PAPER

Decomposition of Radar Target Based on the Scattering Matrix Obtained by FM-CW Radar

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SUMMARY One of the polarimetric radar applications is classification or identification of targets making use of the scattering matrix. This paper presents a decomposition scheme of a scattering matrix into three elementary scattering matrices in the circular polarization basis. The elementary components are a sphere, a diplane (dihedral corner reflector), and a helix. Since a synthetic aperture FM-CW radar provides scattering matrix through a polarimetric measurement, this decomposition scheme was applied to the actual raw data, although the matrix is resulted from a swept frequency measurement. Radar imaging experiments at the *Ku* band (14.5-15.5 GHz) were carried out to obtain a total of 64× 64 scattering matrices in an imaging plane, using flat plates, corner reflectors and wires as elementary radar targets for classification. It is shown that the decomposition scheme has been successfully carried out to distinguish these targets and that the determination of rotation angle of line target is possible if the scattering matrix is classified as a wire.

key words: *decomposition, scattering matrix, radar polarimetry, FM-CW radar*

1. Introduction

One of the attracting applications in radar polarimetry is to retrieve information from a radar target. This "retrieving" includes target classification, identification, recognition, and decomposition. If a scattering matrix or an equivalent matrix pertaining to a radar target is obtained by a polarimetric radar, there is a possibility to extract some information on the target. Decomposition of the polarization matrix aims at separating the matrix into basic components, or matching it with specific types of matrices to determine the best-matching type. Huynen [1] has provided a phenomenological theory to decompose the Mueller matrix into a combination of an average of single (symmetric) and a residual (non-symmetric and irregular noise) components. Cloude [2] decomposes a scattering matrix into orthogonal component matrices (Pauli spin matrices), which define a vector used to form a coherency matrix for eigenvalue analysis. Krogager and Czyz [3] applied the decomposition theorem based on the scattering matrix formulation, leading to a combination of fundamental elements such as a sphere, a diplane (corner reflector), and other component (right or left helix). Good summary on target decomposition theorems has been recently given by Cloude and Pottier [4], however, there is no decisive decomposition technique presented at the present stage.

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Followed by Krogager [3], this paper presents a coherent decomposition algorithm based on the magnitude of scattering matrix elements in the circular polarization basis and shows an experimental decomposition result obtained by a polarimetric synthetic aperture FM-CW radar. The purposes of this paper are to pursuit the polarimetric FM-CW radar capability and to examine experimentally the applicability and possibility of the scattering matrix decomposition, because the theorem provided by Krogager is based on pulse radar systems [5]. After the polarimetric FM-CW radar system calibration operative at the *Ku* band (14.5-15.5 GHz), several targets including flat plates, corner reflectors and line targets were imaged in a laboratory. A two-dimensional image was picked up to retrieve a total amount of 64× 64 scattering matrices. The components of the scattering matrix decomposition are a sphere (which exhibits the same characteristics as flat plate), a diplane (dihedral corner reflector), and a helix (right or left sense). It is shown that the decomposition scheme has been successfully carried out to distinguish these targets and that the determination of the rotation angle of a wire is possible based on the FM-CW radar scattering matrix.

2. Scattering Matrix in the Circular Polarization Basis

The main purpose of a decomposition is to provide means for interpretation and full utilization of polarimetric scattering data. In the coherent scattering case, it is possible to obtain the Sinclair scattering matrix by a synthetic aperture FM-CW radar [6], which represents fundamental polarimetric scattering characteristics of a target and plays an important role in radar polarimetry. Before going into the detail of the decomposition theorem, let us begin with the scattering matrix characteristics [7]. The Sinclair scattering matrix $[S]$ is defined in terms of the Jones vector representation of electric field E in the polarization basis AB as

$$\begin{bmatrix} E_A^r \\ E_B^r \end{bmatrix} = [S] \begin{bmatrix} E_A^t \\ E_B^t \end{bmatrix} = \begin{bmatrix} S_{AA} & S_{AB} \\ S_{BA} & S_{BB} \end{bmatrix} \begin{bmatrix} E_A^t \\ E_B^t \end{bmatrix} \quad (1)$$

where, the superscript t refers to transmitting and r to receiving (see Fig. 1). In the monostatic radar arrangement (backscattering) case in the reciprocal wave propagation medium, the off-diagonal elements of the scattering matrix are identical with each other. This results in a symmetric

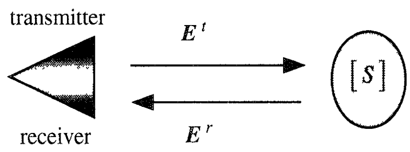


Fig. 1 Scattering matrix.

matrix as

$$[S] = \begin{bmatrix} a+b & c \\ c & a-b \end{bmatrix}, \tag{2}$$

where *a*, *b*, and *c* are complex quantities. This can be further expanded as a combination of three elementary scattering matrices,

$$[S] = a\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + c\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \tag{3}$$

This form is the starting point of coherent scattering matrix decomposition [3]. The problem here is to relate the scattering matrix elements to physically realizable targets or to find out some characteristic features from the scattering matrix structure (3).

It is known that the element of scattering matrix depends on the polarization basis, and that the transformation of scattering matrix from a polarization basis to another one is always possible, because the actual target is the same. Which polarization basis is the most convenient for scattering matrix decomposition? To answer this question, we write down some scattering matrices of elementary target in Table 1 for side by side comparison. In this table, the assumed target is rotated at angle θ with respect to the *H* direction as shown in Fig. 2. This rotation can be expressed in the scattering matrix as

$$\left[S_{(HV)}\right]_{\theta} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^T \begin{bmatrix} S_{HH} & S_{HV} \\ S_{HV} & S_{VV} \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \tag{4}$$

where the superscript *T* denotes transpose.

The transformation from the linear polarization basis *HV* to the circular polarization basis *LR* can be carried out by

$$\left[S_{(LR)}\right] = \begin{bmatrix} S_{LL} & S_{LR} \\ S_{LR} & S_{RR} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix}^T \begin{bmatrix} S_{HH} & S_{HV} \\ S_{HV} & S_{VV} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix}. \tag{5}$$

The typical feature in Table 1 is such that the scattering matrix of sphere or plate remains in the same form throughout the rotation in both bases, whereas wire or diplane element strongly depends on the rotation angle in the linear polarization basis. The scattering matrices keep the same forms with same element magnitude in the circular polarization basis. The rotation phenomenon appears in the phase term only. This fact may be anticipated on the Poincaré sphere [1] where the rotation of target corresponds to moving along the equator while the invariant point stays in the north and south poles (*LHC* and *RHC*). Therefore, it is easier to classify scattering matrix structure (magnitude and phase relation) in the circular polarization basis rather than in the linear basis with respect to the rotation of target, although the physical interpretation is easier in the linear *HV* basis.

Table 1 Scattering matrix in the two polarization bases.

	Linear basis (HV)	Circular basis (LR)
$[S]_{plate}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
$[S]_{diplane}$	$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$	$\begin{bmatrix} e^{j2\theta} & 0 \\ 0 & e^{-j2\theta} \end{bmatrix}$
$[S]_{wire}$	$\begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} e^{j2\theta} & 1 \\ 1 & e^{-j2\theta} \end{bmatrix}$
$[S]_{L\text{-}helix}$	$\frac{1}{2}e^{-j2\theta}\begin{bmatrix} 1 & j \\ j & -1 \end{bmatrix}$	$e^{-j2\theta}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
$[S]_{R\text{-}helix}$	$\frac{1}{2}e^{j2\theta}\begin{bmatrix} 1 & -j \\ -j & -1 \end{bmatrix}$	$e^{j2\theta}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

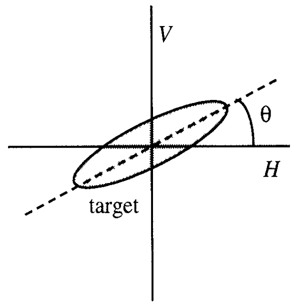


Fig. 2 Target rotation.

3. Decomposition of Scattering Matrix

Based on the Table 1, Krogager and Czyz [3] proposed a 3-component expansion in the circular polarization basis as follows

$$[S] = e^{j\varphi}\left\{ e^{j\varphi_s}K_s[S]_{sphere} + K_d[S]_{diplane} + K_h[S]_{helix} \right\} \tag{6}$$

where the first, the second, and the third term correspond to sphere (or plate), diplane (dihedral corner reflector), and helix, respectively. The factors K_s, K_d, and K_h represent the magnitude contribution. The phase φ_s in the first term is measured relative to the diplane's phase. For right-handed helix target inclusion, (6) has the explicit form,

$$\left[S_{(LR)}\right]=e^{j\varphi}\left\{e^{j\varphi_s}K_s\begin{bmatrix}0&1\\1&0\end{bmatrix}\right.$$

$$\left.+K_d\begin{bmatrix}e^{j2\theta}&0\\0&e^{-j2\theta}\end{bmatrix}+K_h\begin{bmatrix}e^{j2\theta}&0\\0&0\end{bmatrix}\right\},\quad (7a)$$

and for left-handed helix,

$$\left[S_{(LR)}\right]=e^{j\varphi}\left\{e^{j\varphi_s}K_s\begin{bmatrix}0&1\\1&0\end{bmatrix}\right.$$

$$\left.+K_d\begin{bmatrix}e^{j2\theta}&0\\0&e^{-j2\theta}\end{bmatrix}+K_h\begin{bmatrix}0&0\\0&e^{-j2\theta}\end{bmatrix}\right\}.\quad (7b)$$

Matching the elements of the scattering matrix for right helix

$$\left[S_{(LR)}\right]=e^{j\varphi}\begin{bmatrix}(K_d+K_h)\,e^{j2\theta}&K_s\,e^{j\varphi_s}\\K_s\,e^{j\varphi_s}&K_d\,e^{-j2\theta}\end{bmatrix}$$

$$=\begin{bmatrix}|S_{LL}|\,e^{j\varphi_{LL}}&|S_{LR}|\,e^{j\varphi_{LR}}\\|S_{LR}|\,e^{j\varphi_{LR}}&|S_{RR}|\,e^{j\varphi_{RR}}\end{bmatrix}\quad (8a)$$

and for left helix

$$\left[S_{(LR)}\right]=e^{j\varphi}\begin{bmatrix}K_d\,e^{j2\theta}&K_s\,e^{j\varphi_s}\\K_s\,e^{j\varphi_s}&(K_d+K_h)\,e^{-j2\theta}\end{bmatrix}$$

$$=\begin{bmatrix}|S_{LL}|\,e^{j\varphi_{LL}}&|S_{LR}|\,e^{j\varphi_{LR}}\\|S_{LR}|\,e^{j\varphi_{LR}}&|S_{RR}|\,e^{j\varphi_{RR}}\end{bmatrix}.\quad (8b)$$

yields the magnitude K_s, K_d, and K_h in terms of the element of scattering matrix in the circular polarization basis as

$$K_s=|S_{LR}|,\quad K_d=|S_{LL}|,\quad K_h=|S_{RR}|-|S_{LL}|$$
$$\text{for } |S_{RR}|>|S_{LL}|,\quad (9a)$$

$$K_s=|S_{LR}|,\quad K_d=|S_{RR}|,\quad K_h=|S_{LL}|-|S_{RR}|$$
$$\text{for } |S_{RR}|<|S_{LL}|,\quad (9b)$$

Table 2 Contribution of amplitude factors for elementary targets.

	K_s	K_d	K_h
sphere, plate, trihedral	1	0	0
diplane (corner reflector)	0	1	0
line (wire)	0.5	0.5	0
right / left > helix	0	0	1

and the phase components as

$$\theta=\frac{1}{4}\left(\varphi_{LL}-\varphi_{RR}\right)\quad (10)$$

$$\varphi=\frac{1}{2}\left(\varphi_{LL}+\varphi_{RR}\right)\quad (11)$$

$$\varphi_s=\varphi_{LR}-\frac{1}{2}\left(\varphi_{LL}+\varphi_{RR}\right).\quad (12)$$

Theoretical contributions of K_s, K_d, and K_h for elementary targets are listed in Table 2.

4. Decomposition Algorithm

It may be possible to decompose a scattering matrix into three components, with each contribution given by

$$\frac{K_i}{K_s+K_d+K_h}\quad\left(i=s,d,h\right)\quad (13)$$

referring to Table 2. Here we propose a decomposition algorithm based on Table 2. The block diagram of the decomposition algorithm is shown in Fig. 3. This algorithm is based on the magnitude of the scattering matrix element. First, we check the magnitude of off-diagonal element, and classify the scattering matrix into two categories. After examining the magnitude of diagonal elements, we come up to the final elementary scattering matrices (see Fig. 3).

It is interesting to note in Table 2 that a wire target consists of diplane and sphere components with 50% contributions, although the physical shape is quite simple.

$$\left[S\right]_{wire}=\frac{1}{2}\left(\left[S\right]_{diplane}+\left[S\right]_{sphere}\right)\quad (14)$$

If a scattering matrix was recognized as a wire or diplane using (13), there is a possibility to determine the rotation angle by (10). It should be noted that the absolute phase (11) is dependent on the target range due to the phase shift of propagation path, and that the rotation angle (10) is independent of the range, i.e., it can be determined by the relative scattering matrix only. However, there exists uncertainty in the rotation angle such that

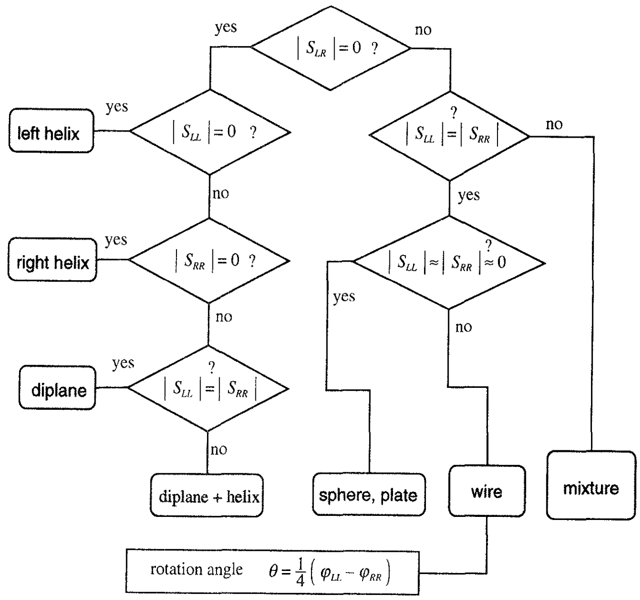


Fig. 3 Block diagram of target decomposition in the circular polarization basis.

$$[S]_{diplane(\theta)}=\begin{bmatrix} e^{j2\theta} & 0 \\ 0 & e^{-j2\theta} \end{bmatrix},$$

$$[S]_{diplane(\theta+\pi/2)}=\begin{bmatrix} e^{j2(\theta+\pi/2)} & 0 \\ 0 & e^{-j2(\theta+\pi/2)} \end{bmatrix}$$

$$=-\begin{bmatrix} e^{j2\theta} & 0 \\ 0 & e^{-j2\theta} \end{bmatrix}.$$

This means the diplanes rotated θ degrees and θ+90 degrees give the same form of scattering matrix in the same range except for the sign. In this case, it is necessary to take the phase (12) of sphere into account. Since there exists uncertainty of $2n\pi$ ($n=0,1,2,3,\cdots$) in the phases θ, φ and φ_s, we eliminate this uncertainty by putting $\varphi_s=0$ for wire target because it is independent of diplane. This elimination results in the determination of the rotation angle θ. This will be verified in the experiment which follows.

5. Experiment

In order to examine how the decomposition theorem and algorithm apply to the FM-CW radar scattering matrix, we carried out imaging and decomposition experiment in a laboratory. Using two orthogonal linear polarized antennas operative at 14.5-15.5 GHz, 2-dimensional image were obtained. The radar specifications are listed in Table 3. The experimental scheme is shown in Fig. 4. After the system calibration [8] and the synthetic aperture processing, it is possible to make a radar image consisting of 64× 64 pixels, each having own scattering matrix. It should be noted that the FM-CW radar scattering matrix is obtained as the result

Table 3 Radar specification.

Frequency	14.5 - 15.5 GHz
Scanning interval	0.8 cm
Scanning points	64×64
Distance to target	120 cm
Polarization	HH, VV, HV

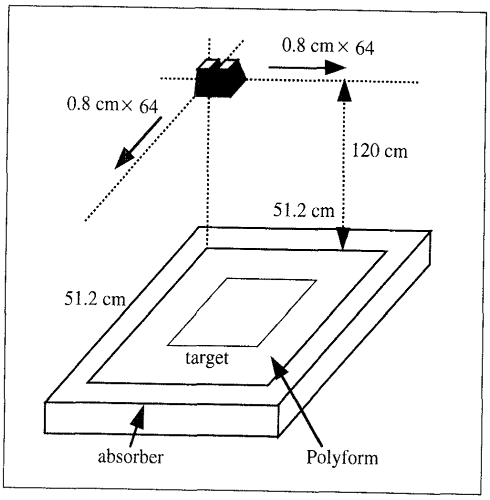


Fig. 4 Experimental scheme.

of swept frequency measurement.

The first experiment aims at classifying targets by the decomposition algorithm. The targets employed were:

three plates (4.5× 20 cm^2, 10× 17 cm^2 × 2),

seven dihedral corner reflectors (10× 10 cm^2, 8× 8 cm^2× 2, 6× 6 cm^2 × 2 , 4× 4 cm^2 × 2),

a line target (10× 8 cm^2) which has been specially designed to act as a wire with large RCS [8].

The arrangement of these targets is shown in Fig. 5. Figure 6 shows the decomposition result of measured scattering matrix for each component (i.e., component distribution) and Fig. 7 shows the overall decomposition result image where each pixel represents the main contribution of four fundamental scattering matrices. The contribution of K_i more than 70% magnitude compared to other K's is displayed in a single pixel. For smaller value of 70% contribution, especially for K_d and K_s, there is a possibility to classify this scattering matrix as a wire. In this case, a magnitude imbalance $K_s:K_d=1{:}2$ *to* 2:1 was allowed to recognize a wire, provided that $K_s+K_d>70\%$.

It is seen in Fig. 7 that the decomposition of targets is quite satisfactory, according to the target arrangement. Although the image itself is somewhat distorted, the overall image is well detected and the pixel component exhibits the actual target configuration. The smallest corner reflector is

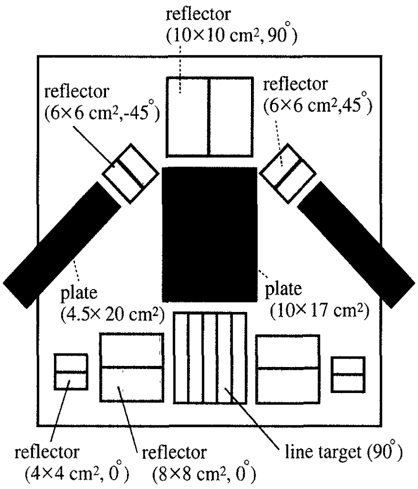


Fig. 5 Target arrangement.

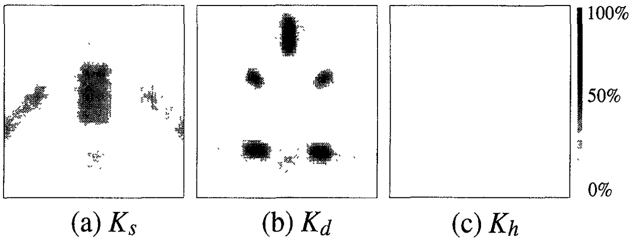


Fig. 6 Three component distribution.

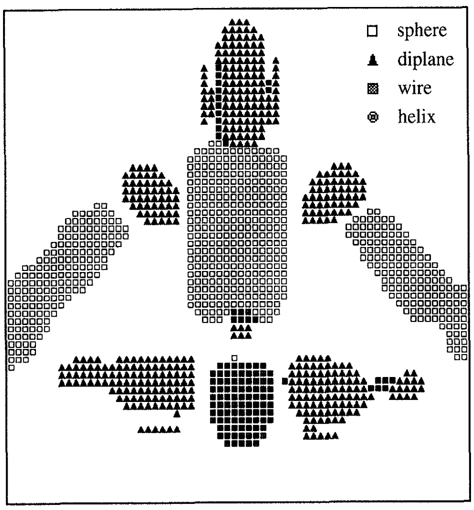


Fig. 7 Decomposition result of Fig. 5.

difficult to detect because of small RCS. However, it is possible to recognize it as a corner reflector component in the decomposition image.

The second experiment is to examine the ability of determining rotation angle of wire. Three line targets were placed at angles of 90, 45, and –45 degrees with respect to the H-direction in the imaging plane as shown in Fig. 8. First, the targets were imaged in the same way as the previous experiment, then, they were recognized as "wire" from the decomposition of scattering matrices (see Fig. 9). At this stage, we can know that the targets are "wire." So, the rotation angle was calculated based on the Eq. (10), yielding

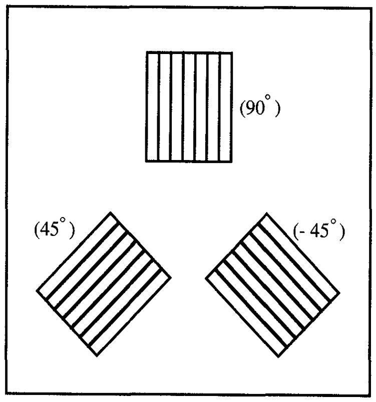


Fig. 8 Line target arrangement.

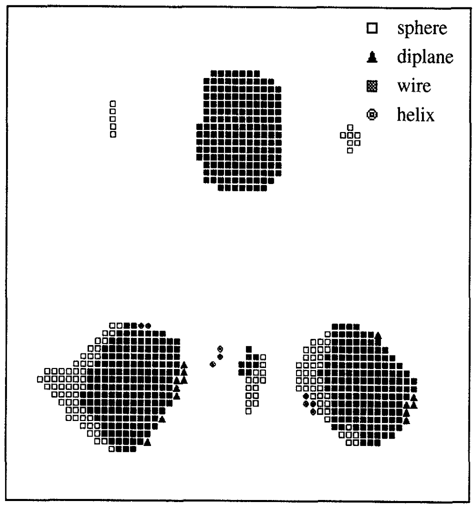


Fig. 9 Decomposition result of Fig. 8.

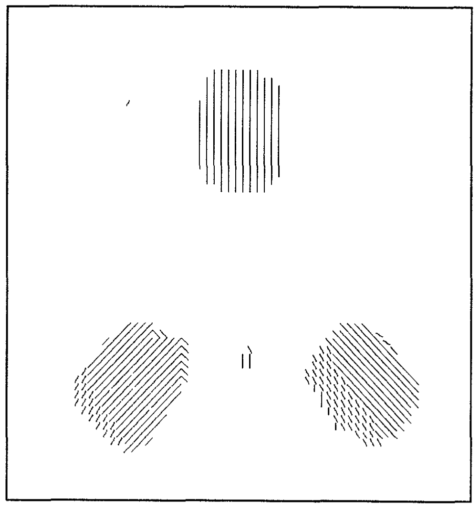


Fig. 10 Detected rotation angles of Fig. 8.

a satisfactory result in Fig. 10.

This determination scheme of rotation angle is completely different from [6] in the sense whether the target is known or unknown. The target is known as wire from the outset in [6]. The rotation angle here is determined after the decomposition of the scattering matrix.

6. Conclusion

In this paper, we proposed a scattering matrix decomposition algorithm in the circular polarization basis and applied the technique to an FM-CW radar scattering matrix. This decomposition scheme was successfully carried out to classify elementary targets in a laboratory experiment. The components of the scattering matrix decomposition are sphere, diplane, and helix. It was shown that the decomposition to FM-CW radar scattering matrix was possible although the scattering matrix was obtained by a swept frequency measurement. If a scattering matrix is classified as wire by taking the phase difference of the spherical and diplane component into account, it is possible to determine the rotation angle with respect to the radar scanning direction. This point was also confirmed in the experiment. Further efficient coherent decomposition technique including the absolute phase will be needed for classifying radar targets, which will be treated in a near future.

References

[1] J.R.Huynen, "Phenomenological theory of radar targets," Doctoral Thesis, Technical University, Delft, The Netherlands, 1970.

[2] S.R.Cloude, "Target decomposition theorems in radar scattering," Electronic Letters, vol.21, no.1, pp.22-24, Jan. 1985.

[3] E.Krogager and Z.H.Czyz, "Properties of the sphere, diplane, helix decomposition," Proc. Third International Workshop on Radar Polarimetry, vol.1, pp.106-114, March 1995.

[4] E.Krogager, J.Dall, and S.N.Madsen, "The sphere, diplane, helix decomposition recent results with polarimetric SAR data," Proc. Third International Workshop on Radar Polarimetry, vol.2, pp.621-625, March 1995.

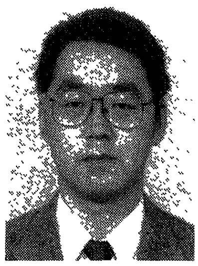
[5] S.R.Cloude and E.Pottier, "A review of target decomposition theorems in radar polarimetry," IEEE Trans. Geoscience Remote Sensing, vol.34, no.2, pp.498-518, March 1996.

[6] Y.Yamaguchi, T.Nishikawa, and M.Sengoku, "Fundamental study on synthetic aperture FM-CW radar polarimetry," IEICE Trans. Commun., vol.E77-B, no.1, pp.73-80, 1994.

[7] W.-M.Boerner, et al. eds., "Direct and Inverse Methods in Radar Polarimetry," NATO-ARW, Part 1 and 2, Netherlands, Kluwer Academic Publishers, 1992.

[8] Y.Takayanagi, Y.Yamaguchi, and H.Yamada, "On the calibration of polarimetric FM-CW radar," IEICE Technical Report, SANE95-75, Oct. 1995.

[9] M.Nakamura, Y.Yamaguchi, and H.Yamada, "Consideration on the decomposition of radar targets by polarimetric scattering matrix," IEICE Technical Report, AP96-34, June 1996.



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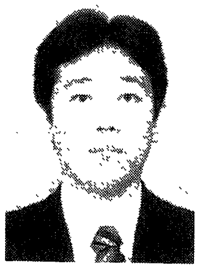

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