PAPER

A Time-Domain Filtering Scheme for the Modified Root-MUSIC Algorithm

Hiroyoshi YAMADA†, Yoshio YAMAGUCHI†, *and* Masakazu SENGOKU†, *Members*

SUMMARY A new superresolution technique is proposed for high-resolution estimation of the scattering analysis. For complicated multipath propagation environment, it is not enough to estimate only the delay-times of the signals. Some other information should be required to identify the signal path. The proposed method can estimate the frequency characteristic of each signal in addition to its delay-time. One method called modified (Root) MUSIC algorithm is known as a technique that can treat both of the parameters (frequency characteristic and delay-time). However, the method is based on some approximations in the signal decorrelation, that sometimes make problems. Therefore, further modification should be needed to apply the method to the complicated scattering analysis. In this paper, we propose to apply a time-domain null filtering scheme to reduce some of the dominant signal components. It can be shown by a simple experiment that the new technique can enhance the estimation accuracy of the frequency characteristic in the Root-MUSIC algorithm.

key words: *modified Root-MUSIC algorithm, time-domain filtering, beamspace Root-MUSIC algorithm, electromagnetic scattering, frequency decay parameter*

1. Introduction

A superresolution technique (a MUSIC algorithm [1]) is one of the most promising solutions for the high-resolution method in the scattering and multipath propagation analysis. The method formulated in the frequency domain [2] has a high resolution capability for resolving delay time of each incident signal [3]. With the recent development of indoor millimeter wave radio systems, a higher resolution capability may be required.

As far as the identification of propagation path is concerned, it is difficult to discriminate each path when the number of signals is large, even if all the signal is resolved. In such a case, some other information is needed. Direction-of-arrivals or frequency characteristics are examples of the further information. In this paper, we focus on the latter information (frequency characteristics) because no modifications to the measurement system are required. The frequency characteristics of incident signals can be treated by the modified (Root) MUSIC algorithm [4]. The method is based on the assumption that the spatial smoothing scheme is approximately applicable. The assumption is not sometimes valid for large subarrays and/or many incidents in the narrow bandwidth estimation. Then, some difficulties arise for selecting the parameters ('M', '$\hat{d}$', etc.) in the analysis. Furthermore, the bandwidth requirement for the frequency characteristic estimation in this method is often wider than that for only delay-time estimation. Using the delay-time information, the frequency characteristics of the signals will be obtained with much narrower bandwidth.

Therefore, we introduce a time-domain null filtering scheme for suppressing some signals whose delay times are almost correctly estimated. DeGroat, et al., utilize the signal information (direction of arrival in their paper) of constraint in the MUSIC estimation [5]. In this paper, we introduce the concept as a filtering scheme, and extend the method to a beamspace-type Root-MUSIC algorithm [6]. In the beamspace Root-MUSIC algorithm, a spatial filtering preprocessing is introduced in the antenna array data to form an adequate beam pattern. On the other hand, the filtering scheme in the proposed method is applied to the frequency domain data to make nulls in the time domain. Since both of the methods are based on the filtering concept, our method can be regarded as a 'beamspace-type' Root-MUSIC algorithm, based on the null steering concept. When the nulls are correctly steered in some signals, the signals are suppressed and the cross-terms, concerning the signals in the correlation matrix, are also diminished. Therefore the signals that are not suppressed can be enhanced in comparison with the filtered signals. In addition, the suppression makes the dimension of the noise subspace large (signal subspace small). It often makes the estimation stable on the parameter settings in the analysis.

In the following, we formulate the problem and propose the new method in Sect. 2. In Sect. 3, we show the effect of the method using numerical and experimental results. Section 4 contains conclusions.

2. Formulation of the New Method

2.1 Data Model

Let us assume that the amplitude and the phase of a received signal can be obtained as a function of frequency by an equipment such as a vector network analyzer. In the high frequency electromagnetic scattering, the total field r_i at the frequency f_i can be modeled as a linear

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†The authors are with the Faculty of Engineering, Niigata University, Niigata-shi, 950–21 Japan.

combination of d dominant signals;

$$r_i = \sum_{j=1}^{d} s_j W_i^{(j)} e^{-j2\pi f_i t_j} + n_i, \tag{1}$$

where $s_j W_i^{(j)}$ denotes j-th signal component at the frequency f_i, and t_j denotes the delay time of the component of each signal. The frequency characteristics of the signals depend on the local shape of scatterer. In Eq. (1), the frequency characteristics are modeled by $W_i^{(j)}$. The additive white Gaussian noise samples n_i are assumed to be statistically independent from sample to sample and have zero mean with variance σ^2.

Here, we write the L uniformly sampled (increment of Δf) frequency-domain data (r_i; $i = 1 \sim L$) using the vector notation as follows:

$$\boldsymbol{r} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n}, \tag{2}$$

where

$$\boldsymbol{r} = [r_1, r_2, \cdots, r_L]^T, \tag{3a}$$

$$\boldsymbol{A} = [\boldsymbol{a}(t_1), \boldsymbol{a}(t_2), \cdots, \boldsymbol{a}(t_d)], \tag{3b}$$

$$\boldsymbol{s} = [s_1, s_2, \cdots, s_d]^T, \tag{3c}$$

$$\boldsymbol{n} = [n_1, n_2, \cdots, n_L]^T, \tag{3d}$$

where T denotes the transpose. The L-dimensional vector $\boldsymbol{a}(t_j)$ ($j = 1 \sim d$) is the mode vector of j-th signal including its frequency characteristic. The i-th element of $\boldsymbol{a}(t_j)$ is $W_i^{(j)} e^{-j2\pi f_i t_j}$.

2.2 Modified Root-MUSIC with Time-Domain Filtering

In the modified Root-MUSIC algorithm [4], all of the signals can be resolved in much narrower bandwidth than that required for the frequency characteristic estimation. In such a case, the delay-time information can be used for the frequency characteristic estimation. The proposed method utilizes the information to make a time-domain null filtering matrix for rejecting some of the dominant components. Let us assume that the estimated number of signals $\hat{d}$ and the estimated delay time of each signal ($\hat{t}_j$; $j = 1 \sim \hat{d}$) have been acquired before the following procedures.

2.2.1 Spatial Smoothing Preprocessing

The procedure in this stage is the same as the modified Root-MUSIC algorithm described in [4]. To simplify the discussions in Sect. 3, we adopt a function $W_i(\alpha)$;

$$W_i(\alpha) = \left(\frac{1}{f_i}\right)^{\alpha}, \tag{4}$$

as a pre-weight function. The weighted data vector $\boldsymbol{r}^{(\alpha)}$ is then written as

$$\boldsymbol{r}^{(\alpha)} = \left[\frac{r_1}{W_1(\alpha)}, \frac{r_2}{W_2(\alpha)}, \cdots, \frac{r_L}{W_L(\alpha)}\right]^T. \tag{5}$$

Partitioning $\boldsymbol{r}^{(\alpha)}$ into M ($M > d$) overlapped subarrays $\boldsymbol{r}_m^{(\alpha)}$, we can define the spatially smoothed correlation matrix [7] as follows:

$$\boldsymbol{R}_{SSP}^{(\alpha)} = \frac{1}{M} \sum_{m=1}^{M} \boldsymbol{r}_m^{(\alpha)} \boldsymbol{r}_m^{(\alpha)H}, \tag{6}$$

where H denotes complex conjugate transpose. Note that $\boldsymbol{R}_{SSP}^{(\alpha)}$ is an $N \times N$ matrix ($N = L - M + 1$).

2.2.2 Time-Domain Filtering

The filtering scheme is applied to the matrix $\boldsymbol{R}_{SSP}^{(\alpha)}$. As described previously, we can use the estimated values of $\hat{t}_j$ ($j = 1 \sim \hat{d}$) in this stage. Here, we formulate the algorithm for extracting the k-th signal whose delay time is t_k. The other estimated delay-times are used to construct the time-domain null filtering matrix. The filtering matrix which makes nulls at the delay time of $\hat{t}_j$ ($j = 1, 2, \cdots, k-1, k+1, \cdots, \hat{d}$) can be given by [5]

$$\boldsymbol{G} = \boldsymbol{I} - \boldsymbol{C}(\boldsymbol{C}^H \boldsymbol{C})^{-1} \boldsymbol{C}^H, \tag{7}$$

where $\boldsymbol{I}$ is the identity matrix. The matrix $\boldsymbol{C}$ is defined by

$$\boldsymbol{C} = [\boldsymbol{c}_1, \boldsymbol{c}_2, \cdots, \boldsymbol{c}_{k-1}, \boldsymbol{c}_{k+1}, \cdots, \boldsymbol{c}_{\hat{d}}], \tag{8a}$$

$$\boldsymbol{c}_i = [e^{-j2\pi f_1 \hat{t}_i}, e^{-j2\pi f_2 \hat{t}_i}, \cdots, e^{-j2\pi f_N \hat{t}_i}]^T. \tag{8b}$$

We can use $\boldsymbol{G}$ as a filtering matrix, however it can be modified further. The $N \times N$ matrix $\boldsymbol{G}$ has the rank of $N - (\hat{d} - 1)$. Thus, the singular value decomposition of $\boldsymbol{G}$ can be also written as

$$\boldsymbol{G} = \boldsymbol{B} \Sigma \boldsymbol{B}^H. \tag{9}$$

The elements of the diagonal matrix Σ denote the non-zero eigenvalues of $\boldsymbol{G}$, and the columns of $\boldsymbol{B}$ contain the corresponding eigenvectors of it. It can be clear that we can use $\boldsymbol{B}^H$ as a filtering matrix.

The filtered data-correlation matrix $\tilde{\boldsymbol{R}}_{SSP}$ is obtained by

$$\tilde{\boldsymbol{R}}_{SSP} = \boldsymbol{B}^H \boldsymbol{R}_{SSP} \boldsymbol{B}. \tag{10}$$

Note that the correlation matrix now becomes $\left(N - (\hat{d} - 1)\right) \times \left(N - (\hat{d} - 1)\right)$.

2.2.3 Generalized Eigenanalysis

In the modified Root-MUSIC algorithm, we should obtain the generalized eigenvectors of $\boldsymbol{R}_{SSP}$ in the metric of $\boldsymbol{R}_N$ where

$$\boldsymbol{R}_N = \frac{1}{M}$$

$$\cdot \sum_{m=1}^{M} \begin{bmatrix} \frac{1}{|W_m(\alpha)|^2} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{|W_{m+N-1}(\alpha)|^2} \end{bmatrix}. \quad (11)$$

On the other hand, we should acquire the generalized eigenvectors of $\tilde{\boldsymbol{R}}_{SSP}$ in the metric of $\boldsymbol{B}^H \boldsymbol{R}_N \boldsymbol{B}$ in the proposed method. Here, we define that λ_i and $\boldsymbol{e}_i$ represent the i-th generalized eigenvectors and corresponding generalized eigenvectors of $\tilde{\boldsymbol{R}}_{SSP}$. The eigenvalues ($\lambda_i$, $i = 1 \sim N - \hat{d} + 1$) are assumed to be indexed in descending order with respect to magnitude. Also, we assume that the number of dominant eigenvalues in $\tilde{\boldsymbol{R}}_{SSP}$ becomes $\hat{d}_f$. If the filtering matrix can completely remove all of the signal components located at t_j ($j = 1, 2, \cdots, k-1, k+1, \cdots, \hat{d}$), one dominant eigenvalue will appear ($\hat{d}_f = 1$). The frequency characteristics of the signals are not considered in this filtering scheme. Moreover, the estimated delay times contain the estimation error in general. Therefore, several dominant eigenvalues may appear. However, if the nulls are located close to the exact delay times of the signals, their components are suppressed effectively. That makes the signal subspace more compact than that in the unfiltering case ($\hat{d}_f < \hat{d}$). The filtering matrix also affects the target signal which locates at t_k in this stage. However, the effect is compensated by the reverse filtering in the root finding procedure [6] (see also Eq. (14)).

2.2.4 Root-Finding Procedure

Now, we construct the Root-MUSIC polynomial. As described in [6], the polynomial of order $2N-2$ can be expressed as

$$p(z) = p_0 + p_1 z + \cdots + p_{N-1} z^{N-1} + \cdots + p_1^* z^{2N-3} + p_0^* z^{2N-2}. \quad (12)$$

Each coefficient of $p(z)$ can be computed as

$$p_j = \sum_{i=0}^{j} \boldsymbol{P}(N-j+i, i+1), j = 0, 1, \cdots, N-1, \quad (13)$$

where $\boldsymbol{P}(i, j)$ denotes the (i, j) element of the following matrix:

$$\boldsymbol{P} = \boldsymbol{B} \left(\sum_{i=\hat{d}_f+1}^{N-\hat{d}+1} \boldsymbol{e}_i \boldsymbol{e}_i^H \right) \boldsymbol{B}^H. \quad (14)$$

$\boldsymbol{B}$ is the filtering matrix defined in Eq. (9), and the vectors, $\boldsymbol{e}_i$ ($i = \hat{d}_f \sim N - \hat{d} + 1$), are the generalized eigenvectors of $\tilde{\boldsymbol{R}}_{SSP}$.

Apparently, the polynomial $p(z)$ has roots of multiplicity two at t_j ($j = 1, 2, \cdots, k-1, k+1, \cdots, \hat{d}$) because columns of $\boldsymbol{B}$ are orthogonal to the vectors $\boldsymbol{c}_j$ in Eq. (8b). Then, we can rewrite Eq. (12) as

$$p(z) = \prod_{j=1, j \neq k}^{\hat{d}} (z - z_j)^2 p'(z), \quad (15)$$

where $z_j = e^{-j2\pi\Delta f \hat{t}_j}$. From these formulations, it can be clear that the polynomial to be evaluated is the $p'(z)$ of order $2(N - \hat{d} + 1) - 2$. Note that the order of $p'(z)$ is independent of $\hat{d}_f$. Another derivation of $p(z)$ is shown in [8]. We can also provide $p(z)$ using the derivation (see appendix for the details).

If the pre-weight function $W_i(\alpha)$ coincides with the frequency decay of the target signal at $\hat{t}_k$, the root pair corresponding to the signal will be on the unit circle in the z-plane. Changing the weight function in various ways (α of Eq. (4) in this paper) and find $W_i^{(j)}$ which makes the roots on the unit circle, it will be the frequency characteristic of the signal at $\hat{t}_k$.

3. Examples

In this section, first, we point out the problems in the modified Root-MUSIC algorithm numerically, and show the application result of time-domain null filtering to the modified Root-MUSIC. In the latter part of this section, we apply the method to the backscattered data by a rectangular flat plate [9]. The signal parameters of the numerical model are selected to approximate the measured data. Figure 1 (a) shows a geometrical coordinate of the target under the considerations. As-

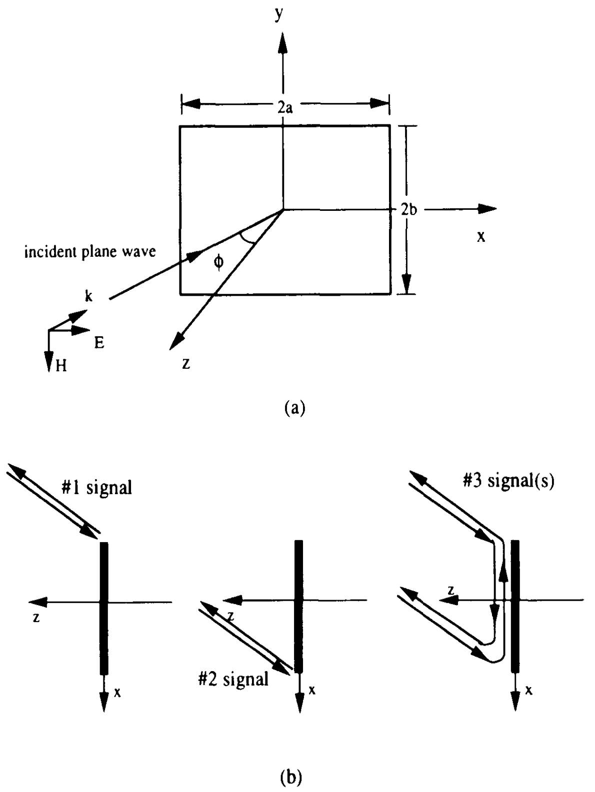

Fig. 1 Plane wave incident on a flat plate. (a) geometry and coordinate system. (b) dominant three ray paths (top view).

pect angle ϕ in Fig. 1 (a) is 30°, and size of the plate is 30 cm × 30 cm ($a = 0.15$ m, $b = 0.15$ m). Polarization of the incident field is also shown in the figure. We select y axis as vertical, then just refer to the polarization as the horizontal polarization in the following discussion. In this case, there exist three dominant scattered fields. Their ray paths are shown in Fig. 1 (b). Note that the single edge diffracted signals in this target are observed with no frequency dependence because of the three dimensional target [10].

3.1 Numerical Study

We show the numerical results for the data model;

$$r_i = \sum_{j=1}^{3} s_j W_i^{(j)} e^{-j2\pi f_i t_j},$$

where delay times of the signals are defined as $t_1 = -0.5$ ns, $t_2 = 0.5$ ns, and $t_3 = 1.0$ ns. The first and second signals have no frequency decay ($W_i^{(1)} = W_i^{(2)} = 1$). The frequency characteristic of the third signal is defined as $(1/f)^{0.5}$ ($= W_i^{(3)}$). The first frequency in the MUSIC analysis is 5 GHz, and the sampling frequency period is 50 MHz ($f_1 = 5$ GHz, $\Delta f = 50$ MHz). The radar cross section (RCS) corresponding to the first, second and third signals at frequency of f_1 are −10 dBsm, −20 dBsm, and −25 dBsm, respectively. The parameters of the three signals are selected almost the same as those appeared in the experimental data discussed later. However, note that this model has only three signals and no noise.

Figure 2 shows an estimation result by the conventional modified Root-MUSIC algorithm. In this estimation, we use the weight function in Eq. (4), then α in the figure corresponds to that in Eq. (4). Each curve expresses the root-pair distance of corresponding signal roots with respect to α. When the weight coincides with, or approximately equals to, the frequency decay of the signal, the root-pair distance becomes minimum. The error in the delay-time estimation is less than 1.5 ps for each signal. As shown in this figure, the frequency decay parameters of #1 and #2 signals are correctly estimated. However, the estimated value of α_3 is 0.3.

When M and $\hat{d}$ are changed, estimated values of decay parameter are also changed. We show some results of #3 signal in Fig. 3. As you see in this figure, the estimated decay parameter is changed in each analysis. However, the delay times can be estimated within 4 ps error for all signals. From the results of $\hat{d} = 5$, we can say that there exist more than three components to be selected as a signal subspace, though the actual number of signals is three. In this case, the data correlation matrix is precisely estimated because there exist no noise. The problem is caused by the spatial smoothing preprocessing (SSP). As reported in [4], the application of the SSP to the non-exponentially-decayed frequency-domain signals is approximately valid, however, the validity of the approximation depends on the parameters (i.e., $N, M, \hat{d}, \Delta f$) in the analysis.

Now, we apply the time-domain filtering to the case of $M = 15, \hat{d} = 3$ in Fig. 3. The delay parameters in the filtering matrix are $\hat{t}_1 = -0.500$ ns, $\hat{t}_2 = 0.499$ ns, and $\hat{t}_3 = 0.998$ ns. They were the estimated values with the conventional modified Root-MUSIC algorithm. The filtering processing in Eq. (10) transform the $N \times N$ data correlation matrix into the $(N - \hat{d} + 1) \times (N - \hat{d} + 1)$ filtered matrix. To simplify the discussions, we select $\hat{d}_f$ which is equal to $\hat{d}$. Surely, we can make $\hat{d}_f$ smaller than $\hat{d}$ when filtered signals are completely removed. The estimation results of the proposed method are shown in Fig. 4. The curves in the figure slightly fluctuate around the points where the filter-nulls are located. However, the global minima appear clearly, and corresponding decay parameters are estimated exactly. Furthermore,

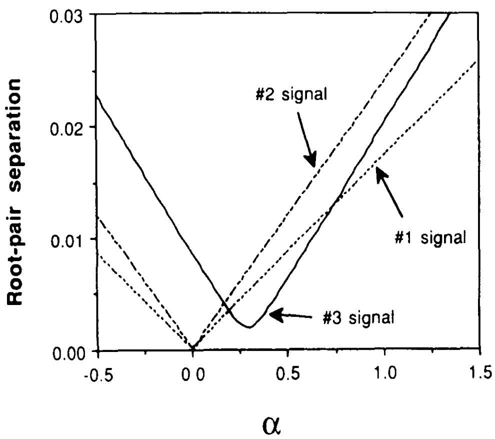

Fig. 2 Root-pair separation of the Root-MUSIC polynomial versus frequency decay parameter α. $f_1 = 5$ GHz, $\Delta f = 50$ MHz, $N = 20$, $M = 15$, $\hat{d} = 3$. True decay parameters are $\alpha_1 = 0.0$, $\alpha_2 = 0.0$, $\alpha_3 = 0.5$.

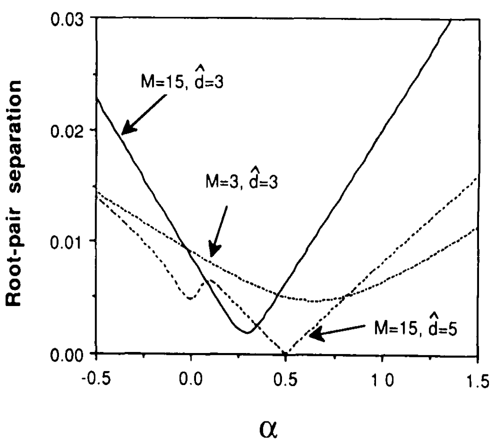

Fig. 3 Root-pair separation of the Root-MUSIC polynomial versus estimated frequency decay parameters of # 3 signal. $f_1 = 5$ GHz, $\Delta f = 50$ MHz, $N = 20$. True decay parameter is $\alpha_3 = 0.5$.

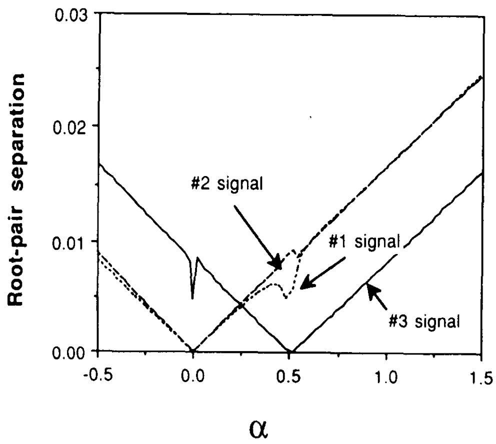

Fig. 4 Estimation results of the modified Root-MUSIC algorithm with time-domain null filtering scheme. $f_1 = 5\,\mathrm{GHz}$, $\Delta f = 50\,\mathrm{MHz}$, $N = 20$, $M = 15$, $\hat{d} = 3$. Delay-time parameters in the filtering scheme are $\hat{t}_1 = -0.500\,\mathrm{ns}$, $\hat{t}_2 = 0.499\,\mathrm{ns}$, $\hat{t}_3 = 0.998\,\mathrm{ns}$.

the root-pair distance can now be almost zero at each global minimum.

From these results, we can conclude that the estimated performance of the modified Root-MUSIC algorithm can be greatly improved when the filter can suppress some of the dominant signals.

3.2 Experimental Study

Now, let us consider the measured backscattered field by a metal flat plate of $0.3 \times 0.3\,\mathrm{m}$ ($a = 0.15\,\mathrm{m}$, $b = 0.15\,\mathrm{m}$ in Fig. 1). The field was measured for a horizontal polarization using a quasi-monostatic RCS measurement system with a network analyzer (HP8510B).

Figure 5 (a) shows the calibrated data and the local scattered field. Each local scattered field was obtained using the time-domain filtering based on the Fourier transform [2]. In Fig. 5 (b), we show the time-domain responses calculated by the inverse Fourier transform. The asymptotical theoretic values by GTD [9] are also plotted in Figs. 5 (a) and (b) as references (the curves marked by "GTD"). In each figure, measured values almost coincide with the GTD results. The delay-times and frequency decay parameters of the three dominant signals are almost the same as the model discussed above. As shown in Fig. 5 (b), the scattering centers of the dominant signals cannot be resolved in the "1.65 GHz span", so we select the bandwidth for evaluating the performance of the MUSIC algorithms.

The estimation results by the modified Root-MUSIC without the filtering scheme are listed in Table 1. Also, one of the results is plotted in Fig. 6. The decay parameters of #2 and #3 signals cannot be detected in the swept range [−0.5, 1.5]. The value "−0.5−" in the table means $\hat{\alpha}$ may appear below −0.5, and the value "1.5+" expresses $\hat{\alpha}$ may exceed 1.5. In such cases, we adopted $\hat{t}$ at the edge of the range as an approximated value of the delay-time. You see that the delay times of the three signals remain stable in the analysis. Then, we may say that the dominant components can be resolved. On the other hand, the decay parameters cannot be correctly estimated. That is obvious because $\hat{\alpha}_2$ has a large minus value, which cannot be permitted physically.

The estimation results of the modified Root-MUSIC algorithm with the proposed filtering scheme are also shown in Fig. 6. The "#1 signal" curve was obtained with the filter which makes null at $\hat{t}_2$ and $\hat{t}_3$.

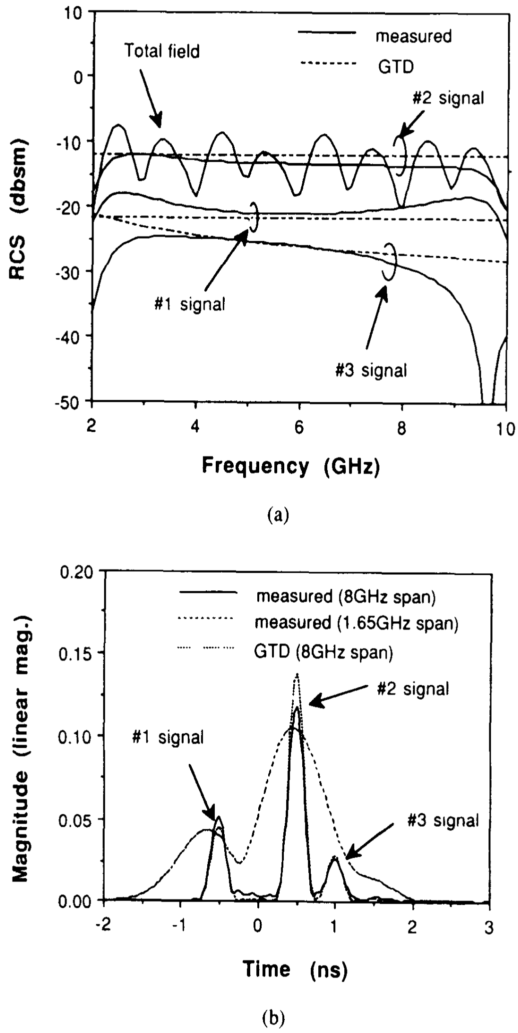

Fig. 5 Scattered fields from the rectangular plate for horizontal polarization. $a = 0.15\,\mathrm{m}$, $b = 0.15\,\mathrm{m}$, $\phi = 30°$. (a) total scattered field and each scattered component. (b) time-domain responses obtained by the inverse Fourier transform. The solid and dotted curves: wideband data (2 − 10 GHz), dashed curve: narrowband data (5 − 6.65 GHz).

Table 1 Estimated values of delay time and decay parameters using modified Root-MUSIC algorithm. $f_1 = 5\,\mathrm{GHz}$, $\Delta f = 50\,\mathrm{MHz}$, $N = 20$, $\hat{d} = 4$.

	#1 signal		*#2 signal*		*#3 signal*	
M	$\hat{t}_1$ (ns)	$\hat{\alpha}_1$	$\hat{t}_2$ (ns)	$\hat{\alpha}_2$	$\hat{t}_3$ (ns)	$\hat{\alpha}_3$
5	−0.528	0.06	0.500	−0.5−	0.990	1.5+
9	−0.529	−0.02	0.485	−0.5−	1.054	1.5+
15	−0.525	−0.14	0.474	−0.5−	1.121	1.5+

Table 2 Estimated values of delay time and decay parameters using modified Root-MUSIC algorithm with and without filtering scheme. $f_1 = 5\,\text{GHz}$, $\Delta f = 50\,\text{MHz}$, $N = 20$, $M = 15$, $\hat{d} = 4$.

	#1 signal		*#2 signal*		*#3 signal*	
	$\hat{t}_1$ (ns)	$\hat{\alpha}_1$	$\hat{t}_2$ (ns)	$\hat{\alpha}_2$	$\hat{t}_3$ (ns)	$\hat{\alpha}_3$
Modified-Root MUSIC	−0.525	−0.14	0.474	−0.5−	1.121	1.5+
Proposed Technique	−0.527	−0.02	0.468	0.08	1.058	0.42

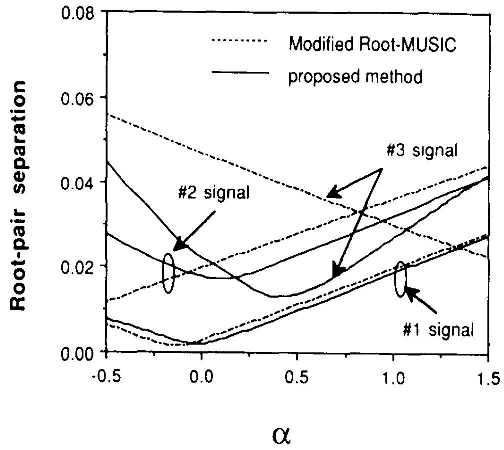

Fig. 6 Estimation results of decay parameter for the three dominant components. $f_1 = 5\,\text{GHz}$, $\Delta f = 50\,\text{MHz}$, $N = 20$, $M = 15$, $\hat{d} = 4$. Delay-time parameters in the filtering scheme are $\hat{t}_1 = -0.525\,\text{ns}$, $\hat{t}_2 = 0.474\,\text{ns}$, $\hat{t}_3 = 1.121\,\text{ns}$.

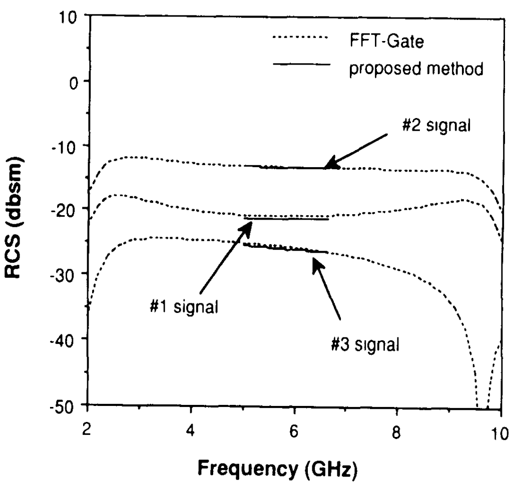

Fig. 7 Estimated frequency response for three dominant scattered components.

The other curves were obtained by the similar manner. As seen in this figure, the curves of #2 and #3 signals make clear minima in this swept range. The estimated delay-time and the decay parameters are also listed in Table 2. Also, the estimated fields of the signals are shown in Fig. 7. The filter suppresses some of the dominant signals, hence the correlation components between the signals and the clutter, and the signals and the non-filtered dominant signal, are diminished. These are the main reason of the estimation accuracy enhancement.

From these results, we can conclude that the estimation accuracy of the modified Root-MUSIC method can be greatly improved by the proposed filtering scheme when the delay-time of dominant signals can be almost resolved. That is, we can estimate both delay time and frequency characteristic in narrower frequency bandwidth than that required by the conventional technique.

4. Conclusions

In this paper, we have proposed the new superresolution technique based on the time-domain null filtering scheme, and have shown its availability through the numerical and experimental study. The model treated here was simple, however, the conventional technique could not evaluate the parameters correctly. In the actual applications such as the indoor multipath propagation analysis, the situations become worse than that in the present model. However, when the data can approximately be modeled by Eq. (1), the proposed technique will work properly. The validity of the method for the actual indoor propagation data will be examined in the near future.

The procedures from Eq. (12) to Eq. (15) are considered to be expressed more compactly using the matrix notation [6]. In addition, we can extend the method to iterative or adaptive algorithm straightforwardly. Further discussions will be treated in the future.

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Appendix: Another Derivation of the Root Polynomial

In this appendix, we show another derivation of the Root-MUSIC polynomial [8]. In the filtering procedure in Sect. 2.2B, the signal mode vector of the target, $\boldsymbol{a}(t_k)$, is also transformed by the filtering matrix, then the orthogonality among the mode vector and eigenvectors, that span noise subspace, can be written as

$$\left(\boldsymbol{B}^H \boldsymbol{a}(t_k)\right)^H \boldsymbol{e}_i = \boldsymbol{a}(t_k)^H \boldsymbol{B}\boldsymbol{e}_i = 0 \qquad i = \hat{d}_f \sim N - \hat{d} + 1. \quad \text{(A·1)}$$

Then, if we define polynomial using the eigenvectors and the filtering matrix, i.e.,

$$S_i(z) = [1, z, z^2, \cdots, z^{N-1}] \cdot \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,N-\hat{d}+1} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,N-\hat{d}+1} \\ \vdots & \vdots & & \vdots \\ b_{N,1} & b_{N,2} & \cdots & b_{N,N-\hat{d}+1} \end{bmatrix} \cdot \begin{bmatrix} e_{1,i} \\ e_{2,i} \\ \vdots \\ e_{N-\hat{d}+1,i} \end{bmatrix} = \sum_{l=1}^{N} \left(\sum_{j=1}^{N-\hat{d}+1} b_{l,j} e_{j,i} \right) z^{l-1}, \qquad i = \hat{d}_f + 1 \sim N - \hat{d} + 1, \quad \text{(A·2)}$$

where $b_{l,j}$ is the (l, j) element of $\boldsymbol{B}$, and $e_{j,i}$ denotes the j-th element of $\boldsymbol{e}_i$, then the signal zero corresponding to the k-th signal is the root of each of the above polynomials. The root appears on the unit circle in the z plane when the pre-weight function coincide with the frequency characteristic of the signal [4].

Using Eq. (A·2), the Root-MUSIC polynomial can also be defined by

$$p(z) = \sum_{i=\hat{d}_f+1}^{N-\hat{d}+1} (S_i(z) S_i^*(1/z^*)), \quad \text{(A·3)}$$

where $*$ denotes complex conjugation.

Note that $\boldsymbol{B}$ has a property:

$$\boldsymbol{c}_i^H \boldsymbol{B} = [0, 0, \cdots, 0]. \quad \text{(A·4)}$$

where $\boldsymbol{c}_i$ is defined in Eq. (8b). To substitute $\boldsymbol{a}(t_k)$ in Eq. (A·1) by $\boldsymbol{c}_i$, we find that Eq. (A·1) still holds. Then, the roots corresponding to the filter nulls are also the zeros of $S_i(z)$. They appear on the unit circle. As a result, the polynomial $p(z)$ brings the null-related-roots of multiplicity two.

Hiroyoshi Yamada was born in Hokkaido Prefecture, Japan, on November 2, 1965. He received the B.S., M.S., and Ph.D. degrees from Hokkaido University, Sapporo, Japan, in 1988, 1990 and 1993, respectively, all in electronic engineering. Since 1993, he has been with Niigata University, where he is currently a lecturer. His current research involves superresolution techniques, electromagnetic wave measurements, and radar signal processings. Dr. Yamada is a member of the IEEE.

Yoshio Yamaguchi was born in Niigata, Japan, on March 12, 1954. He received the B.E. degree in electronics engineering from Niigata University in 1976, and the M.E. and Dr. Eng degrees from Tokyo Institute Technology, Tokyo, Japan, in 1978 and 1983, respectively. In 1978, he joined the Faculty of Engineering, Niigata University, where he is a Professor. From 1988 to 1989, he was a Research Associate at the University of Illinois at Chicago. His interests are in the field of propagation characteristics of electromagnetic wave in lossy medium, radar polarimetry, microwave remote sensing and imaging. Dr. Yamaguchi is a senior member of IEEE and a member the Japan Society for Snow Engineering.

Masakazu Sengoku was born in Nagano Prefecture, Japan, on October 18, 1944. He received the B.E. degree in electrical engineering from Niigata University, Japan, in 1967 and the M.E. and Ph.D. degrees from Hokkaido University in 1969 and 1972, respectively. In 1972, he joined the stuff at Hokkaido University as a Research Associate. In 1978, he was an Associate Professor at Niigata University, where he is presently a Professor. During 1986–1987, he was a Visiting Scholar at the University of California, Berkley, and at the University of Illinois, Chicago. His research interests include transmission of information, network theory, and graph theory. He is a recipient of the best paper award of IEICE in 1992. Dr. Sengoku is a senior member of IEEE, a member of Japan Society of Industrial and Applied Mathematics.