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Analysis of the Reflection Method for Measuring the Radiation Efficiency Using the Transmission Line Model

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SUMMARY The reflection method is an accurate and simple method for measuring the radiation efficiency of a small antenna. However, it takes too long and has the disadvantage of underestimating the efficiency due to resonance in the cavity formed by the straight waveguide and two sliding shorts. To reduce the measurement time, one sliding short can be fixed while the other one is moved. To improve the accuracy of this technique, we can set the antenna to be measured at the center of the two sliding shorts or at a local anti-node of the standing wave in the waveguide. When one of the sliding shorts is fixed, the measured efficiency becomes negative at certain frequencies. We examine these reductions in efficiency using an equivalent transmission line model for the reflection method. We also derive analytical expressions for the overall efficiency in the above cases and verify new procedures that enable measurements to be performed without any drops in the measured efficiency.

key words: *radiation efficiency, reflection method, transmission line model, waveguide, sliding short*

1. Introduction

As antennas are small compared to the wavelength, they have small gains and the magnitude of their reflection coefficients is close to unity; that is, total reflection occurs at the input port of the antenna. As a result of this small gain, the ohmic loss of an antenna's material exceeds the radiation loss of an antenna. In other words, the radiation efficiency, which is defined as the ratio of the radiated power to the input power, is greatly reduced as the dimensions of the antenna are decreased. The radiation efficiency is, therefore, one of the most important parameters for evaluating the performance of small antennas and it is thus vital to establish accurate measurement methods for estimating the radiation efficiency. The pattern integration method is well known as a typical method for measuring the radiation efficiency [1]. In general, it is accurate if an appreciable angular separation is employed. It, however, requires enormous cost and time since it measures the radiated power in an anechoic chamber.

By contrast, in the reflection method discussed in this paper, the radiation efficiency is estimated by measuring the reflection coefficients of an antenna in free space using a waveguide shorted by two sliding shorts [2]. Therefore, this measurement system is inexpensive, compact and rapid. Its validity has been demonstrated by comparing results obtained using it with those obtained using the pattern integration method [2]. However, it suffers from the disadvantage that the measured efficiency is sometimes lower than the actual efficiency due to resonance in the cavity formed by the waveguide and two sliding shorts [3], [4]. Although some papers suggest that this underestimation of the efficiency is due to the resistance of the cavity wall at resonance exceeding the radiation resistance [3], no practical models have been proposed for examining this effect.

In this paper, the antenna in the cavity is modeled using the transmission line model [4]. The two sliding shorts can be modeled by a resistive load having a small resistance so that the loss at the cavity wall can be included in the transmission line model. The expressions for the reflection coefficients (whose magnitudes are ideally unity) of the sliding shorts used in previous studies should be replaced by expressions that include the loss at the cavity wall. After performing extensive calculations, it was found that the efficiencies calculated in earlier studies are the product of the true radiation efficiency and the transmission efficiency in the section formed by the waveguide and the two sliding shorts [5]. If the cavity is lossless, the transmission efficiency is zero so the efficiency is not underestimated. Furthermore, we clarify that the underestimations of the actual efficiency observed in previous studies are due to the resistance of the cavity wall at resonance.

2. Measurement Principle and Experimental System for the Reflection Method

2.1 Measurement Principle in the Original Study

An antenna in free space can be considered to be a linear, passive, reciprocal two-port network, which is fed at port 1 and is connected to free space at port 2. Since the power from the antenna is radiated into free space at port 2, the reflection coefficient of port 2 is zero. In this case, the radiation efficiency $\eta_{\rm ant}$ is given by

$$\eta_{\rm ant} = \frac{|S_{21}|^2}{1-|S_{11}|^2}, \tag{1}$$

where S_{ij}, $i, j = 1, 2$, are S parameters of the network. Now, $|S_{11}|$ can be determined by measuring the magnitude of the antenna's reflection coefficient in free space. In the reflection method, $|S_{21}|$ is determined by measuring the reflection

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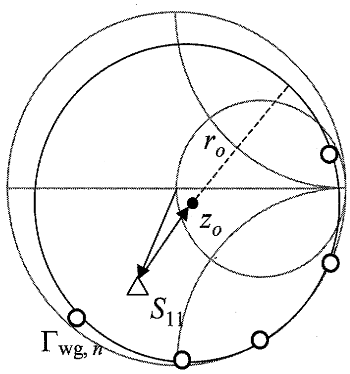

Fig. 1 S_{11} and $\Gamma_{wg,n}$ plotted on a Smith chart.

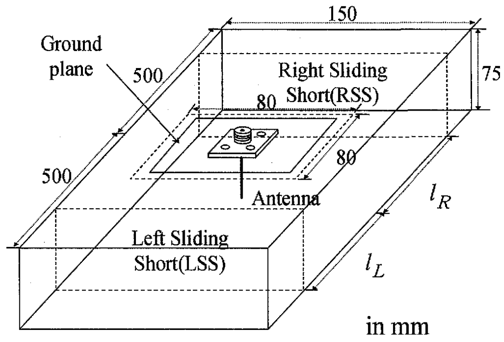

Fig. 2 Rectangular waveguide with a square aperture.

coefficients of the antenna when it is inserted in the short-circuited waveguide for three or more combinations of the positions of the two sliding shorts. These reflection coefficients are denoted by $\Gamma_{wg,n}$, $n = 1, 2, \cdots$. In the following, a short-circuited line refers to the waveguide and two sliding shorts, which are connected to port 2 of the network. If Γ_n denotes the reflection coefficient of the short-circuited line at port 2, the reflection coefficient at port 1 is given by [2]

$$\Gamma_{wg,n} = S_{11} + \frac{S_{21}^2 \Gamma_n}{1 - S_{22}\Gamma_n}. \qquad (2)$$

If the short-circuited line is ideal, then the magnitude of the reflection coefficient will be unity; in other words, Γ_n defines a circle having a center of 0 and a radius of 1 on the Smith chart. That is, it can be described by $\Gamma_n = e^{j\theta_n}$, where θ_n is the phase of the reflection coefficient, Γ_n. In accordance with the bilinear transformation nature of the complex plane, if Γ_n defines a circle, then $\Gamma_{wg,n}$ defines another circle, as can be seen from (2). If $S_{11} + z_o$ and r_o represent the center and the radius of the circle respectively, $\Gamma_{wg,n}$, then $|S_{21}|$ is given by [4]

$$|S_{21}|^2 = r_o - \frac{|z_o|^2}{r_o}. \qquad (3)$$

Then, from (1), the radiation efficiency is given by

$$\eta_{\text{ant}} = \frac{1}{1 - |S_{11}|^2}\left(r_o - \frac{|z_o|^2}{r_o}\right). \qquad (4)$$

And, $S_{11} + z_o$ and r_o can be determined by plotting three or more $\Gamma_{wg,n}$s on a Smith chart and fitting them with a circle by the least-squares method, as shown in Fig. 1 [4].

2.2 Modified Measurement Principle

In practice, the short-circuited line is not ideal. For example, the resistance of the sliding shorts cannot be ignored at or near the resonant frequencies of the cavity. We thus assume that the magnitude of the reflection coefficient $|\Gamma_n|$ is not unity and that Γ_n has a center of z_i and radius of r_i. That is, it can be described by $\Gamma_n = z_i + r_i e^{j\theta}$, where θ is a real number. In this case, $\Gamma_{wg,n}$ defines a circle on a Smith chart, as we can see from (2). If the center and radius of the mapped circle are $S_{11} + z_o$ and r_o, respectively, then $|S_{21}|$ is given by

$$|S_{21}|^2 = \frac{r_o - \dfrac{|z_o|^2}{r_o}}{r_i - \dfrac{|z_i|^2}{r_i}}. \qquad (5)$$

The derivation of (5) is given in the appendix. Substituting (5) into (1), gives the following relation [5]:

$$\eta_{\text{net}} = \eta_{\text{ant}} \cdot \eta_{\text{line}}, \qquad (6)$$

where,

$$\eta_{\text{net}} = \frac{1}{1 - |S_{11}|^2}\left(r_o - \frac{|z_o|^2}{r_o}\right), \qquad (7)$$

$$\eta_{\text{line}} = r_i - \frac{|z_i|^2}{r_i}. \qquad (8)$$

η_{net} given by (7) is the same as (4), which represents the radiation efficiency measured by the reflection method if the short-circuited line is ideal. While the transmission efficiency given by (7) includes the efficiency of the variable short-circuited line given by (8), the original efficiency given by (4) does not include the effect of the short-circuited line. (6) states that η_{net}, which can be determined by measuring $|S_{11}|$ and $\Gamma_{wg,n}$, is given by the product of the true radiation efficiency η_{ant} and the transmission efficiency of the short-circuited line η_{line}. In other words, (4) can be derived if the loss in the short-circuited line is ignored, that is, if $z_i = 0$ and $r_i = 1$, or if $\eta_{\text{line}} = 1$, so that (4) is valid if the loss at the cavity wall is negligible. If it is not negligible, the radiation efficiency must be calculated using (6) instead of (4).

2.3 Measurement System

In the following paragraphs, the measurement system we developed for the reflection method is described [4].

2.3.1 Waveguide

The rectangular waveguide consists of two U-shaped aluminum castings and two aluminum plates, as shown in Fig. 2. These castings and plates are screwed together at

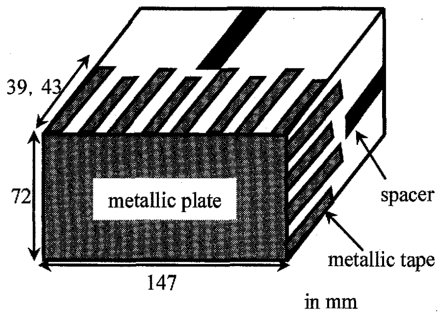

Fig. 3 Sliding short.

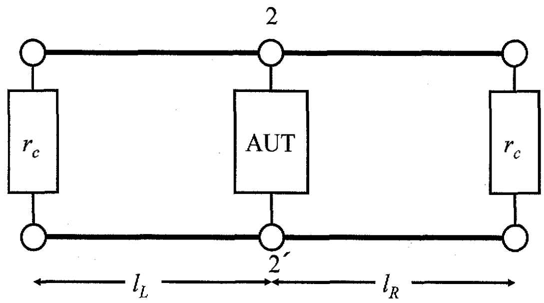

Fig. 4 An equivalent transmission line model.

200 mm intervals. The interior dimensions of its cross-section are 150 mm × 75 mm and its length is 1,000 mm. The antenna to be measured is inserted into the waveguide via a square hole in the broad wall which has an area of 80 mm × 80 mm. This waveguide has single-mode operation in the frequency range 1.00–2.00 GHz.

2.3.2 Sliding Short

The sliding short consists of an aluminum plate attached to the undersurface of a rectangular wooden box which has dimensions of 147 mm × 72 mm × 500 mm, as shown in Fig. 3. To ensure its mobility, insulated spacers with a thickness of 1.5 mm are attached to the sidewalls of the sliding short. To avoid leakage from the clearance, two types of aluminum tapes having lengths of 39 mm or 43 mm and a width of 7 mm are alternately attached to the side walls of the sliding short, as shown in Fig. 3. These tapes function as a choke when the sliding short is inserted into the waveguide [2].

2.3.3 Antenna

In this paper, a monopole antenna with a length of 40 mm and a diameter of 1 mm was used. The antenna was mounted on a grounded plate having an area of 120 mm × 120 mm.

2.3.4 Network Analyzer

S parameters were measured using a network analyzer, Agilent 8720ES, with an averaging factor of 32.

3. Analysis of Radiation Efficiency Using the Transmission Line Model

The sliding shorts can be considered as small resistors having normalized resistances of r_c ($r_c \ll 1$). l_L or l_R represents the distance between the center of the antenna and the left or right sliding short. The measurement system can be equivalently expressed by the transmission line model shown in Fig. 4 [4]. The normalized admittance as viewed from port 2 of the network is given by

$$y_n = \frac{1 + jr_c \tan\beta_g l_L}{r_c + j\tan\beta_g l_L} + \frac{1 + jr_c \tan\beta_g l_R}{r_c + j\tan\beta_g l_R}, \tag{9}$$

where β_g ($= 2\pi/\lambda_g$) is the phase constant and λ_g is the guide wavelength of the TE_{10} mode of the air-filled rectangular waveguide. The corresponding reflection coefficient is given by

$$\Gamma_n = \frac{1 - y_n}{1 + y_n}. \tag{10}$$

We clarified that the underestimation of the efficiency by use of (4) results in a small radius of the circle $\Gamma_{wg,n}$ on the Smith chart. As can be seen from (2), the small radius of $\Gamma_{wg,n}$ leads to the circle Γ_n having a small radius or a shifted center. To examine these effects, we derived analytical expressions for the center z_i and radius r_i of the circle Γ_n and the efficiency in the short-circuited section η_{line} using the transmission line model. In the following, we discuss two methods that we previously proposed [4] for avoiding underestimating the actual efficiency.

3.1 The Case when One Sliding Short is Fixed and the Other is Translated

One problem that is encountered when evaluating the radiation efficiency using the reflection method is the considerable length of time that it takes to measure the reflection coefficients for many positional combinations of the two sliding shorts. To overcome this problem, the authors proposed estimating the radiation efficiency when one sliding short is fixed and the other sliding short is translated.

The situation described above can be analytically modeled using the transmission line model. For example, if the left sliding short is fixed, that is, l_L is constant, then the admittance viewed from the left side of the antenna, denoted by y_L, is given by

$$y_L = \frac{1 + jr_c \tan\beta_g l_L}{r_c + j\tan\beta_g l_L}. \tag{11}$$

Since l_L is arbitrary, Γ_n depicts a circle on the Smith chart. The center z_i and radius r_i of the circle are given by

$$z_i = -\frac{2r_c|y_L|^2 + (1 + r_c)^2 y_L + (1 - r_c)^2 y_L^*}{2r_c|y_L|^2 + (1 + r_c)^2(y_L + y_L^*) + 2(1 + r_c)^2}, \tag{12}$$

$$r_i = \frac{2(1 - r_c^2)}{2r_c|y_L|^2 + (1 + r_c)^2(y_L + y_L^*) + 2(1 + r_c)^2}. \tag{13}$$

Then, from (8), the efficiency in the short-circuited section

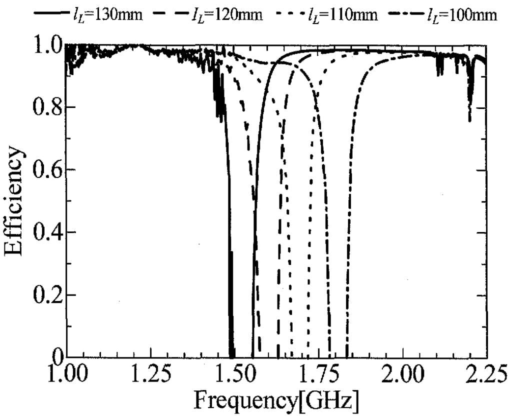

Fig. 5 Measured efficiency η_{net} of 40-mm monopole when the left sliding short is fixed.

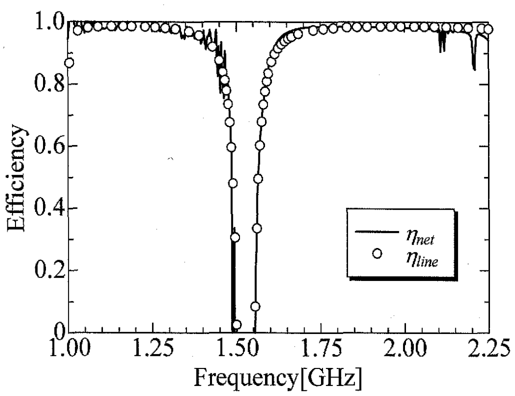

Fig. 6 Comparison of η_{line} and η_{net} for l_L =130 mm.

η_{line} is given by [5]

$$\eta_{\text{line}} = -\frac{2r_c|y_L|^2 + (1-r_c)^2(y_L + y_L^*) - 2(1-r_c^2)}{2(1-r_c^2)}$$
$$= \frac{1-r_c+r_c^2}{1-r_c^2}\frac{r_c(r_c-2)+(1-2r_c)\tan^2\beta_g l_L}{r_c^2+\tan^2\beta_g l_L}. \quad (14)$$

In the corresponding measurement, the left sliding short is fixed and the right sliding short is moved from l_R = 60 mm to 130 mm in 10 mm intervals and the reflection coefficients of the antenna in the waveguide are measured. Figure 5 shows the relationship between the frequency and the overall efficiency η_{net} for various l_Ls. As can be seen from the figure, reductions in the measured efficiency are observed at approximately 1.5 GHz, 1.6 GHz, 1.7 GHz, and 1.8 GHz for l_L = 130 mm, 120 mm, 110 mm and 100 mm, respectively. Thus, we see that the frequency at which the reductions in the measured efficiency occur increases as l_L decreases, that is, as the left sliding short is moved closer to the antenna [4].

Figure 6 shows η_{net} and η_{line} as a function of the frequency. η_{net} can be evaluated using the principle of the original reflection method, whereas η_{line} can be estimated using (14) with r_c = 0.003 [5]. The value of the normalized resistance r_c can be determined by the least-squares method. Figure 6 shows that η_{net} has the same tendency as η_{line}. Then, we infer from the difference between η_{net} and

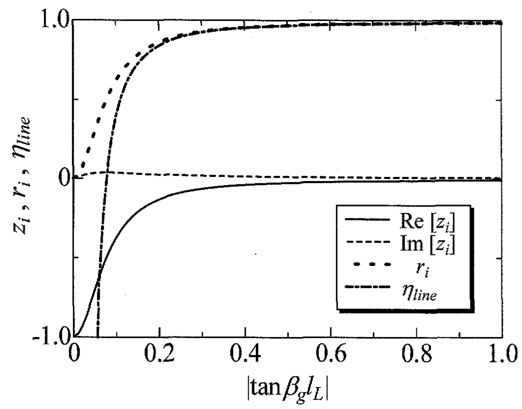

Fig. 7 The center z_i and radius r_i of the circle Γ_n, and η_{line} as function of $|\tan\beta_g l_L|$.

η_{line} at frequencies where there are no reductions in the measured efficiency that the true radiation efficiency is almost unity, so that the behavior, especially, the sharp reductions in the measured efficiency can be determined from the loss at the cavity wall and the cavity resonance. Furthermore, Fig. 6 shows the validity of the proposed transmission line model.

The range for $\eta_{\text{line}} \geq 0$ can be determined from (15):

$$|\tan\beta_g l_L| \geq \sqrt{\frac{r_c(2-r_c)}{1-2r_c}}. \quad (15)$$

The above inequality means that the efficiency of the short-circuited section is positive if the distance between the antenna and the left sliding short l_L is appropriately selected. The above inequality also indicates that a reduction in the measured efficiency can be observed at a certain frequency band. These results confirm the experimental results shown in Fig. 5. The deepest drop in the measured efficiency corresponds to the minimum of (14) which is negative and given by

$$\eta_{\text{line}}|_{\text{min}} = \frac{1-r_c+r_c^2}{1-r_c^2}\frac{r_c-2}{r_c} < 0. \quad (16)$$

The above equation was derived by substituting $\tan\beta_g l_L = 0$ into (14). When $\tan\beta_g l_L = 0$ or $l_L = n\lambda_g$ where n is integer, the antenna is located at the nodes of the standing wave in the short-circuited waveguide [4].

Figure 7 shows the center z_i and the radius r_i of the circle Γ_n and the transmission efficiency in the short-circuited section η_{line} as a function of $\tan\beta_g l_L$. When $|\tan\beta_g l_L| \geq 0.077$, $\eta_{\text{line}} \geq 0$. As the value of $|\tan\beta_g l_L|$ increases, the efficiency becomes more stable in the sense that the measured efficiency is affected less by the loss of the sliding shorts. On the other hand, the efficiency becomes negative or unstable when $\tan\beta_g l_L = 0$. To investigate the effect of selecting $|\tan\beta_g l_L|$ in more detail, the efficiency as a function of the frequency for l_L = 130 mm and the locus of the circle Γ_n on the Smith chart are shown in Fig. 8. For example, $|\tan\beta_g l_L| = 1$ corresponds to 1.326 GHz or 1.750 GHz where η_{net} is nearly equal to unity. And the locus of Γ_n coincides with a unit circle on the Smith chart so that the actual

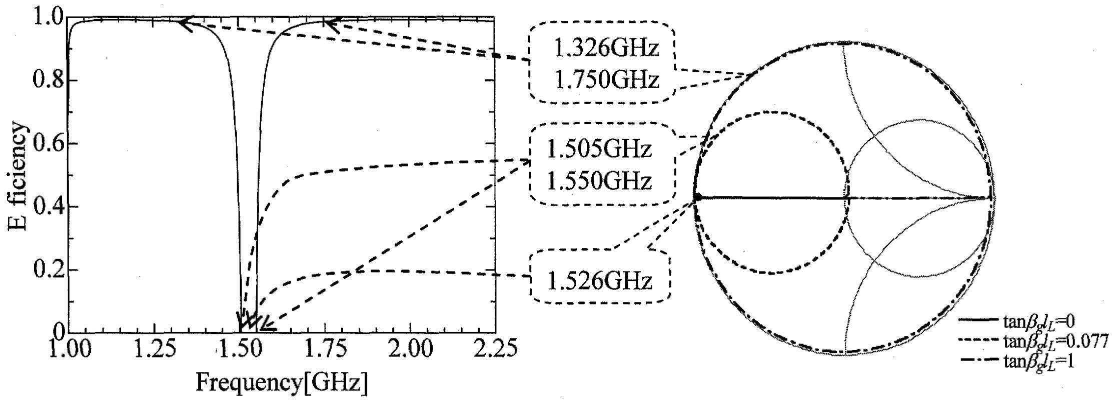

Fig. 8 The relationship between Γ_n and η_{line}.

efficiency can be estimated. When $|\tan\beta_g l_L| = 0.077$, the radius of the circle is less than unity making the estimated efficiency unstable. When $\tan\beta_g l_L = 0$, Γ_n is represented by a single point on the Smith chart so that a circle cannot be determined and the efficiency cannot be estimated. As mentioned above, the measured efficiency becomes more stable as the value of $|\tan\beta_g l_L|$ increases. As a guideline for stability, the efficiency in the section of the sliding shorts is required to be more than 95 %, where the length l_L should be selected such that $|\tan\beta_g l_L| > 0.4$.

3.2 The Case when Both Sliding Shorts are Translated

To avoid the drops in the measured efficiency due to the loss of the cavity wall at resonance, we have proposed that two sliding shorts are translated such that $l_L = l_R$ [4]. This is because when the antenna is located at the maxima or minima of the standing wave, the magnitude of the reflection coefficient is equal to unity. Since $|\Gamma_n| = 1$, drops in the measured efficiency due to the loss of the sliding shorts cannot be observed.

The above situation can be modeled analytically using the transmission line model. When $l = l_L = l_R$, the admittance viewed from the antenna is given by

$$y_n = 2\frac{1 + jr_c \tan\beta_g l}{r_c + j\tan\beta_g l}. \tag{17}$$

Since l is arbitrary, Γ_n is represented by a circle on the Smith chart. The center z_i and radius r_i of this circle are given by

$$z_i = -\frac{(3/2)r_c}{1 + (5/2)r_c + r_c^2}, \tag{18}$$

$$r_i = \frac{1 - r_c^2}{1 + (5/2)r_c + r_c^2}. \tag{19}$$

Then, from (8), the efficiency in the short-circuited section η_{line} is given by [5]

$$\eta_{\text{line}} = \frac{1 - (5/2)r_c + r_c^2}{1 - r_c^2}. \tag{20}$$

It is independent of the frequency and the distance between the antenna and the short, and is a constant that is nearly equal to unity since $r_c \ll 1$. That is, although there is a slight loss due to the resistance of the shorts, large drops in the measured efficiency are not observed.

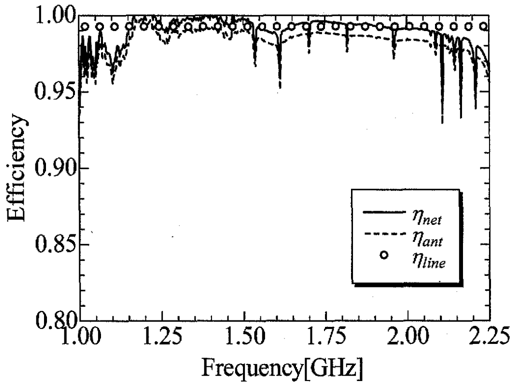

Fig. 9 $\eta_{\text{net}}, \eta_{\text{ant}}, \eta_{\text{line}}$ as a function of frequency for $l_L = l_R$.

In the corresponding measurement, two sliding shorts are moved from l = 60 mm to 130 mm in 10 mm intervals and then the reflection coefficients of the antenna inserted into the waveguide are measured. Figure 9 shows the efficiency η_{net} determined by the above method as well as the radiation efficiency η_{ant} and the efficiency in the section of the sliding shorts η_{line}, whose value is estimated to be 0.99 when $r_c = 0.003$ [5]. Figure 9 confirms that the sharp drops in the measured efficiency are not observed. It also shows some small drops caused when $2l = n\lambda_g/2$, which represents the resonance between the two sliding shorts.

4. Conclusion

A serious problem that occurs when measuring the radiation efficiency of a small antenna using the reflection method is the drops in the resulting measured efficiency that appear at and near the resonant frequency of the cavity formed by the waveguide and two sliding shorts. In this paper, we introduce the transmission line model that includes the wall resistance of the cavity, and we clarify analytically that the measured efficiency is equal to the product of the true radiation efficiency of the antenna and the transmission efficiency in the sliding shorts section. Then, we derive simple expressions for the efficiency in the sliding shorts section for two measurement methods that enable drops in the measured efficiency to be avoided. We found that the drops in the measured efficiency occur when the radius of the circle defined by the locus of the reflection coefficients on the Smith chart is much smaller than unity. As is well known, this is because the resistance of the wall is larger than the radiation resistance when the cavity is resonant. This interpretation is consistent with the findings of other researchers' studies. By estimating the resistance of the walls, we can predict not only the frequency but also the frequency band for the drops in the measured efficiency. Thus, these results confirm the validity of proposed transmission line model for the sliding shorts in the reflection method.

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Appendix: Derivation of (5)

When $z = \Gamma_{\mathrm{wg},n} - S_{11}$, $a = S_{22}$, $b = S_{21}^2$, (2) can be reduced to

$$\Gamma_n = \frac{z}{az + b}. \qquad \text{(A·1)}$$

The circle Γ_n has a center at z_i and a radius of r_i, so that it can be denoted by $\Gamma_n = z_i + r_i e^{j\theta_n}$, where θ_n is real. By substituting Γ_n into (A·1) and solving for $e^{j\theta_n}$, we obtain

$$e^{j\theta_n} = \frac{(1 - az_i)z - bz_i}{ar_i z + br_i} = \frac{Az + B}{Cz + D}, \qquad \text{(A·2)}$$

where $A = 1 - az_i$, $B = -bz_i$, $C = ar_i$, $D = br_i$. Note that $|e^{j\theta_i}|^2 = 1$, (A·2) can be reduced to

$$\left| z - \frac{-A^*B + C^*D}{|A|^2 - |C|^2} \right|^2 = \left| \frac{AD - BC}{|A|^2 - |C|^2} \right|^2. \qquad \text{(A·3)}$$

The circle $\Gamma_{\mathrm{wg},n}$ has a center at $S_{11} + z_o$ and a radius of r_o, so that

$$z_o = \frac{-A^*B + C^*D}{|A|^2 - |C|^2} = b\frac{z_i - a^*(r_i^2 - |z_i|^2)}{|1 - az_i|^2 - |r_i a|^2}, \qquad \text{(A·4)}$$

$$r_o = \left| \frac{AD - BC}{|A|^2 - |C|^2} \right| = \frac{|b| r_i}{|1 - az_i|^2 - |r_i a|^2}. \qquad \text{(A·5)}$$

Solving (A·5) for $|b|$ gives

$$|b| = \frac{r_o}{r_i}\left(|1 - az_i|^2 - |r_i a|^2 \right). \qquad \text{(A·6)}$$

Therefore, we can find $|z_o|$ from (A·4) as follows:

$$|z_o| = \frac{r_o}{r_i}\left| z_i - a^*(r_i^2 - |z_i|^2) \right|. \qquad \text{(A·7)}$$

Denoting the phase of z_o as ϕ, we can find a as follows:

$$a = \frac{1}{r_i^2 - |z_i|^2}\left(z_i^* - r_i \frac{|z_o|}{r_o} e^{-j\phi} \right). \qquad \text{(A·8)}$$

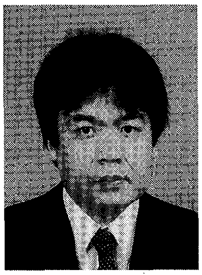
Then,

$$1 - az_i = \frac{r_i}{r_i^2 - |z_i|^2}\left(r_i - z_i \frac{|z_o|}{r_o} e^{-j\phi} \right), \qquad \text{(A·9)}$$

(A·6) can be given by

$$|b| = \frac{r_o}{r_i}\left\{ \left| \frac{r_i}{r_i^2 - |z_i|^2}\left(r_i - z_i \frac{|z_o|}{r_o} e^{-j\phi} \right) \right|^2 - \left| \frac{1}{r_i^2 - |z_i|^2}\left(z_i^* - r_i \frac{|z_o|}{r_o} e^{-j\phi} \right) \right|^2 r_i^2 \right\}. \qquad \text{(A·10)}$$

In the above expression, $|b|$ on the left-hand side corresponds to $|S_{21}|^2$ and the right-hand side can be reduced to (5).



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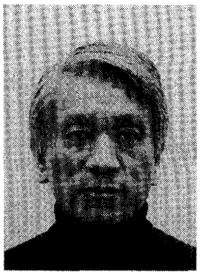

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