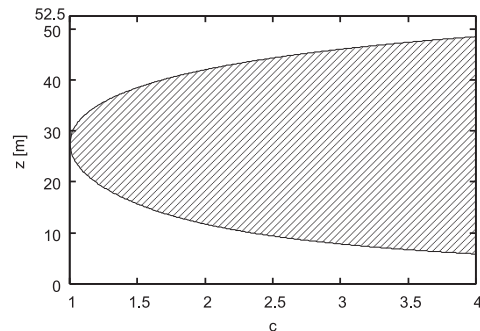


Fig. 7 $I(c)$ for $u(z) = f_1(z)$, where $d = 52.5$ m.

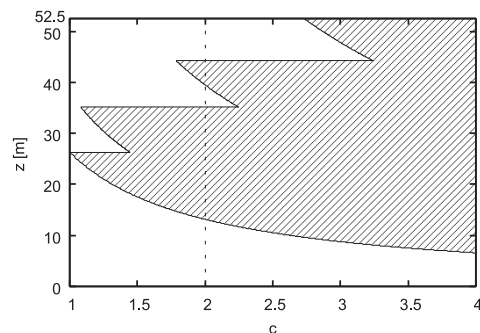


Fig. 8 $I(c)$ for $u(z) = f_4(z)$, where $d = 52.5$ m.

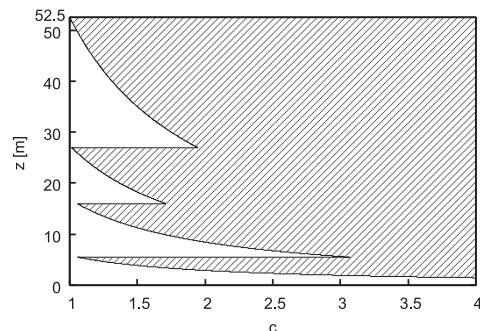


Fig. 9 $I(c)$ for $u(z) = f_5(z)$, where $d = 52.5$ m.

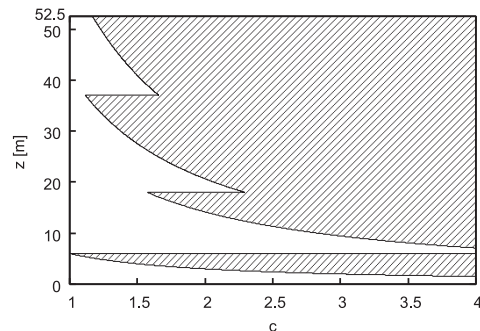


Fig. 10 $I(c)$ for $u(z) = f_6(z)$, where $d = 52.5$ m.

small c , therefore, while using approximate policies to select the next link, we cannot always select a link whose length is included in $I(c)$ as the next link. Furthermore, we can consider various approximate policies to choose the next link

like Policies 1 and 2, and all of these policies do not always consider $I(c)$; therefore, if we use a policy without considering $I(c)$, we may choose a link whose length is not included in $I(c)$.

Suppose candidates for the next link whose lengths are included in $I(c)$. In this case, Policy 2 can automatically choose a link whose length is included in $I(c)$, as seen from Property 5. On the other hand, Policy 1 does not always choose a link whose length is included in $I(c)$, although it automatically chooses a link whose length is included in $I(c)$ if we set $d_s = \max_{z \in I(c)} z$ only if $\eta(c) = 1$, as seen from Property 4.

From these facts, Policy 2 is expected to always well approximate Optimum Routing for both of ETX and MTM. Furthermore, Policy 1 is also expected to well approximate Optimum Routing for both of ETX and MTM if there exists a small constant c_0 such that $|I(c_0)|$ is large and $\eta(c_0) = 1$. Actually, Policy 1 well approximates Optimum Routing if ETX is used as shown in [1] because there exists such a small constant c_0 because $\eta(c)$ is always equal to 1 if ETX is used, as seen from Property 3. As opposed to ETX, we sometimes encounter a situation where we do not have such a small constant c_0 if MTM is used, and it is expected that Policy 1 does not well approximate Optimum Routing in this case. Of course, however, if there exists a small constant c_0 such that $|I(c_0)|$ is large and $\eta(c_0) = 1$, then Policy 1 with $d_s = \max_{z \in I(c_0)} z$ is expected to well approximate Optimum Routing even if MTM is used.

For example, consider $I(c)$ s of $f_1(z)$, $f_4(z)$, $f_5(z)$, and $f_6(z)$, which are represented in Figs. 7, 8, 9, and 10, respectively. Note that $f_1(z)$ is a link ETX function and $f_4(z)$, $f_5(z)$, and $f_6(z)$ are link MTM functions. Suppose that $c_0 = 1.5$. Then $\eta(c_0) = 1$ and $|I(c_0)| \approx 22.8$ m for $f_1(z)$, $\eta(c_0) = 1$ and $|I(c_0)| \approx 17.6$ m for $f_4(z)$, $\eta(c_0) = 4$ and $|I(c_0)| \approx 32.6$ m for $f_5(z)$, and $\eta(c_0) = 3$ and $|I(c_0)| \approx 23.1$ m for $f_6(z)$. As mentioned, for $f_1(z)$, $\eta(c_0) = 1$ and $|I(c_0)|$ is large because $f_1(z)$ is a link ETX function. For $f_5(z)$ and $f_6(z)$, $\eta(c_0)$ is not equal to 1; therefore, it is expected that Policy 1 does not work well for $f_5(z)$ and $f_6(z)$. For $f_4(z)$, $\eta(c_0) = 1$, and $|I(c_0)|$ is not so different from that of $f_1(z)$; therefore, it is expected that Policy 1 works well for $f_4(z)$ even though $f_4(z)$ is a link MTM function.

4.3 Simulation Results of Policies 1 and 2

In this subsection, we compare Policies 1 and 2 using $U(r_{P1})$ and $U(r_{P2})$ obtained by computer simulation. In the computer simulations, we distribute nodes in interval $[0, L]$ based on a Poisson process with intensity λ . We locate S and D at coordinates 0 and L , respectively. If at least one path exists between S and D, then we construct r_{P1} , r_{P2} , and r_O and compute $U(r_{P1})$, $U(r_{P2})$, and $U(r_O)$. Otherwise, we distribute nodes again. We repeat these procedures 10000 times and compute the probability that $U(r_{P1}) \leq xU(r_O)$ and that $U(r_{P2}) \leq xU(r_O)$ for a real number $x \geq 1$. Denote these probabilities by $q_{P1}(x)$ and $q_{P2}(x)$, respectively. We evalu-

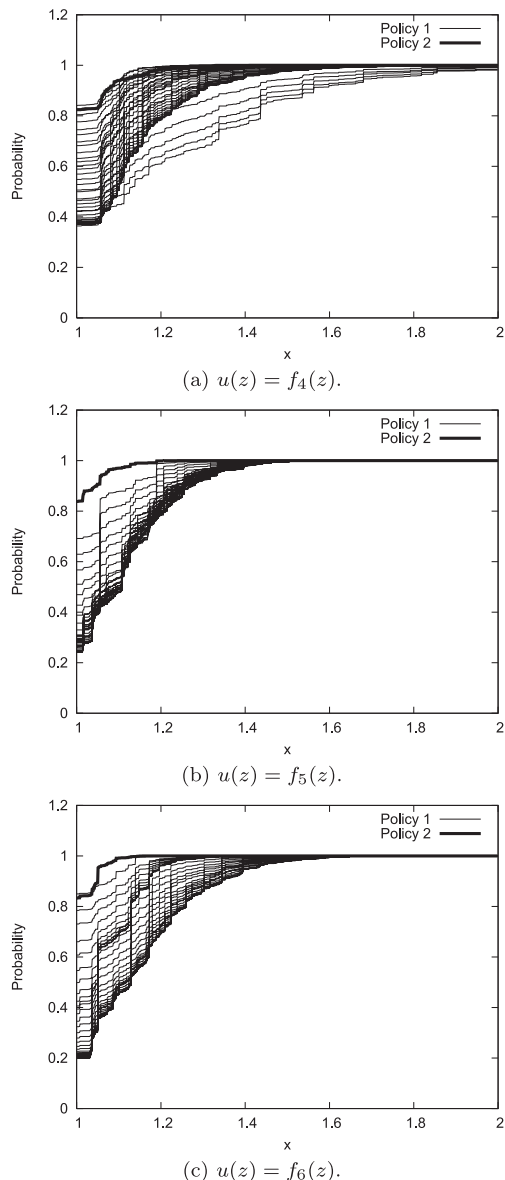


Fig. 11 Probability that $U(r_{P1}) \leq xU(r_O)$ and that $U(r_{P2}) \leq xU(r_O)$, where $d = 52.5$ m, $L = 120$ m, and $\lambda = 0.02$.

ate Policies 1 and 2 by $q_{P1}(x)$ and $q_{P2}(x)$, respectively. Note that Policy 1 well approximates Optimum Routing if $q_{P1}(x)$ is close to 1 for a small x and that Policy 2 well approximates Optimum Routing if $q_{P2}(x)$ is close to 1 for a small x .

To compare Policies 1 and 2, we use $f_4(z)$, $f_5(z)$, and $f_6(z)$ as $u(z)$. As explained in Sect. 4.2, it is expected that Policy 1 well approximates Optimum Routing for $f_4(z)$ although $f_4(z)$ is a link MTM function, that Policy 1 does not work well for $f_5(z)$ and $f_6(z)$, and that Policy 2 well approximates Optimum Routing for $f_4(z)$, $f_5(z)$, and $f_6(z)$. In the following, we show some simulation results and confirm this expectation.

Figures 11 and 12 show the simulation results of $q_{P1}(x)$ and $q_{P2}(x)$ as a function of x . In these figures, $d = 52.5$ m

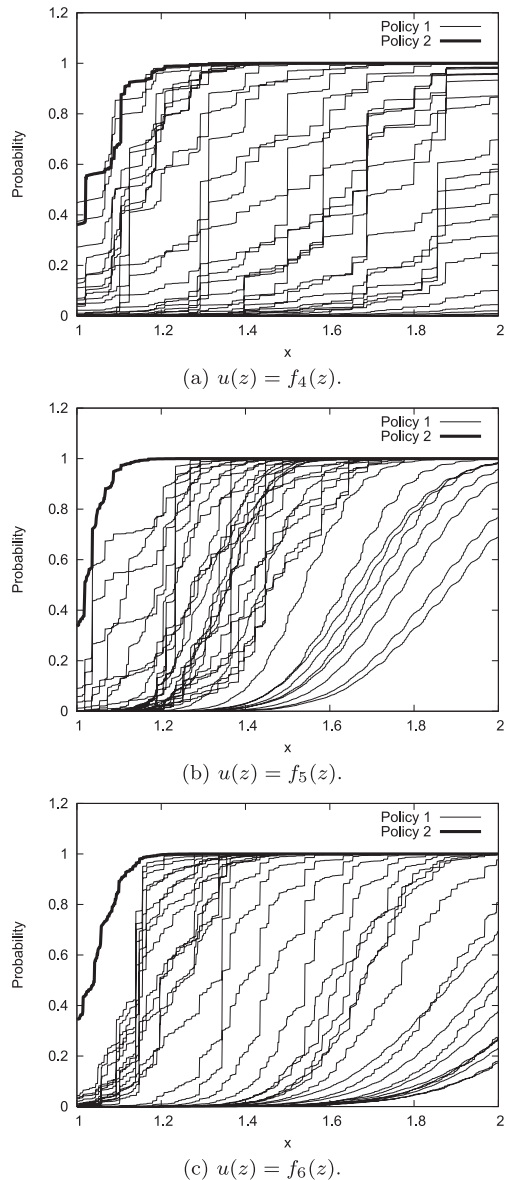


Fig. 12 Probability that $U(r_{P1}) \leq xU(r_O)$ and that $U(r_{P2}) \leq xU(r_O)$, where $d = 52.5$ m, $L = 120$ m, and $\lambda = 0.2$.

and $L = 120$ m. In Fig. 11, $\lambda = 0.02$, and $\lambda = 0.20$ in Fig. 12. $u(z) = f_4(z)$ in Figs. 11(a) and 12(a), $u(z) = f_5(z)$ in Figs. 11(b) and 12(b), and $u(z) = f_6(z)$ in Figs. 11(c) and 12(c). As d_s of Policy 1, we examine 36 values, 0 m, 1.5 m, 3 m, ..., 52.5 m for $f_4(z)$, $f_5(z)$, and $f_6(z)$, and examine 35.1 m other than the above 36 values for $f_4(z)$ because $\max_{z \in I(c_0)} z = 35.1$ m for $f_4(z)$. Note that we cannot find $\max_{z \in I(c_0)} z$ for $f_5(z)$ and $f_6(z)$ because $f_5(z)$ and $f_6(z)$ do not have a small constant c_0 such that $|I(c_0)|$ is large and $\eta(c_0) = 1$. From Figs. 11 and 12, we can confirm that $q_{P2}(x)$ approaches 1 for a small x in all cases. Specifically, $q_{P2}(1.1) \approx 1$ in all cases. Also, for $f_5(z)$ and $f_6(z)$, $q_{P1}(1.1)$ is smaller than $q_{P2}(1.1)$ for any d_s especially for $\lambda = 0.2$. Therefore, we can confirm that Policy 2 always approximates Optimum Routing well although Policy 1 does not always approximate Opti-

imum Routing. Note that Policy 1 sometimes approximates Optimum Routing well. For example, for $f_4(z)$, we have $q_{P1}(1.1) \approx 1$ if we set d_s to be 35.1 m. In such a case, therefore, Policy 1 can approximate Optimum Routing well even for a nondecreasing step-like function. From these results, we can confirm that Policy 2 approximates Optimum Routing better than or as well as Policy 1 in a one-dimensional random network.

5. Analysis of Minimum Route MTM with Policy 2

In this section, we theoretically analyze the mean value of minimum route MTM, denoted by $E(U(r_O))$. In the analysis, we theoretically and precisely analyze the mean value of route MTM of Policy 2, denoted by $E(U(r_{P2}))$. Then we use the formula of $E(U(r_{P2}))$ as an approximation to $E(U(r_O))$.

5.1 Analysis of Policy 2

Let H_{P2} be the number of hops of r_{P2} . Let $X_{P2,i}$ be the coordinate of i th relay node of r_{P2} for $i = 0, 1, \dots, H_{P2}$, where $X_{P2,0} = 0 < X_{P2,1} < \dots < X_{P2,H_{P2}} = L$. Let d_u be the maximum value of z such that $u(z) = u(0)$. Suppose that $L \leq d_u$. In this case, Policy 2 directly connects S and D. Then $E(U(r_{P2})) = u(L)$ if $L \leq d_u$. Suppose that $L > d_u$. In this case, $H_{P2} \leq 2 \lceil \frac{L}{d_u} \rceil - 1$, as proved in Appendix D. If $L > d_u$, therefore

$$E(U(r_{P2})) = \sum_{k=\lceil \frac{L}{d_u} \rceil}^{2\lceil \frac{L}{d_u} \rceil - 1} P(H_{P2} = k | R \neq \emptyset) \times E(U(r_{P2}) | R \neq \emptyset, H_{P2} = k). \quad (2)$$

In the following, we compute $P(H_{P2} = k | R \neq \emptyset)$ and $E(U(r_{P2}) | R \neq \emptyset, H_{P2} = k)$.

Suppose that $H_{P2} = 1$. Define

$$g(x_1, x_2) = \begin{cases} x \in [x_1, L] \left| \frac{u(x - x_1)}{x - x_1} < \frac{u(x_2 - x_1)}{x_2 - x_1} \right. \end{cases}. \quad (3)$$

$H_{P2} = 1$ if and only if $L \leq d$, and there is no node in $g(0, L)$. Hence

$$P(H_{P2} = 1 | R \neq \emptyset) = \begin{cases} e^{-\lambda|g(0,L)|}, & L \leq d, \\ 0, & L > d, \end{cases} \quad (4)$$

$$E(U(r_{P2}) | R \neq \emptyset, H_{P2} = 1) = u(L). \quad (5)$$

Next, we consider $H_{P2} = k$, where $k \geq 2$. Define $f_{P2,k}(x_{P2,1}, \dots, x_{P2,k-1})$ as the joint probability density function of $X_{P2,1}, \dots, X_{P2,k-1}$, where $x_{P2,1}, \dots, x_{P2,k-1}$ are the possible values of $X_{P2,1}, \dots, X_{P2,k-1}$, respectively. Then

$$\begin{aligned} & f_{P2,k}(x_{P2,1}, \dots, x_{P2,k-1}) dx_{P2,1} \dots dx_{P2,k-1} \\ &= P(x_{P2,1} \leq X_{P2,1} < x_{P2,1} + dx_{P2,1}, \dots, \\ & \quad x_{P2,k-1} \leq X_{P2,k-1} < x_{P2,k-1} + dx_{P2,k-1}) \\ &= P(x_{P2,1} \leq X_{P2,1} < x_{P2,1} + dx_{P2,1}, \dots, \\ & \quad x_{P2,k-1} \leq X_{P2,k-1} < x_{P2,k-1} + dx_{P2,k-1}) \end{aligned}$$

$$\begin{aligned}
& |R \neq \emptyset, H_{P_2} = k) \\
& = P(R \neq \emptyset, H_{P_2} = k, \\
& \quad x_{P_2,1} \leq X_{P_2,1} < x_{P_2,1} + dx_{P_2,1}, \dots, \\
& \quad x_{P_2,k-1} \leq X_{P_2,k-1} < x_{P_2,k-1} + dx_{P_2,k-1}) \\
& / P(R \neq \emptyset, H_{P_2} = k). \tag{6}
\end{aligned}$$

Define $C_{P_2,k}$ as the set of $(x_{P_2,1}, x_{P_2,2}, \dots, x_{P_2,k-1})$ such that $x_{P_2,i+1} - x_{P_2,i} \leq d$ for $i = 0, 1, \dots, k-1$, and

$$\frac{u(x_{P_2,i+1} - x_{P_2,i})}{x_{P_2,i+1} - x_{P_2,i}} < \min_{i+2 \leq j \leq k} \frac{u(x_{P_2,j} - x_{P_2,i})}{x_{P_2,j} - x_{P_2,i}} \tag{7}$$

for $i = 0, 1, \dots, k-2$, where $x_{P_2,0} = 0$ and $x_{P_2,k} = L$. Suppose that $(x_{P_2,1}, \dots, x_{P_2,k-1}) \in C_{P_2,k}$. Then $f_{P_2,k}(x_{P_2,1}, \dots, x_{P_2,k-1}) \geq 0$. Also, $R \neq \emptyset, H_{P_2} = k, X_{P_2,1} = x_{P_2,1}, \dots, X_{P_2,k-1} = x_{P_2,k-1}$ if and only if there are $k-1$ nodes at $x_{P_2,1}, x_{P_2,2}, \dots, x_{P_2,k-1}$, respectively, and no node in $g(x_{P_2,i-1}, x_{P_2,i})$ for $i = 1, 2, \dots, k$. Define

$$G(x_{P_2,1}, \dots, x_{P_2,k-1}) = \left| \bigcup_{i=1}^k g(x_{P_2,i-1}, x_{P_2,i}) \right|, \tag{8}$$

where $x_{P_2,0} = 0$ and $x_{P_2,k} = L$. Hence

$$\begin{aligned}
& P(R \neq \emptyset, H_{P_2} = k, \\
& \quad x_{P_2,1} \leq X_{P_2,1} < x_{P_2,1} + dx_{P_2,1}, \dots, \\
& \quad x_{P_2,k-1} \leq X_{P_2,k-1} < x_{P_2,k-1} + dx_{P_2,k-1}) \\
& = \lambda^{k-1} dx_{P_2,1} \dots dx_{P_2,k-1} e^{-\lambda G(x_{P_2,1}, \dots, x_{P_2,k-1})}. \tag{9}
\end{aligned}$$

If $(x_{P_2,1}, \dots, x_{P_2,k-1}) \notin C_{P_2,k}$, then $f_{P_2,k}(x_{P_2,1}, \dots, x_{P_2,k-1}) = 0$. By integrating Eq. (6), we have

$$\begin{aligned}
& P(H_{P_2} = k | R \neq \emptyset) = \int \dots \int_{C_{P_2,k}} \frac{1}{P(R \neq \emptyset)} \\
& \quad \times \lambda^{k-1} e^{-\lambda G(x_{P_2,1}, \dots, x_{P_2,k-1})} dx_{P_2,1} \dots dx_{P_2,k-1}, \tag{10}
\end{aligned}$$

where $P(R \neq \emptyset)$ can be computed as follows [1]:

$$\begin{aligned}
& P(R \neq \emptyset) = 1 + \sum_{i=1}^{\lceil \frac{L}{d} \rceil - 1} \frac{(-1)^i}{i!} e^{-i\lambda d} \{\lambda(L - id)\}^{i-1} \\
& \quad \times \{\lambda(L - id) + i\}. \tag{11}
\end{aligned}$$

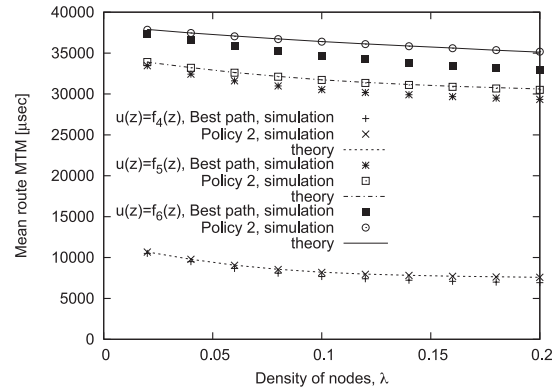
Also

$$\begin{aligned}
& E(U(r_{P_2}) | R \neq \emptyset, H_{P_2} = k) \\
& = \int \dots \int_{C_{P_2,k}} f_{P_2,k}(x_{P_2,1}, \dots, x_{P_2,k-1}) \\
& \quad \times \left\{ \sum_{i=1}^k u(x_{P_2,i} - x_{P_2,i-1}) \right\} dx_{P_2,1} \dots dx_{P_2,k-1}, \tag{12}
\end{aligned}$$

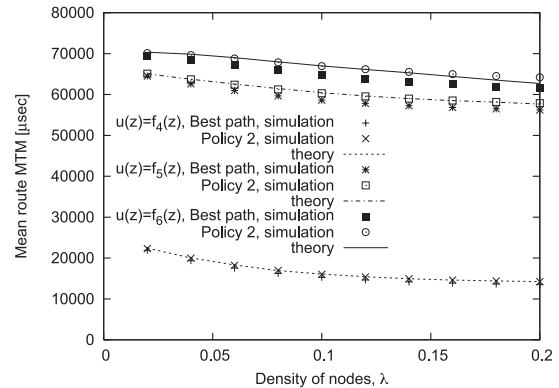
where $x_{P_2,0} = 0$ and $x_{P_2,k} = L$. By substituting Eqs. (10) and (12) into Eq. (2), we can compute $E(U(r_{P_2}))$.

5.2 Numerical Results and Discussion

Figure 13 shows the numerical results of $E(U(r_{P_2}))$ with the simulation results of $E(U(r_0))$ and $E(U(r_{P_2}))$ as a function



(a) $L = 60$ m.



(b) $L = 120$ m.

Fig. 13 Mean route MTM, where $d = 52.5$ m.

of λ . In this figure, the three functions $f_4(z)$, $f_5(z)$, and $f_6(z)$ are used as $u(z)$, and d is set to 52.5 m. In Figs. 13(a) and (b), $L = 60$ m and $L = 120$ m, respectively.

In this figure, the numerical results of $E(U(r_{P_2}))$ agree well with the simulation results of $E(U(r_{P_2}))$. We can confirm that the analysis in Sect. 5.1 is valid. We can also confirm that $E(U(r_{P_2}))$ is close to $E(U(r_0))$ for all cases. These results show that we can approximately compute $E(U(r_0))$ by the theoretical formula of $E(U(r_{P_2}))$. These results also support that Policy 2 well models the best path in addition to the results in Sect. 4.3.

6. Conclusion

In this paper, we analyzed communication quality in a one-dimensional random multi-hop network with MTM as a metric. To analyze route MTM, we considered two policies, Policy 1 and Policy 2, which approximately minimize route MTM. Policy 1 is an ordinary policy proposed in [1], and Policy 2 is a new one proposed here. We showed that Policy 2 approximates Optimum Routing better than Policy 1 from theoretical considerations and simulation results. We also theoretically analyzed the mean value of route MTM for Policy 2 in a one-dimensional random network and showed that the formula can be used as an approximation to the mean value of minimum route MTM from numerical and simulation results. An important future problem is extension

of the results in this paper to two-dimensional networks. Future problems also include characterization of the best path and analysis of communication quality considering interferences and other distributions of nodes.

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Appendix A: Proof of Property 1

To prove Property 1, we use the following lemma:

Lemma 1. $U(r_O) \geq \frac{u(d_0)}{d_0} L$.

Proof. Suppose that r_O is a k -hop path, and the lengths of links in r_O are $Y_{O,1}, Y_{O,2}, \dots, Y_{O,k}$. From the definition of d_0 , $\frac{u(Y_{O,i})}{Y_{O,i}} \geq \frac{u(d_0)}{d_0}$ for $i = 1, 2, \dots, k$. Then

$$\begin{aligned} U(r_O) &= \sum_{i=1}^k u(Y_{O,i}) = \sum_{i=1}^k \frac{u(Y_{O,i})}{Y_{O,i}} Y_{O,i} \\ &\geq \sum_{i=1}^k \frac{u(d_0)}{d_0} Y_{O,i} = \frac{u(d_0)}{d_0} L. \end{aligned} \quad (\text{A} \cdot 1)$$

□

Consider any path r . Suppose that the number of hops of r is k , and the lengths of links in r are Y_1, Y_2, \dots, Y_k . Suppose that the number of kinds of the link lengths in r is k_{length} . Denote these k_{length} lengths by $Y'_1, Y'_2, \dots, Y'_{k_{\text{length}}}$ such that

$Y'_1 < Y'_2 < \dots < Y'_{k_{\text{length}}}$. Suppose that $u(z)$ is defined as the following nondecreasing step-like function:

$$u(z) = \begin{cases} Y'_1, & 0 \leq z \leq Y'_1, \\ Y'_2, & Y'_1 < z \leq Y'_2, \\ \vdots & \vdots \\ Y'_{k_{\text{length}}}, & Y'_{k_{\text{length}}-1} < z \leq Y'_{k_{\text{length}}}. \end{cases} \quad (\text{A} \cdot 2)$$

Then

$$U(r) = \sum_{i=1}^k u(Y_i) = \sum_{i=1}^k Y_i = L \quad (\text{A} \cdot 3)$$

because $u(Y_i) = Y_i$ for $i = 1, 2, \dots, k$. On the other hand, $\frac{u(d_0)}{d_0} = 1$ from Eq. (A·2). Hence, $U(r_O) \geq L$ from Lemma 1. Therefore, we have $U(r) \leq U(r_O)$, which means that $r = r_O$.

Appendix B: Proof of Property 2

Suppose that $Y_i \in I(c)$ for $i = 1, 2, \dots, k$. From the definition of $I(c)$, $u(Y_i) \leq c \frac{u(d_0)}{d_0} Y_i$ for $i = 1, 2, \dots, k$. Then

$$U(r) = \sum_{i=1}^k u(Y_i) \leq \sum_{i=1}^k c \frac{u(d_0)}{d_0} Y_i = c \frac{u(d_0)}{d_0} L. \quad (\text{A} \cdot 4)$$

From this inequality and Lemma 1, we have $U(r) \leq cU(r_O)$.

Appendix C: Proof of Property 3

Since $d_0 \in I(c)$, $I(c) \neq \emptyset$ and $\eta(c) \geq 1$. Consider two real numbers, $z_1 \in I(c)$ and $z_2 \in I(c)$, and suppose that $z_1 \leq z_2$. Then $u(z_1) \leq c \frac{u(d_0)}{d_0} z_1$, and $u(z_2) \leq c \frac{u(d_0)}{d_0} z_2$. Because $u(z)$ is a convex function, $u(\theta z_1 + (1 - \theta)z_2) \leq \theta u(z_1) + (1 - \theta)u(z_2)$ for any real number $\theta \in [0, 1]$. From these inequalities, $u(\theta z_1 + (1 - \theta)z_2) \leq c \frac{u(d_0)}{d_0} \{\theta z_1 + (1 - \theta)z_2\}$ for any $\theta \in [0, 1]$. Hence, $\theta z_1 + (1 - \theta)z_2 \in I(c)$ for any $\theta \in [0, 1]$. This means that $I(c)$ is an interval.

Appendix D: Proof of relation $H_{P2} \leq 2 \left\lceil \frac{L}{d_u} \right\rceil - 1$

In Policy 2, $X_{P2,i+2} > X_{P2,i} + d_u$ for $i = 0, 1, \dots, H_{P2} - 2$. Assume that $H_{P2} \geq 2 \left\lceil \frac{L}{d_u} \right\rceil$. Then

$$\begin{aligned} L &= \sum_{i=1}^{H_{P2}} (X_{P2,i} - X_{P2,i-1}) \geq \sum_{i=1}^{\left\lceil \frac{L}{d_u} \right\rceil} (X_{P2,2i} - X_{P2,2i-2}) \\ &> \sum_{i=1}^{\left\lceil \frac{L}{d_u} \right\rceil} d_u = \left\lceil \frac{L}{d_u} \right\rceil d_u \geq L, \end{aligned} \quad (\text{A} \cdot 5)$$

which is a contradiction. Therefore, $H_{P2} \leq 2 \left\lceil \frac{L}{d_u} \right\rceil - 1$.



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