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LETTER

Generalized Spatial Correlation Equations for Antenna Arrays in Wireless Diversity Reception: Exact and Approximate Analyses

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SUMMARY Multiple antenna systems are promising architectures for overcoming the effects of multi-path interference and increasing the spectrum efficiency. In order to be able to investigate these systems, in this article, we derive generalized spatial correlation equations of a circular antenna array for two typical angular energy distributions: a Gaussian angle distribution and uniform angular distribution. The generalized spatial correlation equations are investigated carefully by exact and approximate analyses.

key words: *spatial correlation, angular spread, antenna array and diversity reception*

1. Introduction

It is well known that correlation in fading across multiple diversity results in a degradation of the diversity gain obtained. Classic work on analysis of diversity with correlated fading channels was done in Refs. [1]–[6]. For simplicity, Refs. [3]–[5] investigated the spatial correlation functions of a linear antenna array. Although recently Ref. [6] has proposed the method to investigate the uniform circular array (UCA), the performance of it has not extensively studied for a Gaussian angle distribution. Also it becomes very complicated in the calculation these correlation equations and also indeed affects the calculation speed in investigating the complex antenna arrays. Then, in this letter, in order to investigate multiple antenna systems with flexible architectures, we derive generalized spatial correlation equations of a circular antenna array for two typical angular energy distributions: a Gaussian angle distribution and uniform angular distribution. The generalized spatial correlation equations are investigated carefully by exact and approximate analyses. These equations are parameterized by radius of circular R, angular spread Δ for uniform angular distribution and the standard deviation σ for Gaussian angle distribution. Approximate the exact analysis with approximation is very good for angular spread about 5–10 degrees that is dependent on the incident angle ϕ. These numerical results are useful to simulate and investigate the performance of macrocell environments.

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2. Array Geometry and Angular Energy Distribution

A. Array geometry

In antenna array system, an antenna array is used at the base station to receive information from users of wireless networking operating under the same or different multiple access schemes such as FDMA, TDMA and CDMA. The antenna array may assume different geometries. In a linear array, the locations of the antennas form a straight line, whereas in a planar array (such as circular array), the positions of the antenna elements are specified by two variables representing polar or Cartesian coordinates. While the propagation delay T between antennas encountered as the signal travels across a linear array is only a function of the elevation angle, both elevation and azimuth angles of arrival define the propagation delay in the case of planar arrays. For simplicity, only azimuth plane is considered in the geometry models. Figure 1 shows the antenna array geometry used in our investigation where we have estimated a circular geometry with a radius R and different antenna element M where the antenna element M lie about at a radius of $r = R$ for our circular antenna array.

The multiple antennas may be omnidirectional or have a nonuniform sensitivity to the angle and frequency signature of the incident waveform. The antennas may be equally or unequally spaced across the array. In wireless communication environment, antenna spacing plays an important role in developing the proper processing method to resist the distortion caused by users accessing the same communication channel and by local and remote scattering of the signal of interest.

B. Angular energy distribution

The first model for a spatial channel is a Gaussian

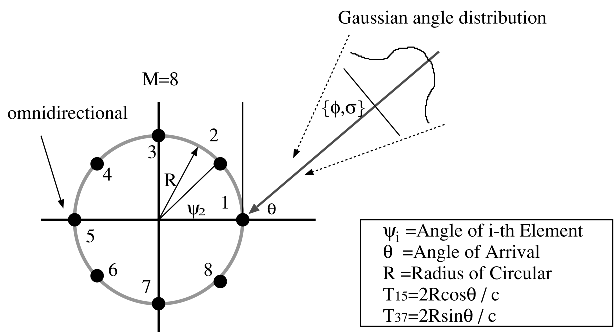

Fig. 1 Signal angle determination in a circular antenna array with 8 elements.

angle distribution which is commonly used [4], [5]. Thus the angular distribution function can be represented as

$$p(\theta) = \frac{\kappa}{\sqrt{2\pi}\sigma} e^{-(\theta-\phi)^2/2\sigma^2} \quad \text{for} \quad \theta \in [-\pi+\phi, \pi+\phi] \tag{1}$$

where ϕ and σ are the mean direction of arrival and the standard deviation of the distribution. κ is the normalization factor, to make $p(\theta)$ a physical density function, i.e.

$$\kappa = \frac{1}{erf(\frac{\pi}{\sqrt{2}\sigma})} \tag{2}$$

where $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ is the error function. Note when the angular spread is small, κ is almost equal to unity.

Another common assumption for angular energy distribution is a uniform distribution. A uniform distribution of angular energy is defined as [6]

$$p(\theta) = \frac{1}{2\Delta}, \quad \text{for} \quad \theta \in [\phi - \Delta, \phi + \Delta] \tag{3}$$

where 2Δ is the range of angles about a central angle-of-arrival ϕ.

3. Spatial Correlated Functions

From the geometry of circular array shown in Fig. 1, the antenna elements are arranged to form a circular with the radius of R. Like the linear array, the array response vector $v_i(\theta)$ can be written as where,

$$v_i(\theta) = e^{-j2\pi \frac{R}{\lambda} \sin(\xi) \cos(\theta - \Psi_i)}, \quad \text{where} \quad i = 1, 2, \cdots, M$$

where R is the circular radius of antenna array shown in Fig. 1 and λ is the wavelength. ξ is the elevation angle where $\xi = 90$ degrees (only azimuth is considered). Ψ_i is the angle of $i-th$ element in azimuth plane.

The array spatial correlation between the m and n antenna element is generally defined as [1]–[3]

$$\rho(m,n) = E[v_m(\theta) v_n(\theta)^*]$$

$$= \int_\theta v_m(\theta) v_n(\theta)^* p(\theta) d\theta \tag{4}$$

where $p(\theta)$ is the probability density function of incident signal.

We wish to show that spatial correlation function for a circular antenna array when the arrival signals are following a uniform angular energy distribution. According to the definition of correlation function in Eq. (4) and the uniform distribution function shown in Eq. (3), we can obtain the correlation function formula with exact analysis as

$$\rho(m,n) = \int_\theta v_m(\theta) v_n(\theta)^* p(\theta) d\theta = \frac{1}{2\Delta} \int_{\phi-\Delta}^{\phi+\Delta} e^{-j2\pi \frac{R}{\lambda} (\cos(\theta - \Psi_m) - \cos(\theta - \Psi_n))} d\theta \tag{5}$$

If we define $Z_1 = 2\pi \frac{R}{\lambda} [\cos(\Psi_m) - \cos(\Psi_n)]$ and $Z_2 = 2\pi \frac{R}{\lambda} [\sin(\Psi_m) - \sin(\Psi_n)]$. So $Z_C = \sqrt{Z_1^2 + Z_2^2}$ can be defined. Let $\sin(\gamma) = Z_1/Z_C$ and $\cos(\gamma) = Z_2/Z_C$, then Eq. (5) can be transformed as

$$\rho(m,n) = \frac{1}{2\Delta} \int_{\phi-\Delta+\gamma}^{\phi+\Delta+\gamma} e^{-jZ_C \sin(\zeta)} d\zeta \tag{6}$$

where $\zeta = \gamma + \theta$. In this formula, $e^{-jZ_C \sin(\zeta)} = \cos(Z_C \sin(\zeta)) - j \sin(Z_C \sin(\zeta))$ can be expressed by the modified Bessel functions as follows

$$\cos(Z_C \sin(\zeta)) = J_0(Z_C) + 2 \sum_{k=1}^{\infty} J_{2k}(Z_C) \cos(2k\zeta)$$
$$\sin(Z_C \sin(\zeta)) = 2 \sum_{k=0}^{\infty} J_{2k+1}(Z_C) \sin((2k+1)\zeta) \tag{7}$$

After substituting Eq. (7) into Eq. (6) and integrating it, the real and imaginary parts of the correlation function $\rho(m,n)$ of circular antenna arrays for a uniform distribution can be expressed as

$$Re[\rho(m,n)] = J_0(Z_C) + 2 \sum_{k=1}^{\infty} J_{2k}(Z_C) \cos(2k(\phi+\gamma)) \text{sinc}(2k\Delta) \tag{8}$$

$$Im[\rho(m,n)] = 2 \sum_{k=0}^{\infty} J_{2k+1}(Z_C) \cdot \sin((2k+1)(\phi+\gamma)) \text{sinc}((2k+1)\Delta) \tag{9}$$

Also, we wish to show that the generalized spatial correlation function for the uniform angular distribution can be approximated according to Eq. (6) for small Δ. We present Eq. (6) and modify it by using $z = \zeta - (\phi + \gamma)$ as

$$\rho(m,n) = \frac{1}{2\Delta} \int_{-\Delta}^{\Delta} e^{-jZ_C \sin(z+\phi+\gamma)} dz$$

$$= \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} e^{-jZ_C(\sin(z)\cos(\phi+\gamma)+\cos(z)\sin(\phi+\gamma))}dz \quad (10)$$

Now for small Δ, we can approximate $\sin(z) \approx z$ and $\cos(z) \approx 1$ that gives the approximate equation as

$$\rho(m,n) \approx \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} e^{-jZ_C(z\cos(\phi+\gamma)+\sin(\phi+\gamma))}dz$$
$$\approx \frac{1}{2\Delta}e^{-jZ_C\sin(\phi+\gamma)}\int_{-\Delta}^{\Delta} e^{-jZ_C z\cos(\phi+\gamma)}dz \quad (11)$$

The integral could be carried out in a closed form applying the mathematic derivation then simplifying Eq. (6) results in

$$\rho(m,n) \approx e^{-jZ_C\sin(\phi+\gamma)}\sin c\big[Z_C\Delta\cos(\phi+\gamma)\big] \quad (12)$$

If the incident signal is not the uniform distribution and the incident signal follows the Gaussian angle distribution by Eq. (1), we can derive correlation function of the circular antenna array for the Gaussian angle distribution. Using the same definitions in Eqs. (4) and (1), the spatial correlation function with exact analysis can be determined as

$$\rho(m,n) = \frac{\kappa}{\sqrt{2\pi}\sigma}\int_{-\pi+\phi}^{\pi+\phi} e^{-(\frac{\theta-\phi}{\sqrt{2}\sigma})^2}e^{-jZ_C\sin(\gamma+\theta)}d\theta \quad (13)$$

If we let $\chi = \frac{\theta-\phi}{\sqrt{2}\sigma}$, then the parameter θ can be transformed as $\theta = \sqrt{2}\sigma\chi + \phi$ and make a change of variables, so $\chi \in [-\frac{\pi}{\sqrt{2}\sigma}, \frac{\pi}{\sqrt{2}\sigma}]$. Therefore we obtain

$$\rho(m,n) = \frac{\kappa}{\sqrt{\pi}}\int_{-\frac{\pi}{\sqrt{2}\sigma}}^{\frac{\pi}{\sqrt{2}\sigma}} e^{-\chi^2}e^{-jZ_C\sin(\gamma+\theta+\sqrt{2}\sigma\chi)}d\chi$$
$$= \frac{\kappa}{\sqrt{\pi}}\int_{-\frac{\pi}{\sqrt{2}\sigma}}^{\frac{\pi}{\sqrt{2}\sigma}} e^{-\chi^2}\cos(Z_C\sin(\gamma+\theta+\sqrt{2}\sigma\chi))d\chi$$
$$+\frac{\kappa}{\sqrt{\pi}}\int_{-\frac{\pi}{\sqrt{2}\sigma}}^{\frac{\pi}{\sqrt{2}\sigma}} e^{-\chi^2}j\sin(Z_C\sin(\gamma+\theta+\sqrt{2}\sigma\chi))d\chi \quad (14)$$

Now, substituting Eq. (7) into Eq. (14), we can get the real and imaginary parts of $\rho(m,n)$ equation shown as

$$Re[\rho(m,n)] = \frac{\kappa}{\sqrt{\pi}}\int_{-\frac{\pi}{\sqrt{2}\sigma}}^{\frac{\pi}{\sqrt{2}\sigma}} e^{-y^2}\Bigg[J_0(Z_C) +2\sum_{k=1}^{\infty}J_{2k}(Z_C)\cos(2k(\gamma+\sqrt{2}\sigma y+\phi))\Bigg]dy \quad (15)$$

$$Im[\rho(m,n)] = \frac{\kappa}{\sqrt{\pi}}\int_{-\frac{\pi}{\sqrt{2}\sigma}}^{\frac{\pi}{\sqrt{2}\sigma}} e^{-y^2}\Bigg[2\sum_{k=0}^{\infty}J_{2k+1}(Z_C) \cdot\sin((2k+1)(\gamma+\sqrt{2}\sigma y+\phi))\Bigg]dy \quad (16)$$

We hope to show that the generalized spatial correlation function for the Gaussian angle distribution can be approximated according to Eq. (13) for small σ. We present Eq. (13) and modify it by defining $z = \frac{\theta-\phi}{\sigma}$ as

$$\rho(m,n) = \frac{\kappa}{\sqrt{2\pi}\sigma}\int_{-\frac{\pi}{\sigma}}^{\frac{\pi}{\sigma}} e^{-\frac{z^2}{2}}e^{-jZ_C\sin(\gamma+\phi+z\sigma)}dz \quad (17)$$

Now for small σ, we can approximate $\sin(z\sigma) \approx z\sigma$ and $\cos(z\sigma) \approx 1$ that gives the approximate equation as

$$\rho(m,n) \approx \frac{\kappa}{\sqrt{2\pi}\sigma}e^{-jZ_C\sin(\gamma+\phi)} \cdot\int_{-\frac{\pi}{\sigma}}^{\frac{\pi}{\sigma}} e^{-\frac{z^2}{2}}e^{-jZ_C\sigma z\cos(\gamma+\phi)}dz \quad (18)$$

By applying the mathematic derivation then simplifying Eq. (18) results in

$$\rho(m,n) \approx \kappa e^{-jZ_C\sin(\gamma+\phi)}e^{-\frac{(Z_C\sigma\cos(\gamma+\phi))^2}{2}} \quad (19)$$

4. Numerical Results

To demonstrate the usefulness of these derivations and to verify the coincidence of the exact and approximate analysis, we consider a circular antenna array as shown in Fig. 1 where these antenna elements are designed as non-directional and arranged in uniform architecture. Our derived formula can be also used in investigation of non-uniform architecture for some peculiar applications in which the directional antenna element could be introduced and the arrangement of these elements may be non-uniform in the circular ring.

As a example, we evaluate the spatial correlation between 1*st* element and 4*th* element, $|\rho(1,4)|$ with the different incident angle ϕ. We plot the correlation as the function of array radius R, and the angle spread Δ and σ. In Figs. 2 and 3, we plot the spatial correlations versus R/λ for different incident angle ϕ and several value of Δ where the incident signal is with uniform angular distribution. As expected, the correlation decreases as R increases. Further, as Δ increases, the correlation decreases. In the investigation, the numerical results are calculated by using Eqs. (8) and (9) for the exact analysis, and Eq. (12) for the approximate analysis. We can see that for Δ up to 5 degrees the approximate analysis is a very good fit to the exact analysis as $\phi = 0$ degree and $R/\lambda < 2$. Figure 3 presents the same results for $\phi = 30$ degrees. We can see that for Δ up to 10 degrees the approximate analysis is a very good fit to the exact analysis. Comparing Figs. 3 and 4, as ϕ increases, the fit is better for moderate value of Δ. One see that with a angular spread Δ less than about 5–10 degrees, the approximate analysis exhibit good agreement with the exact analysis that is dependent on the incident angle ϕ.

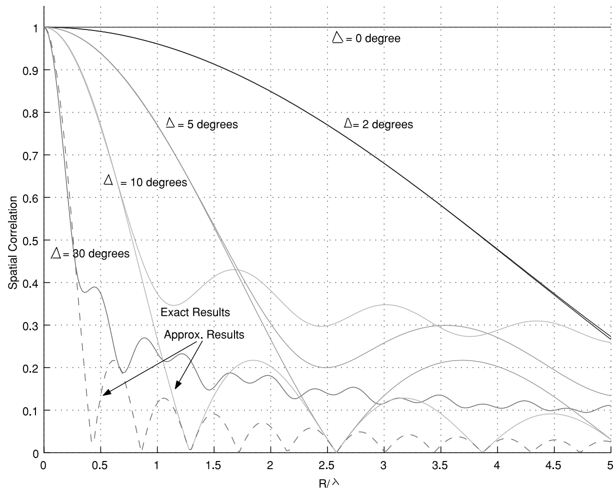

Fig. 2 Spatial correlation between the elements 1 (Ψ_1=0 degree) and 4 (Ψ_4=135 degrees) versus R/λ with uniform distribution of angular energy (ϕ=0 degrees, ξ=90 degrees) for circular array.

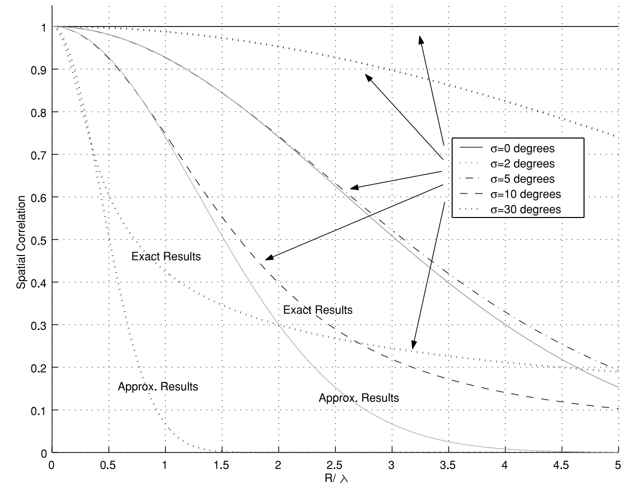

Fig. 4 Spatial correlation between the elements 1 (Ψ_1=0 degree) and 4 (Ψ_4=135 degrees) versus R/λ with Gaussian distribution of angular energy (ϕ=0 degrees, ξ=90 degrees) for circular array.

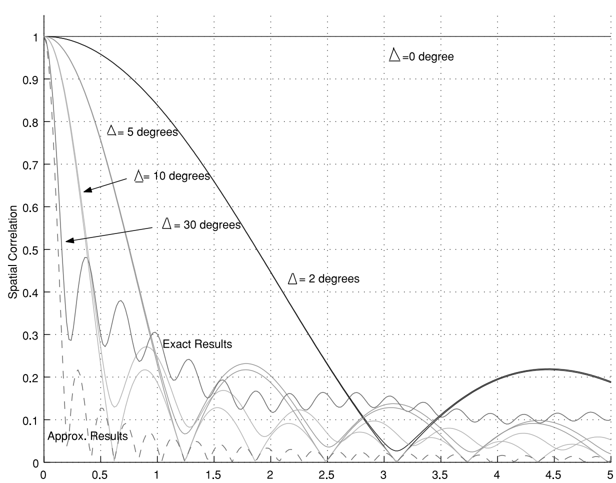

Fig. 3 Spatial correlation between the elements 1 (Ψ_1=0 degree) and 4 (Ψ_4=135 degrees) versus R/λ with uniform distribution of angular energy (ϕ=30 degrees, ξ=90 degrees) for circular array.

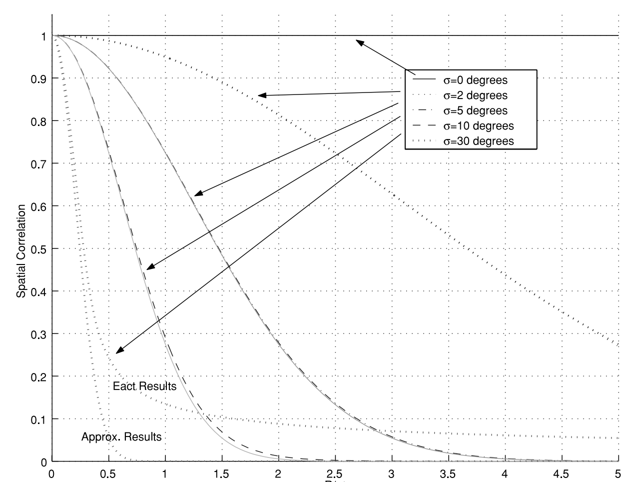

Fig. 5 Spatial correlation between the elements 1 (Ψ_1=0 degree) and 4 (Ψ_4=135 degrees) versus R/λ with Gaussian distribution of angular energy (ϕ=30 degrees, ξ=90 degrees) for circular array.

Figures 4 and 5 show the exact and approximate correlation results for Gaussian angular energy distribution with various standard deviation σ for incident angle $\phi = 0$ and $\phi = 30$ degrees. For Gaussian angular energy distribution, one can see that for σ up to 5 degrees the approximate analysis is a very good fit to the exact analysis as $\phi = 0$ degree and $R/\lambda < 1.5$, and up to 10 degrees as $\phi = 30$ degrees. Comparing Figs. 4 and 5, as ϕ increases, the fit is better for moderate value of σ. From these results, with a standard deviation σ less than about 5–10 degrees, we also can conclude the approximate analysis exhibit good agreement with the exact analysis. If we compare the Gaussian and uniform distributions for several value of ϕ, as expected, the Gaussian distribution decreases more slowly in the main lobe, but lacks the secondary correlation peaks.

5. Conclusions

In this letter, we derived generalized spatial correlation functions for two typical distributions of angular energy for a circular antenna array. These equations are investigated carefully by exact and approximate analyses. The generalized formulas allow the correlation to be found for any practical standard deviation and antenna array geometry. We have shown that the approximations are good for standard deviations of about 10 degrees or less based on the incident angle ϕ. One also can see the Gaussian distribution decreases more slowly in the main lobe, but lacks the secondary correlation peaks contract with uniform distribution. These results also will be used and play the key role in designing smart antenna systems use signal processing methods in conjunction with multiple antennas to achieve significant improvements in capacity and coverage range for mobile communication systems.

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