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PAPER

Analysis and Relative Evaluation of Connectivity of a Mobile Multi-Hop Network

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SUMMARY In mobile multi-hop networks, a source node S and a destination node D sometimes encounter a situation where there is no multi-hop path between them when a message M, destined for D, arrives at S. In this situation, we cannot send M from S to D immediately; however, we can deliver M to D after waiting some time with the help of two capabilities of mobility. One of the capabilities is to construct a connected multi-hop path by changing the topology of the network during the waiting time (Capability 1), and the other is to move M closer to D during the waiting time (Capability 2). In this paper, we consider three methods to deliver M from S to D by using these capabilities in different ways. Method 1 uses Capability 1 and sends M from S to D after waiting until a connected multi-hop path appears between S and D. Method 2 uses Capability 2 and delivers M to D by allowing a mobile node to carry M from S to D. Method 3 is a combination of Methods 1 and 2 and minimizes the waiting time. We evaluate and compare these three methods in terms of the mean waiting time, from the time when M arrives at S to the time when D starts receiving M, as a new approach to connectivity evaluation. We consider a one-dimensional mobile multi-hop network consisting of mobile nodes flowing in opposite directions along a street. First, we derive some approximate equations and propose an estimation method to compute the mean waiting time of Method 1. Second, we theoretically analyze the mean waiting time of Method 2, and compute a lower bound of that of Method 3. By comparing the three methods under the same assumptions using results of the analyses and some simulation results, we show relations between the mean waiting times of these methods and show how Capabilities 1 and 2 differently affect the mean waiting time.

key words: *mobile multi-hop network, connectivity analysis, mobility, epidemic routing*

1. Introduction

A mobile multi-hop network is constructed by two basic functions of mobile devices, namely direct communication and relaying. Here, direct communication is communication between two nodes through a wireless link between them, and relaying means to receive data from an adjacent node and forward the data to another adjacent node immediately. These two functions make it possible to construct a wireless multi-hop path between a source node S and a destination node D. As a result, S can transmit data to D after constructing a multi-hop path by using a certain routing protocol [1], [2]. Let M be a message destined for D. When M arrives at S, we sometimes encounter a situation where S and D have no multi-hop path between them because of the randomness of the positions of mobile nodes and their finite communication ranges. In this case, S cannot send M to D immediately; however, S can send M to D after waiting until a multi-hop path appears between S and D owing to a capability of mobility to reconstruct a connected multi-hop path by changing topology of the network during the waiting time. In this paper, we call this type of data transmission Method 1, and call the above capability of mobility Capability 1.

Even if there is no connected multi-hop path between S and D when M arrives at S, it is also possible to deliver M from S to D by epidemic routing [3]. Epidemic routing delivers M to D by making mobile nodes move M closer to D, and can be achieved in various manners. For example, if S as well as mobile nodes repeatedly distribute a copy of M to other mobile nodes in their communication ranges by direct communication, it is expected that some of the copies will approach D as the mobile nodes move and finally arrive at D in the near future. As can be seen from this example, epidemic routing does not require a connected multi-hop path between S and D to deliver M from S to D. If a node that directly received M from S can enter the communication range of D after moving, this node can move M closer to D by itself and can deliver M to D by direct communication. In this paper, Method 2 is epidemic routing that sends M from S directly to a mobile node and makes this mobile node send M directly to D after moving as in this example. We call the capability of mobility to move M closer to D Capability 2. Then, Method 2 does not use Capability 1 but Capability 2. Likewise, Method 1 uses Capability 1 rather than Capability 2.

Let T_w be the length of a duration from the time when M arrives at S to the time when D starts receiving M. Denote by $E(\cdot)$ the expected value of $\cdot$. If we use Methods 1 or 2 to deliver M from S to D, it is natural to compare the $E(T_w)$s of Methods 1 and 2 to determine which is faster. At the same time, this comparison means a comparison between Capabilities 1 and 2 because Method 1 uses only Capability 1 and Method 2 uses only Capability 2 as mentioned. Furthermore, we can consider a combination of Methods 1 and 2 to reduce T_w. Define Method 3 as a method that combines Methods 1 and 2 to minimize T_w. Then, it is also interesting to investigate how much Method 3 reduces $E(T_w)$ compared with Methods 1 and 2. For the above comparisons, analyses of the $E(T_w)$s of the three methods are needed.

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Connectivity analysis is an important research issue in wireless multi-hop networks because the network is not always connected as mentioned, and the subject has been studied widely as surveyed in [4]. One of the purposes of connectivity analysis is to know in what situations the network is sufficiently connected. For example, this is achieved by finding the critical density of nodes (or the critical communication range) to achieve sufficient connectivity. Other than the density of nodes and communication range, the mobility of relay nodes is an important factor that affects connectivity in mobile multi-hop networks. The $E(T_w)$ of Method 1 is considered as a connectivity measure that reflects the effects of mobility because it means how long S and D have to wait until a connected multi-hop path appears between them due to the conditions of mobility. However, the $E(T_w)$ of Method 1 has never been analyzed theoretically even in a one-dimensional multi-hop network. In addition to this fact, comparing the $E(T_w)$ of Method 1 with those of Methods 2 and 3 has never been done. Therefore, analyses and relative evaluations of the $E(T_w)$s of the three methods are considered as a new approach to connectivity evaluation and are important from the viewpoint of connectivity analysis.

Also, analysis of $E(T_w)$ is important from the viewpoint of applications for the following reason. For applications that require low latency such as ordinary applications used in fixed computer networks, the probability that there is a multi-hop path between S and D, denoted by P_c, is the most appropriate measure of connectivity, and maximizing P_c is the most important task. However, long T_w and low P_c are sometimes acceptable in other kinds of applications that do not strictly require low latency, such as data collection through a sensor network, where S and D correspond to a sensor node and a data center, respectively. In such applications, $E(T_w)$ can be an appropriate measure of connectivity, and it is important to understand the behavior of $E(T_w)$ to guarantee the required connectivity performance.

With this as background, in this paper we analyze the $E(T_w)$s of Methods 1, 2 and 3 supposing that S and D are fixed nodes on a one-dimensional street and that there are two flows of mobile nodes on the street similar to pedestrian flows on a street. First, we approximately analyze the $E(T_w)$ of Method 1 in the one-dimensional network with the pedestrian mobility model because it is not easy to theoretically and exactly analyze the distribution, the mean and other characteristic values of the T_w of Method 1 even in such a simple network. Similar issues of connectivity analyses using the pedestrian mobility model can be seen in [5]–[7]; however, these articles do not include analysis of the $E(T_w)$ of Method 1. Second, we theoretically analyze the $E(T_w)$ of Method 2, and propose a mathematical model to roughly compute a lower bound of the $E(T_w)$ of Method 3. By comparing the $E(T_w)$s of the three methods using the numerical results of the theoretical analyses and some simulation results, we show relations between the mean waiting times of the three methods and show how Capabilities 1 and 2 differently affect the mean waiting time.

In Sect. 2, we give definitions and assumptions. In Sect. 3, we explain background information and the problem statement. In Sect. 4, we analyze the mean waiting time of Method 1. In Sect. 5, we analyze the mean waiting times of Methods 2 and 3. We compare the mean waiting time of Method 1 with those of Methods 2 and 3 in Sect. 6. In Sect. 7, we conclude this paper.

2. Definitions and Assumptions

We consider transmission of a message M from a fixed node S to another fixed node D on a one-dimensional street. The following are definitions and assumptions:

- Suppose that S, D and mobile nodes are on a line. Let $x(n,t)$ be the coordinate of a node n at time t on the line. Let L be the distance from S to D. Suppose that $x(S,t) = 0$ and $x(D,t) = L$ for all t.
- Assume that a mobile node is a pedestrian moving along the line at a constant velocity v without changing direction independently of other nodes as shown in Fig. 1. As represented in this figure, mobile nodes moving left (in the direction from D to S) are represented by black nodes and those moving right (in the direction from S to D) are represented by white nodes.
- Let N_b and N_w be the sets of mobile nodes moving left and those moving right, respectively. Denote a mobile node of N_b by b_i, where i is an integer such that $x(b_i,t) < x(b_{i+1},t)$ for all i and all t. Denote a mobile node of N_w by w_j, where j is an integer such that $x(w_j,t) > x(w_{j+1},t)$ for all j and all t. Note that b_i enters the interval between S and D at $t + \frac{x(b_i,t)-L}{v}$ and leaves this interval at $t + \frac{x(b_i,t)}{v}$ if $x(b_i,t) > L$ and that w_j enters the interval between S and D at $t - \frac{x(w_j,t)}{v}$ and leaves this interval at $t + \frac{L-x(w_j,t)}{v}$ if $x(w_j,t) < 0$.
- Assume that the distributions of nodes of N_b and N_w obey Poisson distributions of intensity $\frac{\lambda}{2}$ at the initial moment $t = 0$, respectively. Note that the same distributions are observed at time $t > 0$ because mobile nodes move independently from other nodes [8].
- Assume that there exists a wireless link between two nodes if the distance between them is not longer than r, where r is a positive constant and $r < L$. Otherwise, there is no link between them.
- Let G_b be the set of (b_i, b_{i+1}) such that $x(b_{i+1},t) - x(b_i,t) > r$. Let G_w be the set of (w_j, w_{j+1}) such that $x(w_j,t) - x(w_{j+1},t) > r$.

Suppose that M arrives at S at $t = \tau_1$ and S tries to send M to D immediately. For this purpose, we consider three methods denoted by Methods 1, 2 and 3 as mentioned in Sect. 1. The following defines the three methods in detail.

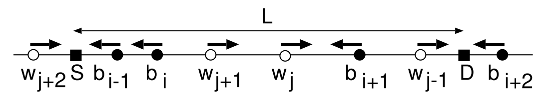

Fig. 1 Fixed nodes S and D, and mobile nodes.

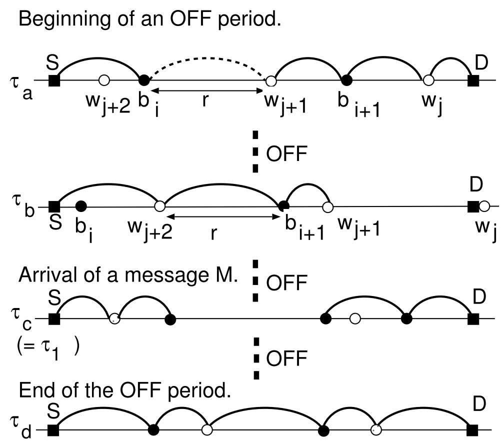

Fig. 2 Example of Method 1, where $\tau_a < \tau_b < \tau_c < \tau_d$.

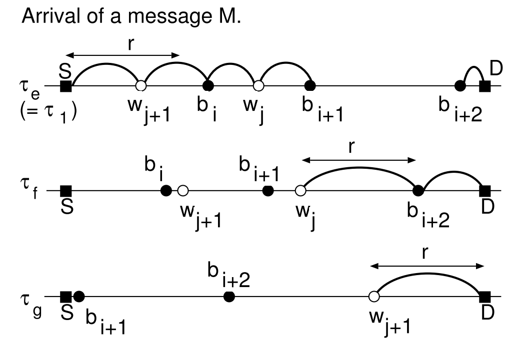

Fig. 3 Example of Methods 2 and 3, where $\tau_e < \tau_f < \tau_g$.

Method 1: If there is a path between S and D at τ_1, S sends M to D immediately. Otherwise, S keeps searching for a multi-hop path to D until it appears due to the change of topology. As soon as a multi-hop path appears, S sends M to D.

This method uses Capability 1 of mobility, which reconstructs a connected multi-hop path by changing topology of the network during the waiting time, as mentioned in Sect. 1. We say that S and D are in the "OFF" state if there is no multi-hop path between S and D, and that they are in the "ON" state otherwise. Denote by T_{off} the length of an OFF period, which is a time period from the beginning of the OFF state to the end of the OFF state. Denote by T_{on} the length of an ON period, which is a time period from the beginning of the ON state to the end of the ON state. We assume that an OFF period ends if a non-zero ON period appears. Define $T_{w,1}$ as the length of a time duration from τ_1 to the time when D starts receiving M by Method 1 given that S and D are in the OFF state at τ_1. Then, $T_{w,1}$ is the time duration from τ_1 to the time when the OFF state ends. Let P_c be the probability that there is a path between S and D. When we represent P_c and $T_{w,1}$ explicitly with parameters L, λ and r, we represent them as $P_c(L, \lambda, r)$ and $T_{w,1}(L, \lambda, r)$, respectively.

Figure 2 shows an example of Method 1. In this example, M arrives at S at $t = \tau_c$ during an OFF period that begins at τ_a due to failure of the link between b_i and w_{j+1} and ends at τ_d. Then, $T_{off} = \tau_d - \tau_a$ and $T_{w,1} = \tau_d - \tau_c$.

Method 2: This method delivers M from S to D by allowing mobile nodes of N_w to carry M from S to D. Suppose that $x(w_j, \tau_1) \geq r$ and $x(w_{j+1}, \tau_1) < r$. At τ_1, S immediately sends M directly to w_{j+1} if $x(w_{j+1}, \tau_1) \geq -r$, and S sends M to w_{j+1} when w_{j+1} enters the communication range of S otherwise. After receiving M, w_{j+1} moves toward D and sends M directly to D when w_{j+1} enters the communication range of D.

This method uses Capability 2 of mobility, which moves M closer to D, and does not use Capability 1 as mentioned in Sect. 1. Note that it is sometimes possible for a mobile node to deliver the message to D while S and D are in an OFF period. In such a case, Method 2 outperforms Method 1.

Method 3: This method maximally utilizes both multi-hop transmission and epidemic routing. At τ_1, S floods M. Mobile nodes that receive M at τ_1 or after τ_1 flood M repeatedly until M is delivered to D. Suppose that a node can send M to every node in a connected component without delay in a flooding. Then, this method minimizes the time to deliver M to D although it may require a lot of redundant transmissions.

Define $T_{w,2}$ as the length of a time duration from τ_1 to the time when D starts receiving M by Method 2, and $T_{w,3}$ as the length of a time duration from τ_1 to the time when D starts receiving M by Method 3. Figure 3 is an example of Methods 2 and 3, where M arrives at S at $t = \tau_e$. In this example, for Method 2, w_{j+1} carries M to D, and $T_{w,2} = \tau_g - \tau_e$. For Method 3, w_j receives M from S by a flooding at τ_e, and it sends M to D by another flooding at τ_f. Then, $T_{w,3} = \tau_f - \tau_e$.

3. Problem Statement

In this paper, we analyze $E(T_{w,1})$ first. If the lengths of OFF periods are independent and identically distributed random variables, the OFF periods comprise a renewal process and we can use the well-known result on the mean residual lifetime in a renewal process [9] as follows:

$$E(T_{w,1}) = \frac{E(T_{off}^2)}{2E(T_{off})}. \tag{1}$$

Although we cannot exactly say that they are independent in our model, we assume independence to simplify the analysis, and use Eq. (1) to approximately compute $E(T_{w,1})$. Before analyzing $E(T_{off})$ and $E(T_{off}^2)$ to compute $E(T_{w,1})$, we explain the reason the analysis is difficult even in the one-dimensional network with the pedestrian mobility model and briefly explain our approach.

Consider a pair of nodes n_1 and n_2. If there is no node between n_1 and n_2 at time t and $|x(n_1, t) - x(n_2, t)| > r$, we say that the interval between n_1 and n_2 is a gap at time t. Here, S, D and mobile nodes can be n_1 and n_2. Then, S and D are in the OFF state if and only if there is a gap between S and D. This property can be seen easily.

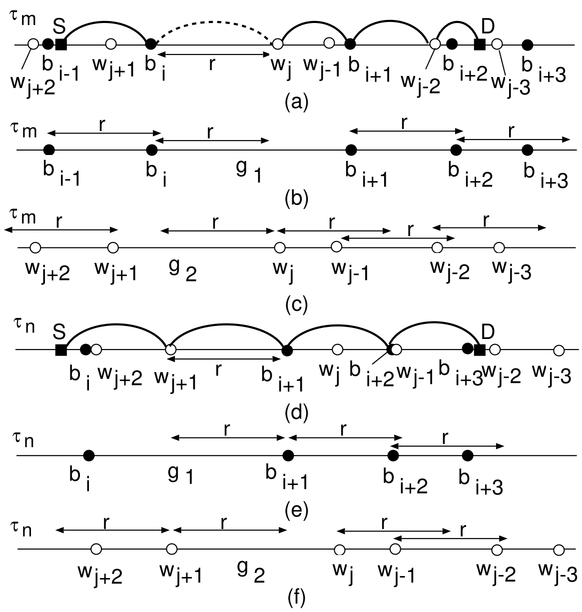

Fig. 4 Example of an OFF period (1).

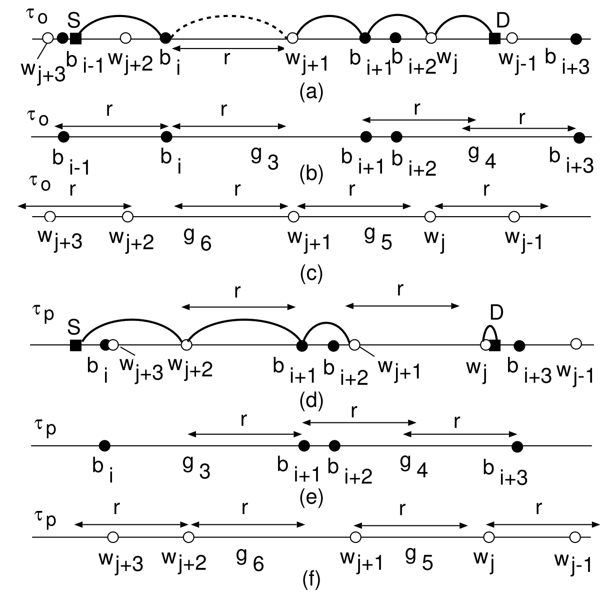

Fig. 5 Example of an OFF period (2).

Consider an example of an OFF period shown in Fig. 4. This OFF period begins right after τ_m and ends at τ_n, where $\tau_m < \tau_n$. Figures 4(a) and (d) represent the networks at τ_m and τ_n, respectively. Figures 4(b) and (e) show positions of black nodes at τ_m and τ_n, respectively, and Figs. 4(c) and (f) show positions of white nodes at τ_m and τ_n, respectively. In this example, $(b_{i-1}, b_i) \notin G_b$, $(b_i, b_{i+1}) \in G_b$, $(b_{i+1}, b_{i+2}) \notin G_b$ and $(b_{i+2}, b_{i+3}) \notin G_b$ as represented in Fig. 4(b), and $(w_{j-3}, w_{j-2}) \notin G_w$, $(w_{j-2}, w_{j-1}) \notin G_w$, $(w_{j-1}, w_j) \notin G_w$, $(w_j, w_{j+1}) \in G_w$ and $(w_{j+1}, w_{j+2}) \notin G_w$ as represented in Fig. 4(c). Denote (b_i, b_{i+1}) and (w_j, w_{j+1}) by g_1 and g_2, respectively.

In Fig. 4(a), although there is a connected path between S and D, the interval between b_i and w_j is becoming a gap because the length of this interval is equal to r and will increase as b_i and w_j move. Then, an OFF period begins right after τ_m. In this example, S and D are connected again and the OFF period ends when the distance between b_{i+1} and w_{j+1} becomes equal to r as represented in Fig. 4(d). The OFF period continues while the length of the overlap of g_1 and g_2 is greater than r, and it ends when the length of the overlap becomes equal to r as can be seen from Figs. 4(d), (e) and (f). Hence, in this example, we can simply represent the length of the OFF period by the relation between g_1 and g_2.

Consider another example of an OFF period depicted in Fig. 5. Figures 5(a) and (d) represent the networks at τ_o and τ_p, respectively, where $\tau_o < \tau_p$. This OFF period begins right after τ_o and still continues at τ_p. Figures 5(b) and (e) show positions of black nodes at τ_o and τ_p, respectively, and Figs. 5(c) and (f) show positions of white nodes at τ_o and τ_p, respectively. In this example, $(b_{i-1}, b_i) \notin G_b$, $(b_i, b_{i+1}) \in G_b$, $(b_{i+1}, b_{i+2}) \notin G_b$ and $(b_{i+2}, b_{i+3}) \in G_b$ as represented in Fig. 5(b), and $(w_{j-1}, w_j) \notin G_w$, $(w_j, w_{j+1}) \in G_w$, $(w_{j+1}, w_{j+2}) \in G_w$ and $(w_{j+2}, w_{j+3}) \notin G_w$ as represented in Fig. 5(c). Denote (b_i, b_{i+1}), (b_{i+2}, b_{i+3}), (w_j, w_{j+1}) and (w_{j+1}, w_{j+2}) by g_3, g_4, g_5 and g_6, respectively.

In Fig. 5(a), an OFF period begins because a gap appears between b_i and w_{j+1} right after τ_o because the length of the overlap between g_3 and g_6 becomes greater than r right after τ_o. Even though the gap between b_i and w_{j+1} disappears at τ_p, the OFF period still continues at τ_p because another gap exists between w_j and w_{j+1} at τ_p because the length of the overlap between g_4 and g_5 is greater than r at τ_p. In this example, g_4 and g_5 overlap successively after g_3 and g_6 overlap, and such successive overlaps generate gaps successively. In this case, therefore, the end of an OFF period cannot be characterized simply as the example in Fig. 4.

As shown in the above examples, it is not easy to simply represent the condition in which an OFF period ends because we have to consider all possible combinations of $(b_i, b_{i+1}) \in G_b$ and $(w_j, w_{j+1}) \in G_w$ that cause successive appearances of gaps. Therefore, it is not easy to theoretically compute $E(T_{off})$ and $E(T_{off}^2)$ even in a one-dimensional network with the pedestrian mobility model.

To overcome this difficulty, we consider two special cases where $L < 2r$ (Case 1) and $L = 2r$ (Case 2). As will be explained later, the successive overlaps do not occur in Case 1. In Case 2, we have simple conditions that are satisfied at the end of an OFF period although the successive overlaps occur. Then, we analyze $E(T_{off})$ and $E(T_{off}^2)$ in Cases 1 and 2 theoretically although we use some approximations. For $L > 2r$ (Case 3), it is difficult to directly compute $E(T_{off})$ and $E(T_{off}^2)$. Then, we try to roughly estimate $E(T_{w,1})$ using a property observed in the relation between P_c and $E(T_{w,1})$.

As mentioned in Sect. 1, connectivity analysis has been conducted widely; however, the above approach to compute $E(T_{w,1})$ has never been taken, although some results of analyses of $E(T_{on})$ and $E(T_{off})$ for the pedestrian mobility model can be seen in [5]–[7]. In [5], the mean lengths of two types of ON periods and a lower bound of the mean length of an OFF period in multi-hop cellular networks were computed. In [7], T_{on} was analyzed in some cases; however, $T_{w,1}$ was not analyzed.

In this paper, there is only a requirement for the length of an ON period, namely T_{on} must be longer than 0 to terminate an OFF period. Although it is our future problem to analyze the waiting time until the appearance of an ON period that is longer than a desired value, it is expected that $E(T_{w,1})$ computed in this paper is a lower bound of the mean of this waiting time.

Next, we theoretically analyze $E(T_{w,2})$ under the same assumptions made for Method 1. Also, we propose a simple mathematical model of Method 3 to compute a lower bound of $E(T_{w,3})$ because the theoretical analysis of $E(T_{w,3})$ is difficult because Method 3 is a generalized version of Method 1. This mathematical model is derived from the results of the analyses of $E(T_{w,1})$. From the numerical results of the analyses and some simulation results, we compare $E(T_{w,2})$ and $E(T_{w,3})$ with $(1-P_c)E(T_{w,1})$ to compare the two capabilities of mobility. Note that $E(T_{w,1})$ is the conditional mean of the waiting time given that S and D are in the OFF state at τ_1; therefore, we multiply $(1-P_c)$ to $E(T_{w,1})$ for unconditioning and use $(1-P_c)E(T_{w,1})$ for the above comparison. The mean waiting time of an epidemic routing in a one-dimensional network where mobile nodes move according to a Brownian motion model has been analyzed in [10]. However, we use the pedestrian mobility model because we analyze $E(T_{w,2})$ and $E(T_{w,3})$ to compare them with $(1-P_c)E(T_{w,1})$ under the same assumptions, and it is more natural to assume the pedestrian mobility model as a mobility model on a street than the Brownian motion model.

In this paper, we observe in what situations $T_{w,1}$ rapidly increases, in what situations Method 2 outperforms Method 1, and how much Method 3 outperforms Method 1. For this purpose, we concentrate on situations where λ is small and $E(T_{w,1})$, $E(T_{w,2})$ and $E(T_{w,3})$ are large. Then, we neglect other short delays caused in a queue of packets or in retransmissions of packets as assumed in Sect. 2.

In this paper, it is also assumed that S can notice the beginning of every ON period in Method 1 and that S and D can notice every node entering their communication ranges in Method 2. Thus, these methods are assumed to be idealized ones in this paper, and it is necessary for S and D to use some additional procedures in actual situations; however, $T_{w,1}$ and $T_{w,2}$ are the minimum waiting times even in actual situations.

4. Mean Waiting Time of Method 1

4.1 Case 1

Suppose that an OFF period begins right after τ_a. In Case 1, there must be a pair of $(b_i, b_{i+1}) \in G_b$ and $(w_j, w_{j+1}) \in G_w$ that satisfies one of the following conditions at τ_a:

A_1: $x(b_i,\tau_a) = L-r$, $x(w_j,\tau_a) \geq L$ and $x(w_{j+1},\tau_a) < L-r$.

A_2: $\frac{L-r}{2} \leq x(b_i,\tau_a) < L-r$, $\frac{L+r}{2} \leq x(w_j,\tau_a) < L$ and $x(w_j,\tau_a) - x(b_i,\tau_a) = r$.

A_3: $x(w_j,\tau_a) = r$, $x(b_i,\tau_a) \leq 0$ and $x(b_{i+1},\tau_a) > r$.

A_4: $0 < x(b_i,\tau_a) < \frac{L-r}{2}$, $r < x(w_j,\tau_a) < \frac{L+r}{2}$ and $x(w_j,\tau_a) - x(b_i,\tau_a) = r$.

Note that these are mutually exclusive and cover all situations where an OFF period begins. In Case 1, an OFF period ends when a node enters the region $[L-r, r]$ because a node has direct links to both S and D simultaneously while it is in $[L-r, r]$. It also ends when a three-hop path is constructed even if there is no node in $[L-r, r]$. We can easily see that we need at most three hops to connect S and D in Case 1. Then, one of the following events has to occur when the OFF period ends:

B_1: b_{i+1} arrives at $x = r$.

B_2: w_{j+1} arrives at $x = L-r$.

B_3: The distance from b_{i+1} to w_{j+1} becomes r.

Note that $T_{on} > \frac{2r-L}{v} > 0$ after each of B_1, B_2 and B_3 occurs. Let T_m be the time from τ_a to the time when B_m occurs for $m = 1, 2$ and 3. Then, $T_{off} = \min(T_1, T_2, T_3)$.

From the above definitions and assumptions, we can see that $P(A_1) = P(A_3)$, $P(A_2) = P(A_4)$, $E(T_{off}^k|A_1) = E(T_{off}^k|A_3)$ and $E(T_{off}^k|A_2) = E(T_{off}^k|A_4)$ because A_1 and A_3 occur in symmetrical situations and so do A_2 and A_4. Also, $P(\cup_{m=1}^4 A_m) = \sum_{m=1}^4 P(A_m)$. Then,

$$E(T_{off}^k) = \sum_{\ell=1}^{4} E(T_{off}^k|A_\ell)P(A_\ell| \cup_{m=1}^4 A_m)$$
$$= \frac{E(T_{off}^k|A_1)P(A_1)}{P(A_1)+P(A_2)} + \frac{E(T_{off}^k|A_2)P(A_2)}{P(A_1)+P(A_2)}. \tag{2}$$

We consider the ratio of $P(A_1)$ to $P(A_2)$ to compute this equation. Before A_2 occurs, there always exists a moment τ_a' such that

A_2': $x(b_i,\tau_a') = L-r$ and $r \leq x(w_j,\tau_a') < L$,

where $\tau_a' < \tau_a$, and an ON period continues from τ_a' until A_2 occurs. Then, this event is equivalent to A_2. We can see that there is only a difference between A_1 and A_2' in the positions of w_j and w_{j+1} when b_i is at $x = L-r$. Also, the distribution of nodes of N_w is independent from that of N_b. Then,

$$P(A_1) : P(A_2) = e^{-\frac{\lambda}{2}r} : \frac{\lambda}{2}(L-r)e^{-\frac{\lambda}{2}r}. \tag{3}$$

From this ratio, we can compute $\frac{P(A_1)}{P(A_1)+P(A_2)}$ and $\frac{P(A_2)}{P(A_1)+P(A_2)}$.

Suppose that an OFF period begins due to A_1. Define that $Y_1 = L-r-x(w_{j+1},\tau_a)$ and $Z_1 = x(b_{i+1},\tau_a) - L$. Then, $T_1 = \frac{Z_1+L-r}{v}$, $T_2 = \frac{Y_1}{v}$ and $T_3 = \frac{Y_1+Z_1}{2v}$. Hence, $T_2 < T_1$ and $T_2 \leq T_3$ if $Y_1 \leq Z_1$, and $T_2 > T_3$ otherwise. Then, $T_{off} = T_2 = \frac{Y_1}{v}$ if $Y_1 \leq Z_1$, and $T_{off} = \min(T_1, T_3) = \frac{Z_1}{v} + \min\left(\frac{Y_1-Z_1}{2v}, \frac{L-r}{v}\right)$ otherwise. Therefore,

$$E(T_{off}|A_1) = \frac{E\{\min(Y_1, Z_1)\}}{v} + P(Y_1 > Z_1)$$

$$\times \frac{E\left\{\min\left(\frac{Y_1-Z_1}{2}, L-r\right)|Y_1 > Z_1\right\}}{v}. \quad (4)$$

In this equation, $\min(Y_1, Z_1)$ is an exponential random variable with the mean $\frac{1}{\lambda}$. If $Y_1 > Z_1$, $Y_1 - Z_1$ is an exponential random variable with the mean $\frac{2}{\lambda}$ because of the memoryless property. Then,

$$E(T_{off}|A_1) = \frac{1}{v}\left\{\frac{3}{2\lambda} - \frac{1}{2\lambda}e^{-\lambda(L-r)}\right\} \quad (5)$$

because the mean of $\min\left(\frac{Y_1-Z_1}{2}, L-r\right)$ given that $Y_1 > Z_1$ is equal to $\frac{1}{\lambda}\left\{1 - e^{-\lambda(L-r)}\right\}$ and $P(Y_1 > Z_1) = \frac{1}{2}$. Also, we have

$$\begin{aligned} E(T_{off}^2|A_1) &= \frac{E\{\min(Y_1,Z_1)^2\}}{v^2} + \frac{P(Y_1 > Z_1)}{v^2} \\ &\times\Bigg[E\left\{2Z_1 \min\left(\frac{Y_1 - Z_1}{2}, L-r\right)\Big|Y_1 > Z_1\right\} \\ &+E\left\{\min\left(\frac{Y_1 - Z_1}{2}, L-r\right)^2 \Big|Y_1 > Z_1\right\}\Bigg] \\ &= \frac{4 - 2e^{-\lambda(L-r)}}{\lambda^2 v^2} - \frac{(L-r)e^{-\lambda(L-r)}}{\lambda v^2} \end{aligned} \quad (6)$$

because $E\{2Z_1 \min\left(\frac{Y_1-Z_1}{2}, L-r\right)|Y_1 > Z_1\}$ equals $2E\{Z_1| Y_1 > Z_1\}E\{\min\left(\frac{Y_1-Z_1}{2}, L-r\right)|Y_1 > Z_1\}$ because Z_1 and $Y_1 - Z_1$ are independent if $Y_1 > Z_1$, and the second moment of $\min\left(\frac{Y_1-Z_1}{2}, L-r\right)$ given that $Y_1 > Z_1$ equals $\frac{2}{\lambda^2} - \frac{2(L-r)}{\lambda}e^{-\lambda(L-r)} - \frac{2}{\lambda^2}e^{-\lambda(L-r)}$.

Next, suppose that an OFF period begins due to A_2. Define that $Y_2 = x(b_i, \tau_a) - x(w_{j+1}, \tau_a)$ and that $Z_2 = x(b_{i+1}, \tau_a) - x(w_j, \tau_a)$. Y_2 and Z_2 are exponential random variables of intensity $\frac{\lambda}{2}$. Let α be $x(b_i, \tau_a) - \frac{L-r}{2}$. Then,

$$\begin{aligned} &T_{off} \\ &= \min\left(\frac{Z_2 + \frac{L-r}{2} + \alpha}{v}, \frac{Y_2 + \frac{L-r}{2} - \alpha}{v}, \frac{Y_2 + Z_2}{2v}\right) \\ &\approx \min\left(\frac{Z_2 + \frac{L-r}{2}}{v}, \frac{Y_2 + \frac{L-r}{2}}{v}, \frac{Y_2 + Z_2}{2v}\right) \\ &= \frac{\min(Y_2, Z_2) + \min(\frac{|Y_2-Z_2|}{2}, \frac{L-r}{2})}{v}, \end{aligned} \quad (7)$$

where we assume that $\alpha \ll Y_2$ and $\alpha \ll Z_2$ because $0 \le \alpha < \frac{L-r}{2}$ and our main attention is paid to T_{off} for a small λ as mentioned. From Eq. (7), in the same manner as the derivations of $E(T_{off}|A_1)$ and $E(T_{off}^2|A_1)$, we have

$$E(T_{off}|A_2) \approx \frac{1}{v}\left\{\frac{2}{\lambda} - \frac{1}{\lambda}e^{-\frac{\lambda(L-r)}{2}}\right\}. \quad (8)$$

$$E(T_{off}^2|A_2) \approx \frac{6 - 4e^{-\frac{\lambda(L-r)}{2}}}{\lambda^2 v^2} - \frac{(L-r)e^{-\frac{\lambda(L-r)}{2}}}{\lambda v^2}. \quad (9)$$

Consequently, we can approximately compute $E(T_{w,1})$ by Eqs. (1), (2), (3), (5), (6), (8) and (9).

4.2 Case 2

In this subsection, we consider Case 2, where $L = 2r$. In Case 2, a two-hop path appears when a node is at $x = r$; however, even if this two-hop path connects S and D, its lifetime is equal to 0. Hence, we neglect such an ON period as mentioned. This means that a two-hop path cannot connect S and D in Case 2. Also, we can easily see that we need at most three hops to connect S and D in Case 2. Then, S and D are in the ON state if and only if there is a three-hop path between S and D. Due to this difference from Case 1, the mean waiting time is affected by the successive appearance of gaps in Case 2 as explained below.

Suppose that an OFF period begins right after τ_a. Then, there must be a pair of $(b_i, b_{i+1}) \in G_b$ and $(w_j, w_{j+1}) \in G_w$ that satisfies one of the following conditions:

A_5: $x(b_i, \tau_a) = r$, $x(w_j, \tau_a) \ge L$ and $0 < x(w_{j+1}, \tau_a) < r$.
A_6: $0 \le x(b_{i-1}, \tau_a) < r$, $x(b_i, \tau_a) = r$, $x(w_j, \tau_a) \ge L$ and $x(w_{j+1}, \tau_a) \le 0$.
A_7: $\frac{r}{2} \le x(b_i, \tau_a) < r$, $\frac{3r}{2} \le x(w_j, \tau_a) < L$ and $x(w_j, \tau_a) - x(b_i, \tau_a) = r$.
A_8: $x(w_j, \tau_a) = r$, $x(b_i, \tau_a) \le 0$ and $r < x(b_{i+1}, \tau_a) < L$.
A_9: $r < x(w_{j-1}, \tau_a) \le L$, $x(w_j, \tau_a) = r$, $x(b_i, \tau_a) \le 0$ and $x(b_{i+1}, \tau_a) \ge L$.
A_{10}: $0 < x(b_i, \tau_a) < \frac{r}{2}$, $r < x(w_j, \tau_a) < \frac{3r}{2}$ and $x(w_j, \tau_a) - x(b_i, \tau_a) = r$.

Note that these events are mutually exclusive and cover all situations where an OFF period begins. An ON period is terminated by link failure between a relay node and D when A_5 or A_6 occurs, by link failure between a relay node and S when A_8 or A_9 occurs, and by link failure between two relay nodes when A_7 or A_{10} occurs. In the same manner as in Case 1, we have

$$E(T_{off}^k) = \sum_{m=5}^{7} \frac{E(T_{off}^k|A_m)P(A_m)}{P(A_5) + P(A_6) + P(A_7)}, \quad (10)$$

and

$$\begin{aligned} &P(A_5) : P(A_6) : P(A_7) \\ &= 1 - e^{-\frac{\lambda}{2}r} : e^{-\frac{\lambda}{2}r}(1 - e^{-\frac{\lambda}{2}r}) : \frac{\lambda}{2}r. \end{aligned} \quad (11)$$

First, we consider $E(T_{off}|A_6)$ and $E(T_{off}^2|A_6)$. Suppose that A_6 occurs. Then, while $0 < x(b_i, t) < r$, there is no white node in $[r, L)$ and $x(b_{i+1}, t) - x(b_i, t) > r$; therefore, b_i cannot be included in any three-hop paths between S and D after τ_a. Then, only b_k and w_ℓ for $k \ge i + 1$ and $\ell \ge j + 1$ are candidate relay nodes to connect S and D after τ_a. For $k \ge i + 1$ and $\ell \ge j + 1$, we have the following properties.

Property 1: b_k and b_{k+1} construct a three-hop path with non-zero lifetime to connect S and D from $t = \tau_a + \frac{x(b_k,\tau_a)-r}{v}$ if $x(b_{k+1}, \tau_a) - x(b_k, \tau_a) \le r$, and they cannot construct such a three-hop path otherwise.
Proof. If $x(b_{k+1}, \tau_a) - x(b_k, \tau_a) \le r$, b_k and b_{k+1} always have

a direct link between them and have direct links to S and D, respectively, after b_k arrives at $x = r$ until b_{k+1} arrives at $x = r$. Otherwise, b_k and b_{k+1} have no direct link between them; therefore, they cannot construct a three-hop path. Therefore, the above property holds. □

Property 2: w_ℓ and $w_{\ell+1}$ construct a three-hop path with non-zero lifetime to connect S and D from $t = \tau_a + \frac{r-x(w_\ell,\tau_a)}{v}$ if $x(w_\ell, \tau_a) - x(w_{\ell+1}, \tau_a) \le r$, and they cannot construct such a three-hop path otherwise.

This can be proved in the same manner as Property 1. Also, we have:

Property 3: b_k and w_ℓ construct a three-hop path with non-zero lifetime to connect S and D from $t = \tau_{k,\ell} = \tau_a + \frac{x(b_k,\tau_a)-x(w_\ell,\tau_a)-r}{2v}$ if $|\{-x(w_\ell, \tau_a)\} - \{x(b_k, \tau_a) - 2r\}| < r$. Otherwise, they cannot construct such a three-hop path.

Proof. From the definition of $\tau_{k,\ell}$, we have $x(b_k, \tau_{k,\ell}) = \frac{x(b_k,\tau_a)+x(w_\ell,\tau_a)}{2} + \frac{r}{2}$ and $x(w_\ell, \tau_{k,\ell}) = \frac{x(b_k,\tau_a)+x(w_\ell,\tau_a)}{2} - \frac{r}{2}$. From these relations, if $|\{-x(w_\ell, \tau_a)\} - \{x(b_k, \tau_a) - 2r\}| < r$, then $r < x(b_k, \tau_{k,\ell}) < 2r$ and $0 < x(w_\ell, \tau_{k,\ell}) < r$. Also, we have $x(w_\ell, \tau_{k,\ell}) + r = x(b_k, \tau_{k,\ell})$. At $t = \tau_{k,\ell}$, therefore, b_k connects w_ℓ and D, and w_ℓ connects b_k and S. The lifetime of this three-hop path via b_k and w_ℓ is not equal to 0 because it does not disappear at least until b_k or w_ℓ arrives at $x = r$. If $|\{-x(w_\ell, \tau_a)\} - \{x(b_k, \tau_a) - 2r\}| > r$, then $x(b_k, \tau_{k,\ell}) < r$ or $x(b_k, \tau_{k,\ell}) > 2r$. From the definition of $\tau_{k,\ell}$, b_k continues being linked directly to w_ℓ from $t = \tau_{k,\ell}$ until $t = \tau_{k,\ell} + \frac{r}{v}$. From $t = \tau_{k,\ell}$ to $\tau_{k,\ell} + \frac{r}{v}$, however, S is connected to neither b_k nor w_ℓ if $x(b_k, \tau_{k,\ell}) > 2r$, and D is connected to neither b_k nor w_ℓ if $x(b_k, \tau_{k,\ell}) < r$. After $t = \tau_{k,\ell}+\frac{r}{v}$, there is no link between b_k and w_ℓ. Therefore, b_k and w_ℓ cannot construct any three-hop paths to connect S and D if $|\{-x(w_\ell, \tau_a)\}-\{x(b_k, \tau_a)-2r\}| > r$. If $|\{-x(w_\ell, \tau_a)\} - \{x(b_k, \tau_a) - 2r\}| = r$, b_k and w_ℓ seem to construct a three-hop path to connect S and D at $\tau_{k,\ell}$ because $x(b_k, \tau_{k,\ell}) = r$ or $x(b_k, \tau_{k,\ell}) = 2r$; however, this path disappears right after $\tau_{k,\ell}$ and the lifetime of this path equals 0. □

Let s_1 and s_2 be a sequence $x(b_{i+1}, \tau_a)-2r$, $x(b_{i+2}, \tau_a)-2r$, ..., and a sequence $-x(w_{j+1}, \tau_a)$, $-x(w_{j+2}, \tau_a)$, ..., respectively. We merge s_1 and s_2 into a single sorted sequence $s_3 = y_1, y_2, ...$ in ascending order. Let n_m be the mth node that enters $(0, L)$ after τ_a, where $m = 1, 2, ...$. If y_m corresponds to $x(b_k, \tau_a) - 2r$, where $k \ge i + 1$, n_m is identical to b_k. If y_m corresponds to $-x(w_\ell, \tau_a)$, where $\ell \ge j + 1$, n_m is identical to w_ℓ. Denote by d_m the direction of n_m. Define that $U_0 = y_1$ and $U_m = y_{m+1} - y_m$ for $m = 1, 2, ...$. Then, U_m is an exponential random variable with intensity λ for all m.

We define the following events:

$C_{m,1}$**:** $U_m > r$;
$C_{m,2}$**:** $U_m = r, d_m \ne d_{m+1}$;
C_m**:** $C_{m,1} \cup C_{m,2}$.

Then, from Properties 1 to 3, n_m and n_{m+1} construct a three-hop path that connects S and D after τ_a if $\overline{C_m}$ occurs, and they cannot construct such a path if C_m occurs. Define that $W = U_0$ if $\overline{C_1}$ occurs, and that $W = \sum_{m=0}^{M} U_m$ if $\{\cap_{m=1}^{M} C_m\} \cap \overline{C_{M+1}}$ occurs for $M \ge 1$. Here, $P(C_m) = P(U_m > r) = P(U_m \ge r) = e^{-\lambda r}$ for all m, and $E(U_m|U_m > r) = E(U_m|U_m \ge r) = r + \frac{1}{\lambda}$ for all m. Then,

$$\begin{aligned} E(W) &= E(U_0)P(U_1 \le r) \\ &+ \sum_{M=1}^{\infty} \{E(U_0) + ME(U_1|U_1 > r)\} P(U_1 > r)^M \\ &\times P(U_{M+1} \le r) = \frac{r + \frac{1}{\lambda}}{1 - e^{-\lambda r}} - r. \end{aligned} \tag{12}$$

We can compute $E(W^2)$ as follows because $U_0, U_1, ..., U_{M+1}$ are independent:

$$\begin{aligned} E(W^2) &= E(U_0^2)P(U_1 \le r) + \sum_{M=1}^{\infty} \Big[E(U_0^2) \\ &+ME(U_1^2|U_1 > r) + 2ME(U_0)E(U_1|U_1 > r) \\ &+M(M-1)\{E(U_1|U_1 > r)\}^2\Big]P(U_1 > r)^M \\ &\times P(U_{M+1} \le r) \\ &= \frac{2}{\lambda^2} + \frac{\left(r + \frac{2}{\lambda}\right)^2}{e^{\lambda r} - 1} + \frac{2\left(r + \frac{1}{\lambda}\right)^2}{(e^{\lambda r} - 1)^2}. \end{aligned} \tag{13}$$

This formula is derived by using $\sum_{M=1}^{\infty} M(e^{-\lambda r})^M = \frac{e^{-\lambda r}}{(1-e^{-\lambda r})^2}$ and $\sum_{M=1}^{\infty} M^2(e^{-\lambda r})^M = \frac{e^{-\lambda r}(1+e^{-\lambda r})}{(1-e^{-\lambda r})^3}$.

Suppose that $\{\cap_{m=1}^{M} C_m\} \cap \overline{C_{M+1}}$ occurs. Then, from Properties 1 to 3, any pair of nodes selected from n_1, n_2, ..., n_{M+1} cannot construct any three-hop paths to connect S and D, and n_{M+1} and n_{M+2} can construct a three-hop path to connect S and D. If $d_{M+1} \ne d_{M+2}$, the OFF period ends when the distance between n_{M+1} and n_{M+2} becomes equal to r, and $T_{off} = \frac{W+\frac{r+U_{M+1}}{2}}{v}$. If $d_{M+1} = d_{M+2}$, n_{M+1} and n_{M+2} construct a three-hop path at $t = \tau_a + \frac{W+r}{v}$; however, n_{M+1} may construct a three-hop path before $t = \tau_a + \frac{W+r}{v}$ together with $n_{M+2+\beta}$, where $\beta \ge 1$, if $d_{M+1} \ne d_{M+2+\beta}$ and $y_{M+2+\beta} - y_{M+1} < r$. Then, $\frac{W}{v} < T_{off} \le \frac{W+r}{v}$ in this case. For a small λ, W tends to be much larger than r as can be seen from Eqs. (12) and (13), and our attention is paid to T_{off} for a small λ. Hence, we approximately compute T_{off} as $T_{off} \approx \frac{W+r}{v}$ if $d_{M+1} = d_{M+2}$. Then,

$$\begin{aligned} E(T_{off}|A_6) &\approx \frac{E\left(W + \frac{r+U_{M+1}}{2}|U_{M+1} < r\right)}{v} \\ &\times P(d_{M+1} \ne d_{M+2}) + \frac{E(W + r)}{v} \\ &\times P(d_{M+1} = d_{M+2}) = \frac{5e^{\lambda r} + 3\lambda r e^{\lambda r} - 1}{4\lambda v(e^{\lambda r} - 1)}, \end{aligned} \tag{14}$$

where $E(W)$ is given in Eq. (12), $P(d_{M+1} \ne d_{M+2}) = \frac{1}{2}$, and $E(U_m|U_m < r) = \frac{1}{\lambda} - \frac{r}{e^{\lambda r}-1}$ for all m. Because W and U_{M+1} are independent, we can compute $E(T_{off}^2|A_6)$ with Eqs. (12) and (13) as follows:

$$E(T_{off}^2|A_6) = \frac{2 + e^{\lambda r}\{7\lambda r(\lambda r + 2) - 8\}}{8\lambda^2 v^2(e^{\lambda r} - 1)^2}$$

$$+\ \frac{e^{2\lambda r}\{\lambda r(5\lambda r+14)+22\}}{8\lambda^2 v^2(e^{\lambda r}-1)^2}. \qquad (15)$$

This equation is derived from $E(W)$, $E(W^2)$ and $E(U_m^2|U_m < r) = \frac{2}{\lambda^2} - \frac{2r}{\lambda(e^{\lambda r}-1)} - \frac{r^2}{e^{\lambda r}-1}$. In Eqs. (14) and (15), W is a main factor that reflects the effects of successive overlaps of gaps, and such a factor does not appear in Case 1.

Next, we consider $E(T_{off}|A_7)$ and $E(T_{off}^2|A_7)$. In this case, an OFF period ends in various manners, and computations of $E(T_{off}|A_7)$ and $E(T_{off}^2|A_7)$ are more complicated than those of $E(T_{off}|A_6)$ and $E(T_{off}^2|A_6)$. To simplify the computation, we assume that $x(b_i,\tau_a) = r/2$ and $x(w_j,\tau_a) = 3r/2$ when A_7 occurs, similar to the assumption made to compute $E(T_{off}|A_2)$ and $E(T_{off}^2|A_2)$. Owing to the above approximation, only b_{i+1}, b_{i+2}, ... and w_{j+1}, w_{j+2}, ... are candidates for relay nodes to connect S and D again. We redefine s_1, s_2 and n_i as follows: Let s_1 and s_2 be a sequence $x(b_{i+1},\tau_a) - \frac{3r}{2}$, $x(b_{i+2},\tau_a) - \frac{3r}{2}$, ... and a sequence $\frac{r}{2} - x(w_{j+1},\tau_a)$, $\frac{r}{2} - x(w_{j+2},\tau_a)$, ..., respectively. Let n_m be the mth node that enters $(\frac{r}{2}, \frac{3r}{2})$ after τ_a. For s_3, d_m, U_m and W, we use the same definitions made for A_6. Then, we can represent the condition to terminate an OFF period in a similar manner to A_6, and the length of an OFF period is as follows:

$$T_{off} \approx \begin{cases} \frac{W+\frac{U_{M+1}}{2}}{v}, & d_{M+1} \neq d_{M+2}, \\ \frac{W+\frac{r}{2}}{v}, & d_{M+1} = d_{M+2}. \end{cases} \qquad (16)$$

Then,

$$E(T_{off}|A_7) \approx E(T_{off}|A_6) - \frac{r}{2v} = \frac{5e^{\lambda r} + \lambda r e^{\lambda r} + 2\lambda r - 1}{4\lambda v(e^{\lambda r}-1)}. \qquad (17)$$

$$E(T_{off}^2|A_7) \approx E(T_{off}^2|A_6) - \frac{r\{4E(W) + 2r + E(U_{M+1}|U_{M+1} \le r)\}}{4v^2}$$
$$= \frac{2 + 2\lambda r(\lambda r - 1) + e^{2\lambda r}\{\lambda r(\lambda r + 4) + 22\}}{8\lambda^2 v^2 (e^{\lambda r}-1)^2} + \frac{e^{\lambda r}\{\lambda r(9\lambda r + 26) - 8\}}{8\lambda^2 v^2(e^{\lambda r}-1)^2}. \qquad (18)$$

Next, let us consider the expected values of T_{off} given that A_5 occurs. In this case, b_i and w_{j+1} construct a three-hop path again when w_{j+1} arrives at $x = r$. Also, w_{j+1} and b_{i+1} may construct it when the distance between them becomes equal to r. Let Y_3 and Z_3 be $r - x(w_{j+1},\tau_a)$ and $x(b_{i+1},\tau_a) - 2r$, respectively. Then,

$$T_{off} = \min\left(\frac{Y_3}{v}, \frac{Y_3 + Z_3}{2v}\right), \qquad (19)$$

where Y_3 and Z_3 are exponential random variables of intensity $\frac{\lambda}{2}$ and $Y_3 < r$. Unlike A_6 and A_7, T_{off} does not include W in this case. Then, with some calculations, we have

$$E(T_{off}|A_5) = \frac{4 - e^{-\frac{\lambda}{2}r} - 3e^{\frac{\lambda}{2}r} + \lambda r}{2\lambda v(1 - e^{\frac{\lambda}{2}r})}, \qquad (20)$$

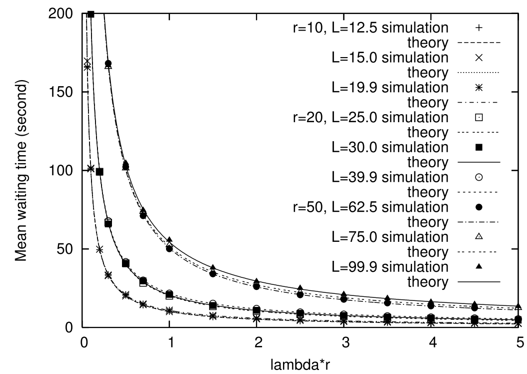

Fig. 6 $E(T_{w,1})$ in Case 1, where horizontal axis is λr.

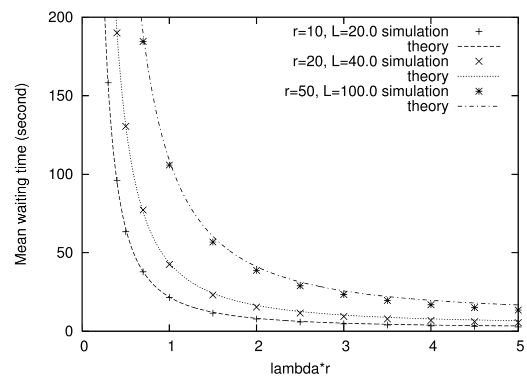

Fig. 7 $E(T_{w,1})$ in Case 2, where horizontal axis is λr.

$$E(T_{off}^2|A_5) = \frac{e^{-\frac{\lambda}{2}r}}{4\lambda^2 v^2 (e^{\frac{\lambda}{2}r} - 1)}\Big[8 + 16e^{\lambda r} + 4\lambda r - e^{\frac{\lambda}{2}r}\{24 + \lambda r(8 + \lambda r)\}\Big]. \qquad (21)$$

In Case 2, we approximately compute $E(T_{w,1})$ by Eqs. (1), (10), (11), (14), (15), (17), (18), (20) and (21).

4.3 Numerical Results in Cases 1 and 2

The numerical results of the above equations in Cases 1 and 2 are shown in Figs. 6 and 7, respectively, together with computer simulation results to verify the equations. We compute $E(T_{w,1})$ for $r = 10$ m, 20 m and 50 m with $v = 1$ m/seconds, and three values of L for each r are evaluated in Case 1. It is intuitively expected that $E(T_{w,1})$ increases as λ decreases or r decreases. We use the mean number of nodes in the communication range r, which is equal to λr, in the horizontal axis to observe effects of λ and r on $E(T_{w,1})$ simultaneously. In the computer simulations, we decide positions of mobile nodes randomly according to the same assumptions made in Sect. 2. If the system is in the OFF state at the initial moment, we observe the system at small intervals denoted by Δt and compute $T_{w,1}$ as the time length from the initial moment until the end of the OFF state. Otherwise, we initialize the positions of mobile nodes again. We repeat these procedures until we examine N different OFF periods, and compute the mean. In this paper,

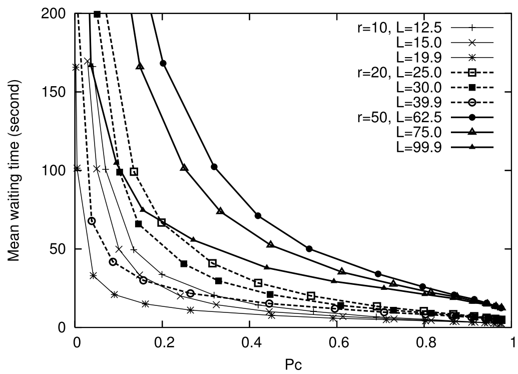

Fig. 8 Simulation results of $E(T_{w,1})$ in Case 1, where horizontal axis is P_c.

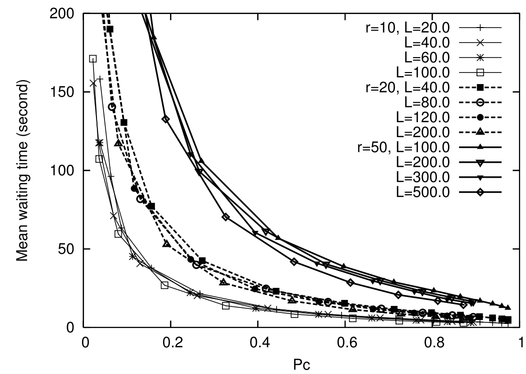

Fig. 9 Simulation results of $E(T_{w,1})$ in Cases 2 and 3, where horizontal axis is P_c.

$\Delta t = 0.05$ seconds and $N = 10000$. Because of iterations to observe the system state in each of the N OFF periods, the running time of a simulation is much longer than the time to compute the above equations.

From Fig. 6, we can confirm that the numerical results agree well with the computer simulation results in Case 1 although our analyses include some approximations. Also, from this figure, we can see that $E(T_{w,1})$ remains almost the same even if L changes. From Fig. 7, we can confirm that, also in Case 2, the numerical results agree well with the computer simulation results and describe well the behavior of $T_{w,1}$ which rapidly increases as λ decreases.

From Figs. 6 and 7, there is a big difference in $E(T_{w,1})$ between Case 1 and Case 2 even if the difference in L is small. For example, we can observe such a result by comparing $E(T_{w,1})$ with $r = 10$ m and $L = 19.9$ m and that with $r = 10$ m and $L = 20.0$ m. It is considered that such a rapid increase of $E(T_{w,1})$ at the boundary between Case 1 and Case 2 is caused by W, which is a main factor in rapidly increasing $E(T_{w,1})$ in Case 2.

4.4 Case 3

As mentioned in Sect. 3, it is generally difficult to analyze $E(T_{w,1})$ in Case 3, where $L > 2r$. Hence, we try to characterize $E(T_{w,1})$ in Case 3 from a property observed in the simulation results. While we show $E(T_{w,1})$ in Case 1 with the horizontal axis of λr in Fig. 6, we show the same simulation results of $E(T_{w,1})$ with the horizontal axis of P_c in Fig. 8. Also, we show simulation results in Cases 2 and 3 with the horizontal axis of P_c in Fig. 9, where $r = 10$ m, 20 m and 50 m, and $L = 2r$, $4r$, $6r$ and $10r$.

By using P_c in the horizontal axis, we can find an interesting property as follows. In Fig. 8, we can see that $E(T_{w,1})$ decreases as L increases in Case 1 while P_c remains the same. In Cases 2 and 3, however, $E(T_{w,1})$ does not change so much even if L increases, and is close to $E(T_{w,1})$ in Case 2 as shown in Fig. 9. Although it is intuitively expected that the tendency in Case 1 is different from those of Cases 2 and 3 because of the effects of the successive appearances of gaps, it remains as a future problem to find theoretical reasons the above property holds.

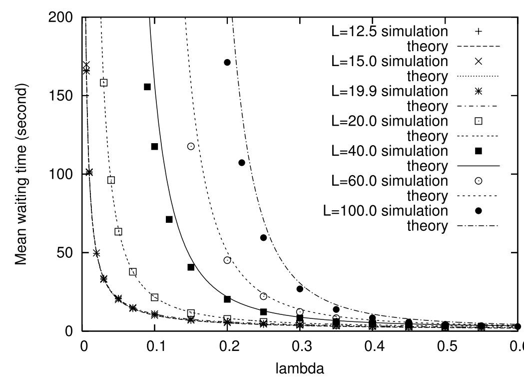

Fig. 10 Numerical results of $E(T_{w,1})$ in Cases 1, 2 and 3, where horizontal axis is λ and $r = 10$ m.

By assuming that the above property holds, it is expected that $E\{T_{w,1}(L, \lambda, r)\} \approx E\{T_{w,1}(2r, \lambda_0, r)\}$ for $L > 2r$ if $P_c(L, \lambda, r) = P_c(2r, \lambda_0, r)$. We can compute $P_c(L, \lambda, r)$ by Eq. (A· 2) because the distribution of all mobile nodes obeys a Poisson distribution of intensity λ for $t \geq 0$ because mobile nodes move independently [8]. Then, we can estimate $E\{T_{w,1}(L, \lambda, r)\}$ for $L > 2r$ as follows:

1. Compute $P_c(L, \lambda, r)$ by Eq. (A· 2).
2. Solve $P_c(2r, \lambda_0, r) = P_c(L, \lambda, r)$ for λ_0.
3. Compute $E\{T_{w,1}(2r, \lambda_0, r)\}$ by the equations derived in Sect. 4.2. This value is expected to be close to $E\{T_{w,1}(L, \lambda, r)\}$.

Figure 10 shows the numerical results of $E(T_{w,1})$ computed by the above procedure together with simulation results, where $r = 10$ m. In this figure, we use λ in the horizontal axis because only one value of r is examined as opposed to Figs. 6 and 7. From the results, we can confirm that the numerical results agree well with the simulation results. Hence, using the above procedure, we can compute $E(T_{w,1})$ in Case 3 approximately but quickly compared with the computer simulation.

5. Mean Waiting Times of Methods 2 and 3

We consider the case of delivering M from S to D by Method 2. Suppose that a message for D arrives at S at time τ_1. Suppose that $x(w_{j-1}, \tau_1) \geq r$ and $x(w_j, \tau_1) < r$. Let X_4 be $r - x(w_j, \tau_1)$. Then, X_4 is an exponential random variable with intensity $\frac{\lambda}{2}$. For $L < 2r$, $T_{w,2} = 0$ if $x(w_j, \tau_1) \geq L - r$ and $T_{w,2} = \frac{X_4-(2r-L)}{v}$ otherwise. For $L \geq 2r$, $T_{w,2} = \frac{X_4+L-2r}{v}$. Therefore,

$$E(T_{w,2}) = \begin{cases} \frac{1}{v} \cdot \frac{2}{\lambda} e^{-\frac{\lambda}{2}(2r-L)}, & L < 2r, \\ \frac{1}{v}\left(\frac{2}{\lambda} + L - 2r\right), & L \geq 2r. \end{cases} \quad (22)$$

Next, we consider delivering a message from S to D by Method 3. Suppose that $L < 2r$ and that a message M for D arrives at S at τ_1. If S and D are in an ON period at τ_1, both Method 1 and Method 3 can send M to D immediately. Otherwise, Method 1 has to wait until the OFF period ends, and Method 3 tries to send a copy of M to the mobile nodes. Suppose again that the OFF period begins at τ_a due to one of A_1 to A_4 defined in Sect. 4.1, and it ends due to one of B_1 to B_3. Define N_S and N_D as $\{S, b_i, b_{i-1}, ..., w_{j+1}, w_{j+2}, ...\}$ and $\{D, b_{i+1}, b_{i+2}, ..., w_j, w_{j-1}, ...\}$, respectively. Then, there is no path from nodes of N_S to those of N_D during the OFF period, and a path appears between them when B_1, B_2 or B_3 occurs. As a result, nodes of N_D cannot receive M during the OFF period, and receive M when B_1, B_2 or B_3 occurs. If $L < 2r$, therefore, Method 3 cannot deliver M to D faster than Method 1. Hence, Method 1 is equivalent to Method 3 if $L < 2r$.

Using the above property, we consider a simple model of Method 3 to characterize $E(T_{w,3})$ for $L \geq 2r$ because it is not easy to compute because Method 3 is a generalized version of Method 1. Denote this model by Method 3a. Suppose that there are K fixed nodes between S and D other than mobile nodes of intensity λ. Denote the fixed nodes by fn_1, fn_2, ..., fn_K. Let L' be a positive constant, and suppose that $L' < 2r$. Suppose that $x(fn_i, t) = iL'$ for $i = 1, ..., K$ and that $KL' \leq L$ and $(K+1)L' > L$. In Method 3a, S sends M to fn_1, fn_i sends M to fn_{i+1} after receiving M for $1 \leq i \leq K-1$, and fn_K sends it to D. The transmission of the message between adjacent fixed nodes is done by Method 1. Let $T_{w,3a}$ be the length of the time interval from τ_1 to the time when M is delivered to D by Method 3a.

As can be seen from the above discussion, Method 1 finishes the message transmission between adjacent fixed nodes as fast as Method 3 because $L' < 2r$. Furthermore, the message transmission is assisted by K additional fixed nodes as opposed to Method 3. Therefore, it is expected that $E(T_{w,3a}) < E(T_{w,3})$, and we can use $E(T_{w,3a})$ as a lower bound of $E(T_{w,3})$. We set L' as $2r - \Delta x$ to minimize K, where Δx is a positive and small constant. We approximately compute $E(T_{w,3a})$ by the equations to compute $E(T_{w,1})$ in Case 1 assuming that the message transmission between one pair of fixed nodes is independent from that between another pair of fixed nodes. Then,

$$E(T_{w,3a}) = K\{1 - P_c(L', \lambda, r)\}E\{T_{w,1}(L', \lambda, r)\} + \{1 - P_c(L - KL', \lambda, r)\} \times E\{T_{w,1}(L - KL', \lambda, r)\}, \quad (23)$$

where $L' = 2r - \Delta x$ and $K = \left\lfloor \frac{L}{2r-\Delta x} \right\rfloor$.

6. Comparison between Methods 1, 2 and 3

In this section, we compare the mean waiting time of Method 1 with those of Methods 2 and 3. For Method 1, we use $(1 - P_c)E(T_{w,1})$ as the mean waiting time because $E(T_{w,1})$ is the mean waiting time given that S and D are not connected at τ_1. We compute $(1-P_c)E(T_{w,1})$ by the methods shown in Sect. 4 and by computer simulation. We compute $E(T_{w,2})$ by Eq. (22), $E(T_{w,3})$ by computer simulation, and $E(T_{w,3a})$ by Eq. (23).

Figure 11 shows $(1 - P_c)E(T_{w,1})$, $E(T_{w,2})$ and $E(T_{w,3})$ for $L = 15$ m and $r = 10$ m. All figures in this section use λ in the horizontal axis in the same manner as Fig. 10. This figure shows the relation between these three mean values for $L < 2r$. In this figure, $(1 - P_c)E(T_{w,1})$ is the same as $E(T_{w,3})$ and is smaller than $E(T_{w,2})$. We can confirm that Method 1 can minimize the mean waiting time without any assistance from Capability 2 of mobility if $L < 2r$.

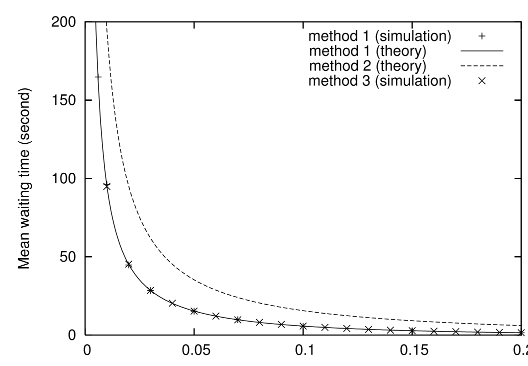

Fig. 11 $(1 - P_c)E(T_{w,1})$, $E(T_{w,2})$ and $E(T_{w,3})$, where $L = 15$ m and $r = 10$ m.

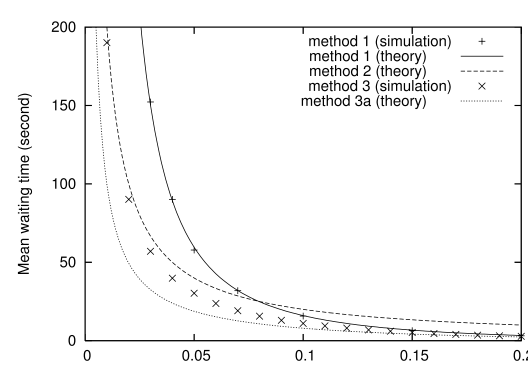

Fig. 12 $(1 - P_c)E(T_{w,1})$, $E(T_{w,2})$, $E(T_{w,3})$ and $E(T_{w,3a})$, where $L = 20$ m and $r = 10$ m.

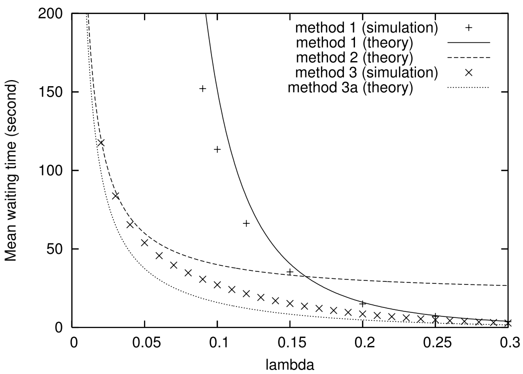

Fig. 13 $(1 - P_c)E(T_{w,1})$, $E(T_{w,2})$, $E(T_{w,3})$ and $E(T_{w,3a})$, where $L = 40$ m and $r = 10$ m.

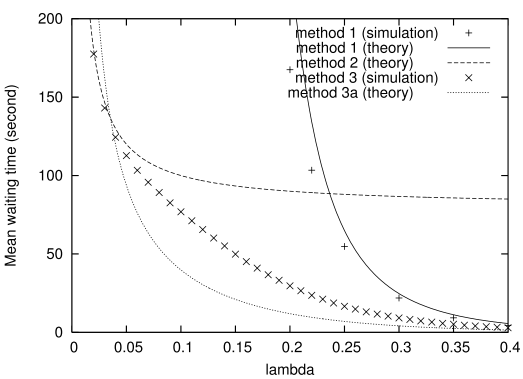

Fig. 14 $(1-P_c)E(T_{w,1})$, $E(T_{w,2})$, $E(T_{w,3})$ and $E(T_{w,3a})$, where $L = 100$ m and $r = 10$ m.

Figures 12 to 14 show $(1 - P_c)E(T_{w,1})$, $E(T_{w,2})$ and $E(T_{w,3})$ together with $E(T_{w,3a})$, where $L = 20$ m, $L = 40$ m and $L = 100$ m, respectively, and $r = 10$ m. These figures show relations between the four mean values for $L \geq 2r$. From these figures, we can see that $(1 - P_c)E(T_{w,1})$, $E(T_{w,2})$ and $E(T_{w,3})$ increase as λ decreases and that $E(T_{w,3})$ is smaller than $(1 - P_c)E(T_{w,1})$ and $E(T_{w,2})$ for all λ. For a large λ, $(1 - P_c)E(T_{w,1})$ is almost the same as $E(T_{w,3})$ and is smaller than $E(T_{w,2})$; however, it rapidly increases as λ decreases and exceeds $E(T_{w,2})$ at a certain λ. As λ keeps decreasing, $E(T_{w,2})$ approaches $E(T_{w,3})$.

We describe the above relations with constants Λ_1, Λ_2 and Λ_3. Suppose that $E(T_{w,2})$ and $E(T_{w,3})$ are considered as almost the same if $\lambda \leq \Lambda_1$, that $\{1 - P_c(L, \Lambda_2, r)\}E\{T_{w,1}(L, \Lambda_2, r)\} = E\{T_{w,2}(L, \Lambda_2, r)\}$ and that $(1 - P_c)E(T_{w,1})$ and $E(T_{w,3})$ are considered as almost the same if $\lambda \geq \Lambda_3$. Then, $E(T_{w,2})$ is almost the smallest for $\lambda \leq \Lambda_1$, $E(T_{w,3}) < E(T_{w,2}) < (1 - P_c)E(T_{w,1})$ for $\Lambda_1 < \lambda < \Lambda_2$, $E(T_{w,3}) < (1 - P_c)E(T_{w,1}) < E(T_{w,2})$ for $\Lambda_2 < \lambda < \Lambda_3$, and $(1 - P_c)E(T_{w,1})$ is almost the smallest for $\lambda \geq \Lambda_3$. From these results, if $\lambda \geq \Lambda_3$, Capability 1 works sufficiently well without any assistance from Capability 2. The reverse is true if $\lambda \leq \Lambda_1$. These results also show that we should utilize both Capability 1 and Capability 2 to reduce the waiting time if $\Lambda_1 < \lambda < \Lambda_3$. Otherwise, Method 1 is inferior to a simple method like Method 2 if $\lambda < \Lambda_2$.

From Figs. 12 to 14, we can confirm that the numerical results of Eq. (23) can be used as a lower bound of $E(T_{w,3})$ in almost all cases although they become slightly greater than $E(T_{w,3})$ in some cases because of the assumption that the multi-hop transmission between adjacent fixed nodes is independent from that between other adjacent fixed nodes. These figures also show that $E(T_{w,3a})$ is close to $E(T_{w,3})$ while $\lambda \leq \Lambda_1$ or $\lambda \geq \Lambda_3$. Then, from the numerical results of $(1 - P_c)E(T_{w,1})$, $E(T_{w,2})$ and $E(T_{w,3a})$, we can estimate Λ_1 and Λ_3, which determine a range of λ in which both capabilities of mobility should be used, in addition to the fact that Λ_2 can be computed by numerically solving an equation $(1 - P_c)E(T_{w,1}) = E(T_{w,2})$ for λ.

7. Conclusions

In this paper, we considered the issue of connectivity of a mobile multi-hop network. As a measure of connectivity, we focused on the mean waiting time for delivery of a message from one fixed node to another, and considered the mean waiting times of three methods, Methods 1, 2 and 3. Method 1 uses Capability 1 of mobility, which reconstructs a connected multi-hop path by changing topology of the network during the waiting time. Method 2 uses Capability 2 of mobility, which moves a message closer to D during the waiting time. Method 3 maximally utilizes Capabilities 1 and 2 to minimize the waiting time.

We theoretically computed the mean waiting time of Method 1 in two cases and proposed a method to estimate it in another case. We theoretically computed the mean waiting time of Method 2, and proposed a model of Method 3 to roughly compute a lower bound of the mean waiting time of Method 3. We characterized the mean waiting time of Method 1 by comparing it with those of the other two methods using the results of the theoretical analyses and some computer simulation results. From this comparison, we showed that Method 1 minimizes the mean waiting time if $L < 2r$. We also showed that there are four kinds of situations in which relations between Methods 1, 2 and 3 are different, and that effects of the two capabilities of mobility on reducing the mean waiting time are different between these situations. Using the numerical results of the above three theoretical analyses, we can estimate in what situations Method 1 and Method 2 can sufficiently reduce the mean waiting time, and in what situations a hybrid method such as Method 3 is necessary.

Other than the future problems described in previous sections, various extensions to other models such as two-dimensional network models and other mobility models are considered as future works.

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Appendix: Computation of P_c

Consider a random variable X that has the following density function given that $X > r$:

$$f_X(x) = \frac{\lambda}{e^{\lambda r} - 1} \times \left[1 + \sum_{j=1}^{\lfloor (x/r)-1 \rfloor} \frac{(-1)^j}{j!} \{\lambda(x-(j+1)r)\}^{j-1} \times e^{-j\lambda r}\{\lambda(x-(j+1)r)+j\}\right], \qquad (A\cdot 1)$$

where $\lfloor \cdot \rfloor$ is the integer part of $\cdot$. This density function is that of the length of a clump defined in [11]. Then, for $L > r$,

$$P_c(L, \lambda, r) = 1 - P(X \leq L) = (1 - e^{-\lambda r})\left(1 - \int_r^L f_X(x)dx\right). \qquad (A\cdot 2)$$

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