PAPER

Theoretical Analysis of Route Expected Transmission Count in Multi-Hop Wireless Networks

Kazuyuki MIYAKITA[†], ***Student Member***, **Keisuke NAKANO**[†a)], ***Member***, **Masakazu SENGOKU**[†], ***and*** **Shoji SHINODA**[††], ***Fellows***

SUMMARY In multi-hop wireless networks, communication quality depends on the route from a source to a destination. In this paper, we consider a one-dimensional multi-hop wireless network where nodes are distributed randomly and theoretically analyze the relation between communication quality and routing policy using a measure called the Expected Transmission Count (ETX), which is the predicted number of data transmissions required to send a packet over that link, including retransmissions. First, we theoretically analyze the mean length of links, the mean number of hops, and the mean route ETX, which is the sum of the ETXs of all links in a route, of Longest Path Routing (LPR), and Shortest Path Routing (SPR). Second, we propose Adjustable Routing (AR), an approximation to Optimum Routing (OR), which minimizes route ETX. We theoretically compute the above characteristic values of AR. We also theoretically compute a lower bound of the mean route ETX of OR. We compare LPR, SPR, and OR using the results of analyses and show differences between these algorithms in the route ETX.
key words: *multi-hop wireless networks, Expected Transmission Count, theoretical analysis*

1. Introduction

In multi-hop wireless networks [1], [2], a source node sends data to a destination node through a multi-hop path consisting of relay nodes. The multi-hop path is selected by the routing algorithm used in the network. Although the minimum hop algorithm is often used as a routing algorithm, this algorithm tends to choose long links, and, as a result, such a selection may cause frequent link failures and many retransmissions. One countermeasure is to choose shorter links to construct a path; however, they may cause more hops. Due to the increase of hops, the sum of retransmissions over all links in the path may become larger than the minimum hop algorithm. Therefore, it is important to construct a path that simultaneously realizes appropriately short links and a small number of hops.

The delay over a link can be characterized by such metrics as Per-hop Round Trip Time (RTT) [3], Expected Transmission Count (ETX) [4], Medium Time Metric (MTM) [5], Expected Transmission Time Metric (ETT) [6], and so on. If the value of each of these metrics of a link is large, delay over this link will be large. The values of these metrics generally increase as the length of a link increases. If we use ETX as a metric, we can evaluate the quality of a path from the sum of the ETXs of all links included in the path. This sum is called the route ETX. Namely, selecting a path with a minimum value of route ETX is important. Of course, RTT, MTM, and ETT can be used in the same manner.

Suppose that nodes are randomly distributed and the communication range is finite. Then, source node S and destination node D sometimes have multiple candidates for the path between them. If S and D have candidate paths, then the routing algorithm used in the network chooses a path from the candidates. Then since the quality of a path depends on the routing policy in the network, knowing the effects of the routing policy on the quality of route is important. However, to our knowledge, the theoretical analysis of the quality of routes in networks where nodes are randomly distributed has never been done.

In this paper, we theoretically analyze the characteristics of a path of a Longest Path Routing (LPR) and a Shortest Path Routing (SPR) using route ETX as a metric in a one-dimensional multi-hop wireless network where nodes are randomly distributed. Also, we theoretically analyze Optimum Routing (OR), which minimizes the route ETX, using an approximate method and a lower bound because it is difficult to directly analyze OR theoretically. As an approximate method, we propose Adjustable Routing (AR) as a model of OR, theoretically analyze the AR characteristics, and use this value as an approximation of OR characteristics. We show that OR characteristics are well described by both the approximate method and the lower bound. Also, we demonstrate how LPR and SPR behave differently from OR and that OR greatly reduces route ETX compared with LPR and SPR.

The rest of this paper is organized as follows. In Sect. 2, we explain the definitions and assumptions. In Sect. 3, we theoretically analyze the mean length of links, the mean number of hops, and the mean route ETX of LPR. In Sect. 4, we theoretically analyze these characteristic values of SPR. In Sect. 5, we propose AR to analyze the mean link length, the mean number of hops, and the mean route ETX of OR. In Sect. 6, we give a lower bound of the mean route ETX of OR. In Sect. 7, we compare the above characteristics of LPR, SPR, OR, and AR using simulation and numerical results. Sect. 8 concludes this paper.

Manuscript received May 31, 2007.
Manuscript revised February 24, 2008.
[†]The authors are with the Department of Information Engineering, Niigata University, Niigata-shi, 950-2181 Japan.
[††]The author is with the Department of Electrical, Electronic and Communication Engineering, Chuo University, Tokyo, 112-8551 Japan.
a) E-mail: nakano@ie.niigata-u.ac.jp
DOI: 10.1093/ietcom/e91–b.8.2533

2. Definitions and Assumptions

We consider a one-dimensional multi-hop wireless network consisting of nodes randomly distributed along a straight line, as shown in Fig. 1. Assume that the number of nodes between S and D obeys a Poisson distribution with intensity λ and that all nodes are stationary. In this paper, we consider a one-dimensional multi-hop wireless network for the following two reasons: First, a one-dimensional multi-hop wireless network can be used as a model of a street multi-hop wireless network. Second, analysis in a one-dimensional multi-hop wireless network is not easy even though it is simpler than that in a two-dimensional case. Then, as a first step we analyze the route ETX in a one-dimensional multi-hop wireless network.

Define S and D as source and destination nodes, respectively. Suppose there are N nodes v_1, v_2, …, v_N between S and D, as shown in Fig. 1, where $N = 7$. For generalization, S and D are also represented as v_0 and v_{N+1}, respectively. For $i = 0, 1, \ldots, N + 1$, let X_i be the position of v_i, where $X_i \le X_{i+1}$. Suppose $X_0 = 0$ and $X_{N+1} = \ell$, where ℓ is the distance between S and D. Let Y_i be the distance between v_{i-1} and v_i, where $i = 1, 2, \ldots, N + 1$. Let d be the maximum radio transmitting range of a node. Namely, two nodes can be directly linked if the distance between them is not longer than d, and they are not linked otherwise.

Let $u(z)$ be the function of ETX of a link, where z is the length of the link. We assume that $u(z)$ is a convex monotonically increasing function and $u(0) > 0$. Route ETX is the sum of the ETXs of all links in the route. Let R be the set of all routes that can connect S and D in the multi-hop wireless network. Suppose that $r \in R$. Let $L(r)$ be the mean length of links included in r. Let $H(r)$ be the number of hops of r. Let $U(r)$ be the route ETX of r. For example, Fig. 1 shows a 4-hop route r that consists of S, v_2, v_5, v_6, and D, and $H(r) = 4$, $L(r) = \frac{\ell}{4}$, and $U(r) = u(X_2)+u(X_5-X_2)+u(X_6-X_5)+u(\ell-X_6)$ in this example.

In this paper, we consider four kinds of routing policies:

- Longest Path Routing (LPR): This method minimizes the length of each link in the path by maximizing the number of hops.
- Shortest Path Routing (SPR): This method chooses the node furthest from v_0 and within d of v_0 as a first relay node, and as a second relay node, it chooses the node furthest from the first relay node and within d of the first relay node. In the same manner, all relay nodes are chosen. As a result, the number of hops is minimized.
- Optimum Routing (OR): This method minimizes the route ETX.
- Adjustable Routing (AR): This method approximately minimizes the route ETX.

Figure 2 shows examples of the paths selected by LPR, SPR, and AR in a one-dimensional multi-hop wireless network. In the following sections, we analyze the characteristic values of LPR, SPR, OR, and AR.

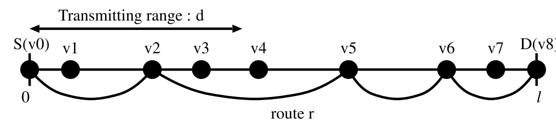
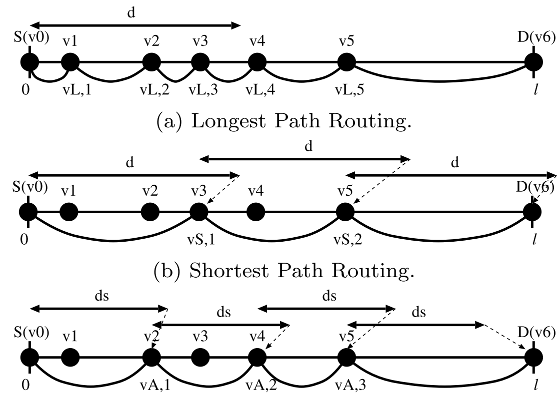


Fig. 1 Street multi-hop wireless network.



(a) Longest Path Routing.

(b) Shortest Path Routing.

(c) Adjustable Routing.

Fig. 2 Routing algorithms.

3. Analysis of Longest Path Routing

LPR minimizes the length of each link in the path by maximizing the number of hops. It selects a multi-hop path that includes all of the N relay nodes between S and D. Suppose that at least one path exists between S and D. Let r_L be the path selected by LPR from R. Let $v_{L,0}$, $v_{L,1}$, …, $v_{L,N+1}$ be the nodes included in r_L, where $v_{L,0} = S$ and $v_{L,N+1} = D$. Then $v_{L,i} = v_i$ for $i = 1, \ldots, N$ as shown in Fig. 2(a). For $i = 0, 1, \ldots, N + 1$, let $X_{L,i}$ be the position of $v_{L,i}$, where $X_{L,i} \le X_{L,i+1}$. Let $Y_{L,i}$ be the distance between $v_{L,i-1}$ and $v_{L,i}$, where $i = 1, 2, \ldots, N + 1$.

In the following, we theoretically compute $E(L(r_L))$, $E(H(r_L))$, and $E(U(r_L))$. These mean values can be represented as follows:

$$E(L(r_L)) = \sum_{k=\lceil \frac{\ell}{d} \rceil}^{\infty} P(H(r_L) = k | R \neq \emptyset) \frac{\ell}{k}, \tag{1}$$

$$E(H(r_L)) = \sum_{k=\lceil \frac{\ell}{d} \rceil}^{\infty} P(H(r_L) = k | R \neq \emptyset) k, \tag{2}$$

$$\begin{aligned} E(U(r_L)) = &\sum_{k=\lceil \frac{\ell}{d} \rceil}^{\infty} P(H(r_L) = k | R \neq \emptyset) \\ &\times E(U(r_L) | R \neq \emptyset, H(r_L) = k), \end{aligned} \tag{3}$$

because $H(r_L) \ge \left\lceil \frac{\ell}{d} \right\rceil$. In the following, we compute $P(H(r_L) = k | R \neq \emptyset)$ and $E(U(r_L) | R \neq \emptyset, H(r_L) = k)$.

First, we compute $P(H(r_L) = k | R \neq \emptyset)$. We have

$$\begin{aligned} P(H(r_L) = k | R \neq \emptyset) &= P(N = k-1 | R \neq \emptyset) \\ &= \frac{P(N = k-1) P(R \neq \emptyset | N = k-1)}{P(R \neq \emptyset)}. \end{aligned} \tag{4}$$

Because N obeys a Poisson distribution with intensity λ, we have

$$P(N = k-1) = \frac{(\lambda\ell)^{k-1}}{(k-1)!}e^{-\lambda\ell}. \tag{5}$$

Also, we have

$$P(R \neq \emptyset | N = k-1) = 1 + \sum_{i=1}^{\lceil \frac{\ell}{d} \rceil - 1} (-1)^i \binom{k}{i} \left(1 - i\frac{d}{\ell}\right)^{k-1}. \tag{6}$$

This equation is given in [7]. We briefly explain the derivation of this equation in Appendix A. From Eqs. (5) and (6), we have

$$\begin{aligned} P(R \neq \emptyset) &= \sum_{k=1}^{\infty} P(N = k-1) P(R \neq \emptyset | N = k-1) \\ &= 1 + \sum_{i=1}^{\lceil \frac{\ell}{d} \rceil - 1} \frac{(-1)^i}{i!} e^{-i\lambda d} \{\lambda(\ell - id)\}^{i-1} \\ &\times \{\lambda(\ell - id) + i\}. \end{aligned} \tag{7}$$

By substituting Eqs. (5), (6), and (7) into Eq. (4), we can compute $P(H(r_L) = k | R \neq \emptyset)$. Note that if $\ell \leq d$, then $P(H(r_L) = k | R \neq \emptyset) = P(N = k-1)$ because $P(R \neq \emptyset | N = k-1) = 1$ and $P(R \neq \emptyset) = 1$.

Next, we compute $E(U(r_L) | R \neq \emptyset, H(r_L) = k)$. If $k = 1$, then r_L is a 1-hop path that directly connects S and D, and $E(U(r_L) | R \neq \emptyset, H(r_L) = 1) = u(\ell)$. If $k \geq 2$, we have

$$\begin{aligned} &E(U(r_L) | R \neq \emptyset, H(r_L) = k) \\ &= \sum_{i=1}^{k} E\left(u(Y_{L,i}) | R \neq \emptyset, H(r_L) = k\right). \end{aligned} \tag{8}$$

To compute Eq. (8), we need the probability density function of $Y_{L,i}$ given that $R \neq \emptyset$ and $H(r_L) = k$ for $i = 1, 2, \ldots, k$. Denote the probability density function of $Y_{L,i}$ given that $R \neq \emptyset$ and $H(r_L) = k$ by $f_{Y_{L,i}}(y | R \neq \emptyset, H(r_L) = k)$, where y is a possible value of $Y_{L,i}$. It is considered that $Y_{L,1}$, $Y_{L,2}$, ..., $Y_{L,k}$ are identically distributed when $R \neq \emptyset$ and $H(r_L) = k$ because of the following reasons: In [8], it is assumed that η points are distributed independently and uniformly in the interval $(0, 1)$, and it is shown that the lengths of $\eta + 1$ divisions formed by these η points are identically distributed. In our model, N means the number of nodes in $[0, \ell]$ and is a Poisson random variable. As mentioned in Appendix A, if we observe the positions of nodes in the interval $[0, \ell]$ when $N = \eta$, where η is a non-negative integer, then these η nodes are distributed independently and uniformly in the interval $[0, \ell]$. From the above property in [8], therefore, we can easily see that Y_1, Y_2, ..., Y_k are identically distributed when $N = k-1$. In addition to this fact, the condition $R \neq \emptyset$ can be represented as $Y_1 \leq d, Y_2 \leq d, \ldots, Y_k \leq d$, and it is considered that the condition $R \neq \emptyset$ equally affects Y_1, Y_2, ..., Y_k when $N = k-1$; therefore, it is considered that Y_1, Y_2, ..., Y_k are identically distributed when $N = k-1$ and $R \neq \emptyset$. Furthermore, $Y_{L,i} = Y_i$ if $R \neq \emptyset$, and $R \neq \emptyset$ and $H(r_L) = k$ if and only if $R \neq \emptyset$ and $N = k-1$ when LPR is used. Therefore, it is considered that $Y_{L,1}$, $Y_{L,2}$, ..., $Y_{L,k}$ are identically distributed when $R \neq \emptyset$ and $H(r_L) = k$. Hence,

$$\begin{aligned} &E(u(Y_{L,1} | R \neq \emptyset, H(r_L) = k)) \\ &= E(u(Y_{L,2} | R \neq \emptyset, H(r_L) = k)) \\ &= \cdots \\ &= E(u(Y_{L,k} | R \neq \emptyset, H(r_L) = k)). \end{aligned} \tag{9}$$

Therefore, from Eqs. (8) and (9),

$$\begin{aligned} &E(U(r_L) | R \neq \emptyset, H(r_L) = k) \\ &= kE\left(u(Y_{L,1}) | R \neq \emptyset, H(r_L) = k\right) \\ &= k \int_0^{\ell} f_{Y_{L,1}}(y | R \neq \emptyset, H(r_L) = k) u(y) dy. \end{aligned} \tag{10}$$

Also, we have

$$\begin{aligned} &P(R \neq \emptyset | N = k-1) f_{Y_1}(y | R \neq \emptyset, N = k-1) dy \\ &= f_{Y_1}(y | N = k-1) dy \\ &\times P(R \neq \emptyset | N = k-1, Y_1 = y), \end{aligned} \tag{11}$$

where $f_{Y_1}(y | N = k-1)$ is the probability density function of Y_1 given that $N = k-1$, and this function is given in [7] as follows:

$$f_{Y_1}(y | N = k-1) = \begin{cases} \frac{k-1}{\ell}\left(1 - \frac{y}{\ell}\right)^{k-2}, & 0 \leq y \leq \ell, \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

We briefly explain derivation of this function in Appendix A. Obviously, we have

$$\begin{aligned} f_{Y_1}(y | R \neq \emptyset, N = k-1) &= f_{Y_{L,1}}(y | R \neq \emptyset, N = k-1) \\ &= f_{Y_{L,1}}(y | R \neq \emptyset, H(r_L) = k). \end{aligned} \tag{13}$$

Also, $P(R \neq \emptyset | N = k-1, Y_1 = y)$ equals the probability that S and v_1 are directly linked, and v_1 and D have a multi-hop path consisting of $k-2$ nodes in $[y, \ell]$. Then from Eq. (6),

$$\begin{aligned} &P(R \neq \emptyset | N = k-1, Y_1 = y) \\ &= 1 + \sum_{i=1}^{\lceil \frac{\ell - y}{d} \rceil - 1} (-1)^i \binom{k-1}{i} \left(1 - \frac{id}{\ell - y}\right)^{k-2} \end{aligned} \tag{14}$$

if $0 \leq y \leq d$, and $P(R \neq \emptyset | N = k-1, Y_1 = y) = 0$ otherwise. We can compute $f_{Y_{L,1}}(y | R \neq \emptyset, H(r_L) = k)$ by substituting Eqs. (6), (12), (13), and (14) into Eq. (11). Note that if $\ell \leq d$, then $f_{Y_{L,1}}(y | R \neq \emptyset, H(r_L) = k) = f_{Y_1}(y | N = k-1)$ because $P(R \neq \emptyset | N = k-1) = 1$ and $P(R \neq \emptyset | N = k-1, Y_1 = y) = 1$.

By substituting Eqs. (4) and (10) into Eqs. (1), (2), and (3), $E(L(r_L))$, $E(H(r_L))$, and $E(U(r_L))$ can be computed as follows: If $\ell \leq d$, then

$$E(L(r_L)) = \frac{1 - e^{-\lambda\ell}}{\lambda}, \tag{15}$$

$$E(H(r_L)) = \lambda\ell + 1, \tag{16}$$

$$E(U(r_L)) = e^{-\lambda\ell}u(\ell) + \int_0^{\ell} u(y)\lambda e^{-\lambda y}\{\lambda(\ell - y) + 2\}dy, \tag{17}$$

and

$$E(L(r_L)) = \frac{\ell}{P(R \neq \emptyset)} \sum_{i=0}^{\lceil \frac{\ell}{d} \rceil - 1} \frac{(-1)^i}{i!} e^{-i\lambda d} \{\lambda(\ell - id)\}^{i-1}, \tag{18}$$

$$E(H(r_L)) = \frac{1}{P(R \neq \emptyset)} \sum_{i=0}^{\lceil \frac{\ell}{d} \rceil - 1} \frac{(-1)^i}{i!} e^{-i\lambda d} \times \{\lambda(\ell - id)\}^{i-1} \left[\{\lambda(\ell - id) + i\}^2 + \lambda(\ell - id)\right], \tag{19}$$

$$E(U(r_L)) = \frac{\lambda}{P(R \neq \emptyset)} \int_0^d \sum_{i=0}^{\lceil \frac{\ell - y}{d} \rceil - 1} \frac{(-1)^i}{i!} \times e^{-\lambda(y+id)} \{\lambda(\ell - y - id)\}^{i-1} \times \left[\{\lambda(\ell - y - id) + (i+1)\}^2 - (i+1)\right] u(y) dy \tag{20}$$

if $\ell > d$, where $P(R \neq \emptyset)$ is given in Eq. (7).

4. Analysis of Shortest Path Routing

SPR selects a multi-hop path that includes the minimum number of relay nodes. In this paper, we suppose that SPR chooses relay nodes in the following manner: it chooses the node furthest from v_0 and within d of v_0 as a first relay node, and as a second relay node, it chooses the node furthest from the first relay node and within d of the first relay node. In the same manner, all relay nodes are chosen. Suppose that at least a path exists between S and D. Let r_S be the path selected by SPR. Let $v_{S,0}$, $v_{S,1}$, …, $v_{S,H(r_S)}$ be the nodes included in r_S, where $v_{S,0} = S$ and $v_{S,H(r_S)} = D$. For $i = 0, 1, \ldots, H(r_S)$, let $X_{S,i}$ be the position of $v_{S,i}$, where $X_{S,i} \leq X_{S,i+1}$. Let $Y_{S,i}$ be the distance between $v_{S,i-1}$ and $v_{S,i}$, where $i = 1, 2, \ldots, H(r_S)$. Let t_i be the number of nodes in the interval $(X_i, X_i + d]$. If v_i is selected as the jth relay node by SPR, v_{i+t_i} is selected as the $j+1$th relay node, where $j = 0, 1, \ldots, H(r_S)-1$. This means that $v_{S,j+1}$ is the node furthest from $v_{S,j}$ and within d of $v_{S,j}$ for $j = 0, 1, \ldots, H(r_S)-1$, and, as a result, r_S minimizes the number of hops. An example of r_S is shown in Fig. 2(b).

First, we consider a case where $\ell \leq d$. Here, SPR directly connects S and D. Then $E(L(r_S)) = \ell$, $E(H(r_S)) = 1$, and $E(U(r_S)) = u(\ell)$.

Next, we consider a case where $\ell > d$. In SPR, $H(r_S) \leq 2\left\lceil \frac{\ell}{d} \right\rceil - 1$, as proved in Appendix B. Therefore, $E(L(r_S))$, $E(H(r_S))$, and $E(U(r_S))$ can be represented as follows:

$$E(L(r_S)) = \sum_{k=\lceil \frac{\ell}{d} \rceil}^{2\lceil \frac{\ell}{d} \rceil - 1} P(H(r_S) = k | R \neq \emptyset) \frac{\ell}{k}, \tag{21}$$

$$E(H(r_S)) = \sum_{k=\lceil \frac{\ell}{d} \rceil}^{2\lceil \frac{\ell}{d} \rceil - 1} P(H(r_S) = k | R \neq \emptyset) k, \tag{22}$$

$$E(U(r_S)) = \sum_{k=\lceil \frac{\ell}{d} \rceil}^{2\lceil \frac{\ell}{d} \rceil - 1} P(H(r_S) = k | R \neq \emptyset) \times E(U(r_S) | R \neq \emptyset, H(r_S) = k). \tag{23}$$

In the following, we compute $P(H(r_S) = k | R \neq \emptyset)$ and $E(U(r_S) | R \neq \emptyset, H(r_S) = k)$. Define $f_{S,k}(x_{S,1}, \ldots, x_{S,k-1})$ as the joint probability density function of $X_{S,1}$, $X_{S,2}$, …, $X_{S,k-1}$, where $x_{S,1}$, …, $x_{S,k-1}$ are possible values of $X_{S,1}$, …, $X_{S,k-1}$, respectively. Then

$$\begin{aligned} & f_{S,k}(x_{S,1}, \ldots, x_{S,k-1}) dx_{S,1} \ldots dx_{S,k-1} \\ &= P(x_{S,1} \leq X_{S,1} < x_{S,1} + dx_{S,1}, \ldots, \\ &\quad x_{S,k-1} \leq X_{S,k-1} < x_{S,k-1} + dx_{S,k-1}) \\ &= P(x_{S,1} \leq X_{S,1} < x_{S,1} + dx_{S,1}, \ldots, \\ &\quad x_{S,k-1} \leq X_{S,k-1} < x_{S,k-1} + dx_{S,k-1} \\ &\quad | R \neq \emptyset, H(r_S) = k) \\ &= P(R \neq \emptyset, H(r_S) = k, \\ &\quad x_{S,1} \leq X_{S,1} < x_{S,1} + dx_{S,1}, \ldots, \\ &\quad x_{S,k-1} \leq X_{S,k-1} < x_{S,k-1} + dx_{S,k-1}) \\ &\quad / \ P(R \neq \emptyset, H(r_S) = k). \end{aligned} \tag{24}$$

Define $C_{S,k}$ as the set of $(x_{S,1}, x_{S,2}, \ldots, x_{S,k-1})$ such that

$$\begin{cases} 0 \leq x_{S,1} \leq d, \\ d < x_{S,2} \leq x_{S,1} + d, \\ x_{S,1} + d < x_{S,3} \leq x_{S,2} + d, \\ \ldots \\ x_{S,k-3} + d < x_{S,k-1} \leq x_{S,k-2} + d, \\ x_{S,k-2} + d < \ell \leq x_{S,k-1} + d. \end{cases} \tag{25}$$

Suppose that $(x_{S,1}, \ldots, x_{S,k-1}) \in C_{S,k}$. Then $f_{S,k}(x_{S,1}, \ldots, x_{S,k-1}) \geq 0$. Also, $R \neq \emptyset$, $H(r_S) = k$, $X_{S,1} = x_{S,1}$, …, $X_{S,k-1} = x_{S,k-1}$ if and only if there are $k-1$ nodes at $x_{S,1}$, …, $x_{S,k-1}$, respectively, and no node in the following intervals: $(x_{S,1}, d]$, $(x_{S,2}, x_{S,1} + d]$, …, $(x_{S,k-1}, x_{S,k-2} + d]$ because SPR selects the furthest node within d as the next relay node. Note that these intervals do not overlap, and the sum of the lengths of these intervals is $(k-1)d - x_{S,k-1}$. If $(x_{S,1}, \ldots, x_{S,k-1}) \notin C_{S,k}$, then, $f_{S,k}(x_{S,1}, \ldots, x_{S,k-1}) = 0$. Hence,

$$\begin{aligned} & P(R \neq \emptyset, H(r_S) = k, \\ &\quad x_{S,1} \leq X_{S,1} < x_{S,1} + dx_{S,1}, \ldots, \\ &\quad x_{S,k-1} \leq X_{S,k-1} < x_{S,k-1} + dx_{S,k-1}) \\ &= \lambda^{k-1} dx_{S,1} \ldots dx_{S,k-1} e^{-\lambda\{(k-1)d - x_{S,k-1}\}}. \end{aligned} \tag{26}$$

By integrating Eq. (24), we have

$$P(H(r_S) = k | R \neq \emptyset) = \frac{1}{P(R \neq \emptyset)} \int \cdots \int_{C_{S,k}} \lambda^{k-1} e^{-\lambda\{(k-1)d - x_{S,k-1}\}} dx_{S,1} \ldots dx_{S,k-1}, \tag{27}$$

where $P(R \neq \emptyset)$ can be computed by Eq. (7). Also,

$$\begin{aligned} & E(U(r_S) | R \neq \emptyset, H(r_S) = k) \\ &= \int \cdots \int_{C_{S,k}} f_{S,k}(x_{S,1}, \ldots, x_{S,k-1}) \\ &\quad \times \left\{ \sum_{i=1}^{k} u(x_{S,i} - x_{S,i-1}) \right\} dx_{S,1} \ldots dx_{S,k-1}, \end{aligned} \tag{28}$$

where $x_{S,0} = 0$ and $x_{S,k} = \ell$. Although link lengths are identically distributed for LPR as mentioned above, link lengths have different probability density functions for SPR. Then Eq. (28) cannot be represented as a simple form like Eq. (10). By substituting Eqs. (27) and (28) into Eqs. (21), (22), and (23), we can numerically compute $E(L(r_S))$, $E(H(r_S))$, and $E(U(r_S))$.

5. Analysis of Optimum Routing with Adjustable Routing

5.1 Approximation to Optimum Routing by Adjustable Routing

OR selects a multi-hop path that minimizes the route ETX. Suppose that at least one path exists between S and D. Let r_O be the path selected by OR. Let $v_{O,0}, v_{O,1}, \ldots, v_{O,H(r_O)}$ be the nodes included in r_O, where $v_{O,0} = S$ and $v_{O,H(r_O)} = D$. For $i = 0, 1, \ldots, H(r_O)$, let $X_{O,i}$ be the position of $v_{O,i}$, where $X_{O,i} \leq X_{O,i+1}$. Let $Y_{O,i}$ be the distance between $v_{O,i-1}$ and $v_{O,i}$, where $i = 1, 2, \ldots, H(r_O)$.

Because it is difficult to precisely analyze $E(L(r_O))$, $E(H(r_O))$, and $E(U(r_O))$, we consider an approximate method. In this approximate method, we define a simple model whose nature is expected to resemble OR's. We call this model Adjustable Routing (AR). Let r_A be the path selected by AR. We theoretically analyze the mean length of links, the mean number of hops, and the mean route ETX of AR, which are denoted by $E(L(r_A))$, $E(H(r_A))$, and $E(U(r_A))$, respectively, and use these values as approximations to $E(L(r_O))$, $E(H(r_O))$, and $E(U(r_O))$. We try to verify that ETX of AR can be used as an approximation to ETX of OR based on some relations between ETXs of LPR, SPR, OR, and AR. Also, comparison between the numerical results of theoretical analysis of AR and simulation results of OR will be done in Sect. 7 to show the above approximation method is valid. Furthermore, we consider a lower bound of $E(U(r_O))$ in the following section.

The following explains relay nodes selected by AR. Suppose that at least one path exists between S and D. Let $v_{A,0}, v_{A,1}, \ldots, v_{A,H(r_A)}$ be the nodes included in r_A, where $v_{A,0} = S$ and $v_{A,H(r_A)} = D$. For $i = 0, 1, \ldots, H(r_A)$, let $X_{A,i}$ be the position of $v_{A,i}$, where $X_{A,i} \leq X_{A,i+1}$. Let $Y_{A,i}$ be the distance between $v_{A,i-1}$ and $v_{A,i}$, where $i = 1, 2, \ldots, H(r_A)$. Define d_s as a positive constant and $d_s \leq d$. Let t'_i be the number of nodes in the interval $(X_i, X_i + d_s]$. Suppose that v_i is selected as a jth relay node by AR. If $t'_i \geq 1$, $v_{i+t'_i}$ is selected as a $j+1$th relay node, and v_{i+1} is selected as a $j+1$th relay node if $t'_i = 0$, where $j = 0, 1, \ldots, H(r_A) - 1$. An example of r_A is shown in Fig. 2(c).

In general, if λ is small, LPR, SPR, OR, and AR tend to select similar paths because there are only a few candidate paths between S and D. Then for small λ, d_s is not expected to be so sensitive to make $E(U(r_A))$ close to $E(U(r_O))$. For large λ, however, we need to carefully decide d_s to minimize $U(r_A)$ because there are many candidate paths. In this paper, we assume that $u(z) \geq 2u(\frac{d}{2})$ and set d_s to be $d_{s,opt}$, which satisfies the following condition:

$$u(d_{s,opt}) = 2u\left(\frac{d_{s,opt}}{2}\right). \tag{29}$$

We have:

Property 1: If $u(z)$ is a convex monotonically increasing function, $u(0) > 0$, and $u(d) \geq 2u(\frac{d}{2})$, then a unique solution exists for Eq. (29).

The proof is given in Appendix C. If $d_s = d_{s,opt}$, we also have the following three properties on relations between $U(r_A)$, $U(r_L)$, $U(r_S)$, and $U(r_O)$.

Property 2: If $d_s = d_{s,opt}$, then $U(r_A) \leq U(r_L)$.

Property 3: If $\frac{d_{s,opt}}{2} \leq Y_{A,i} \leq d_{s,opt}$ for $i = 1, 2, \ldots, H(r_A)$, and $Y_{S,i} > d_{s,opt}$ for $i = 1, 2, \ldots, H(r_S)$, then $U(r_A) < U(r_S)$.

Property 4: If $\frac{d_{s,opt}}{2} \leq Y_{A,i} \leq d_{s,opt}$ for $i = 1, 2, \ldots, H(r_A)$, then $U(r_A) \leq 2U(r_O)$.

The proofs of these properties are provided in Appendices D, E, and F. From Property 2, we have $E(U(r_A)) \leq E(U(r_L))$ if $d_s = d_{s,opt}$ for any λ. If λ is large, $Y_{A,i}$ tends to satisfy $\frac{d_{s,opt}}{2} \leq Y_{A,i} \leq d_{s,opt}$ for $i = 1, 2, \ldots, H(r_A)$, and $Y_{S,i}$ tends to satisfy $Y_{S,i} > d_{s,opt}$ for $i = 1, 2, \ldots, H(r_S)$ because there are many candidate paths for a large λ, and, as a result, the link length tends to be close to $d_{s,opt}$ for AR and to d for SPR. Then $E(U(r_A)) < E(U(r_S))$ is expected for a large λ from Property 3. Hence, $E(U(r_A))$ is expected to be smaller than $E(U(r_L))$ and $E(U(r_S))$ even for large λ in addition to the fact that only a small difference exists between $E(U(r_A))$, $E(U(r_L))$, and $E(U(r_S))$ for small λ. Furthermore, we have $E(U(r_A)) \leq 2U(r_O)$ from Property 4. Consequently, it is expected that $E(U(r_A))$ is smaller than $E(U(r_L))$ and $E(U(r_S))$ and is close to $E(U(r_O))$ for large λ by setting d_s to be $d_{s,opt}$.

5.2 Analysis of Adjustable Routing

In this subsection, we theoretically compute $E(L(r_A))$, $E(H(r_A))$, and $E(U(r_A))$. Suppose that $\ell \leq d_s$. In this case, AR directly connects S and D. Then $E(L(r_A)) = \ell$, $E(H(r_A)) = 1$, and $E(U(r_A)) = u(\ell)$ if $\ell \leq d_s$. Suppose that $\ell > d_s$. In this case, $H(r_A) \leq 2\left\lceil \frac{\ell}{d_s} \right\rceil - 1$, which can be proved in the same manner as in SPR, which is shown in Appendix B. If $\ell > d_s$, therefore,

$$E(L(r_A)) = \sum_{k=\lceil \frac{\ell}{d} \rceil}^{2\lceil \frac{\ell}{d_s} \rceil - 1} P(H(r_A) = k | R \neq \emptyset) \frac{\ell}{k}, \tag{30}$$

$$E(H(r_A)) = \sum_{k=\lceil \frac{\ell}{d} \rceil}^{2\lceil \frac{\ell}{d_s} \rceil - 1} P(H(r_A) = k | R \neq \emptyset) k, \tag{31}$$

$$E(U(r_A)) = \sum_{k=\lceil \frac{\ell}{d} \rceil}^{2\lceil \frac{\ell}{d_s} \rceil - 1} P(H(r_A) = k | R \neq \emptyset) \times E(U(r_A) | R \neq \emptyset, H(r_A) = k). \tag{32}$$

We compute $P(H(r_A) = k|R \neq \emptyset)$ and $E(U(r_A)|R \neq \emptyset, H(r_A) = k)$ in the same manner as $P(H(r_S) = k|R \neq \emptyset)$ and $E(U(r_S)|R \neq \emptyset, H(r_S) = k)$.

Suppose that $H(r_A) = 1$. Then obviously

$$P(H(r_A) = 1|R \neq \emptyset) = \begin{cases} e^{-\lambda\ell}, & \ell \leq d, \\ 0, & \ell > d. \end{cases} \tag{33}$$

$$E(U(r_A)|R \neq \emptyset, H(r_A) = 1) = u(\ell). \tag{34}$$

Next, we consider $H(r_A) = k$ where $k \geq 2$. Define $f_{A,k}(x_{A,1}, \ldots, x_{A,k-1})$ as the joint probability density function of $X_{A,1}, X_{A,2}, \ldots, X_{A,k-1}$, where $x_{A,1}, \ldots, x_{A,k-1}$ are possible values of $X_{A,1}, \ldots, X_{A,k-1}$, respectively. Then

$$\begin{aligned} & f_{A,k}(x_{A,1}, \ldots, x_{A,k-1}) dx_{A,1} \ldots dx_{A,k-1} \\ &= P(x_{A,1} \leq X_{A,1} < x_{A,1} + dx_{A,1}, \ldots, \\ &\quad x_{A,k-1} \leq X_{A,k-1} < x_{A,k-1} + dx_{A,k-1}) \\ &= P(x_{A,1} \leq X_{A,1} < x_{A,1} + dx_{A,1}, \ldots, \\ &\quad x_{A,k-1} \leq X_{A,k-1} < x_{A,k-1} + dx_{A,k-1} \\ &\quad |R \neq \emptyset, H(r_A) = k) \\ &= P(R \neq \emptyset, H(r_A) = k, \\ &\quad x_{A,1} \leq X_{A,1} < x_{A,1} + dx_{A,1}, \ldots, \\ &\quad x_{A,k-1} \leq X_{A,k-1} < x_{A,k-1} + dx_{A,k-1}) \\ &\ / \ P(R \neq \emptyset, H(r_A) = k). \end{aligned} \tag{35}$$

Define $C_{A,k}$ as the set of $(x_{A,1}, x_{A,2}, \ldots, x_{A,k-1})$ such that

$$\begin{cases} 0 \leq x_{A,1} \leq d, \\ \max\{d_s, x_{A,1}\} < x_{A,2} \leq x_{A,1} + d, \\ \max\{x_{A,1} + d_s, x_{A,2}\} < x_{A,3} \leq x_{A,2} + d, \\ \ldots \\ \max\{x_{A,k-3} + d_s, x_{A,k-2}\} < x_{A,k-1} \\ \leq x_{A,k-2} + d, \\ \max\{x_{A,k-2} + d_s, x_{A,k-1}\} < \ell \leq x_{A,k-1} + d. \end{cases} \tag{36}$$

Suppose that $(x_{A,1}, \ldots, x_{A,k-1}) \in C_{A,k}$. Then $f_{A,k}(x_{A,1}, \ldots, x_{A,k-1}) \geq 0$. Also, $R \neq \emptyset$, $H(r_A) = k$, $X_{A,1} = x_{A,1}$, ..., $X_{A,k-1} = x_{A,k-1}$ if and only if there are $k-1$ nodes at $x_{A,1}$, $x_{A,2}$, ..., $x_{A,k-1}$, respectively, and no node in $g(x_{A,i-1}, x_{A,i})$ for $i = 1, 2, \ldots, k$, where

$$g(x_{A,i-1}, x_{A,i}) = \begin{cases} (x_{A,i}, x_{A,i-1} + d_s], & x_{A,i} - x_{A,i-1} \leq d_s, \\ (x_{A,i-1}, x_{A,i}), & \text{otherwise}. \end{cases} \tag{37}$$

Define that

$$G(x_{A,1}, \ldots, x_{A,k-1}) = \left| \left\{ \bigcup_{i=1}^{k} g(x_{A,i-1}, x_{A,i}) \right\} \cap [0, \ell] \right|, \tag{38}$$

where $x_{A,0} = 0$ and $x_{A,k} = \ell$. If $(x_{A,1}, \ldots, x_{A,k-1}) \notin C_{A,k}$, then $f_{A,k}(x_{A,1}, \ldots, x_{A,k-1}) = 0$. Hence,

$$\begin{aligned} & P(R \neq \emptyset, H(r_A) = k, \\ &\quad x_{A,1} \leq X_{A,1} < x_{A,1} + dx_{A,1}, \ldots, \\ &\quad x_{A,k-1} \leq X_{A,k-1} < x_{A,k-1} + dx_{A,k-1}) \\ &= \lambda^{k-1} dx_{A,1} \ldots dx_{A,k-1} e^{-\lambda G(x_{A,1}, \ldots, x_{A,k-1})}. \end{aligned} \tag{39}$$

By integrating Eq. (35), we have

$$\begin{aligned} P(H(r_A) = k|R \neq \emptyset) &= \int \cdots \int_{C_{A,k}} \frac{1}{P(R \neq \emptyset)} \\ &\lambda^{k-1} e^{-\lambda G(x_{A,1}, \ldots, x_{A,k-1})} dx_{A,1} \ldots dx_{A,k-1}, \end{aligned} \tag{40}$$

where $P(R \neq \emptyset)$ can be computed by Eq. (7). Also

$$\begin{aligned} & E(U(r_A)|R \neq \emptyset, H(r_A) = k) \\ &= \int \cdots \int_{C_{A,k}} f_{A,k}(x_{A,1}, \ldots, x_{A,k-1}) \\ &\quad \times \left\{ \sum_{i=1}^{k} u(x_{A,i} - x_{A,i-1}) \right\} dx_{A,1} \ldots dx_{A,k-1}, \end{aligned} \tag{41}$$

where $x_{A,0} = 0$ and $x_{A,k} = \ell$. By substituting Eqs. (40) and (41) into Eqs. (30), (31), and (32), we can numerically compute $E(L(r_A))$, $E(H(r_A))$, and $E(U(r_A))$.

6. Lower Bound of Mean Route ETX of OR

In this section, we compute a lower bound of $E(U(r_O))$. Let d_0 be a value of z such that $\frac{u(z)}{z}$ is the smallest. Define that

$$\hat{u}(z) = \begin{cases} \dfrac{u(d_0)}{d_0} z, & z \leq d_0, \\ u(z), & z > d_0. \end{cases} \tag{42}$$

Define that

$$U_{low} = \sum_{i=1}^{N+1} \hat{u}(Y_i). \tag{43}$$

Then $U_{low} \leq U(r_O)$, as proved in Appendix G, and we can use $E(U_{low})$ as a lower bound of $E(U(r_O))$. We can compute $E(U_{low})$ by Eqs. (17) and (20) after replacing $u(z)$ with $\hat{u}(z)$ because Y_i is the distance between v_{i-1} and v_i.

Suppose that $\ell = nd_0$ where n is a positive integer. When λ is large, it is expected that we can construct an n-hop path consisting of n links whose lengths are close to d_0 because there are many candidate paths. Consider a situation where we can construct an n-hop path consisting of n links whose lengths are equal to d_0. This path minimizes route ETX from Lemma 6 in Appendix F. Then $U(r_O) = nu(d_0)$. Also in this situation,

$$U_{low} = u(d_0) \sum_{i=1}^{N+1} \frac{Y_i}{d_0} = nu(d_0) \tag{44}$$

because $Y_i \leq d_0$ for $i = 1, 2, \ldots, N+1$. Therefore, U_{low} is identical to $U(r_O)$ in the above situation, and similar situations are expected to appear when λ is large.

Conversely, as λ decreases, the number of candidate paths between S and D decreases, and the number of candidate hops decrease and approach $\left\lceil \frac{\ell}{d} \right\rceil$. Consider a situation where there is only one candidate path between S and D, and the number of hops of the path is $\left\lceil \frac{\ell}{d} \right\rceil$. Then $U(r_O)$ and U_{low} are represented as follows:

$$U(r_O) = \sum_{i=1}^{\lceil \frac{\ell}{d} \rceil} u(Y_i), \tag{45}$$

$$U_{low} = \sum_{i=1}^{\lceil \frac{\ell}{d} \rceil} \hat{u}(Y_i). \tag{46}$$

Define that $I = \{i \,|\, i \in \{1, 2, \ldots, \lceil \ell/d \rceil\}, Y_i < d_0\}$ and suppose that $|I| = m$. Then

$$U(r_O) - U_{low} = \sum_{i \in I} \{u(Y_i) - \hat{u}(Y_i)\}. \tag{47}$$

Also, if $Y_i < d_0$, then $u(Y_i) - \hat{u}(Y_i) \leq \left(1 - \frac{Y_i}{d_0}\right) u(0)$. Hence,

$$\begin{aligned} U(r_O) - U_{low} &\leq \sum_{i \in I} \left(1 - \frac{Y_i}{d_0}\right) u(0) \\ &\leq \left\{ m - \frac{\ell - \left(\left\lceil \frac{\ell}{d} \right\rceil - m\right) d}{d_0} \right\} u(0) \leq u(0). \end{aligned} \tag{48}$$

Therefore, U_{low} is close to $U(r_O)$ in the above situation. It is expected that similar situations will appear when λ is small. Hence, it is expected that U_{low} will be close to $U(r_O)$ for a small λ as well as for a large λ.

7. Comparisons between LPR, SPR, OR, and AR

In this section, we compute the characteristic values of LPR, SPR, OR, and AR numerically and by computer simulation. We compare the results to confirm the validity of the theoretical analyses and observe the differences between the four methods.

In computer simulations, OR selects a multi-hop path that minimizes route ETX as follows: Suppose that at least one path exists between S and D. Consider a graph, which consists of the set of nodes $V = \{v_i | i = 0, 1, \ldots, N+1\}$ and the set of links $E = \{(v_i, v_j) |$ the distance between v_i and v_j is not greater than $d\}$. Define the weight of a link as its ETX. Then OR can find the path with the minimum ETX between S and D using the Dijkstra algorithm. Also, in computer simulations, we decide the number of nodes and positions of the nodes according to the assumptions in Sect. 2. If at least one path exists between S and D, we construct r_L, r_S, r_A, and r_O by LPR, SPR, AR, and OR, respectively, and compute the mean length of links, the number of hops, and the route ETX of each path. Otherwise, we decide the number of nodes and the positions of these nodes again. We repeat these procedures 100000 times and compute the mean of each characteristic.

We use $u_1(z)$ and $u_2(z)$ in Fig. 3 as functions of the ETX of a link. $u_1(z)$ and $u_2(z)$ are assumed at a one-sided patio and a concrete canyon, respectively, and the computations of these functions are provided in Appendix H. Also, we set d to 20. If $d = 20$, then $u_1(d) \geq 2u_1(\frac{d}{2})$ and $u_2(d) \geq 2u_2(\frac{d}{2})$. Hence, we can use Property 1 by setting d to 20 while we use $u_1(z)$ and $u_2(z)$ in Fig. 3. Note that $u_1(d) = u_2(d) = 20$, both $u_1(z)$ and $u_2(z)$ are convex monotonically increasing functions, and $u_2(z) \geq u_1(z)$ for $0 \leq z \leq d$. According to Eq. (29), $d_{s,opt} = 16.5$ for $u_1(z)$, and $d_{s,opt} = 11.4$ for $u_2(z)$. As mentioned, we set d_s to be $d_{s,opt}$ for AR.

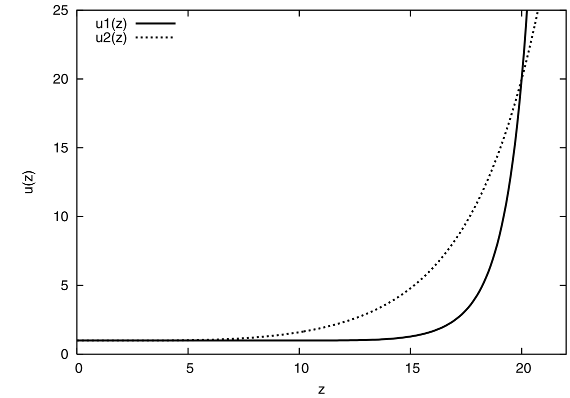
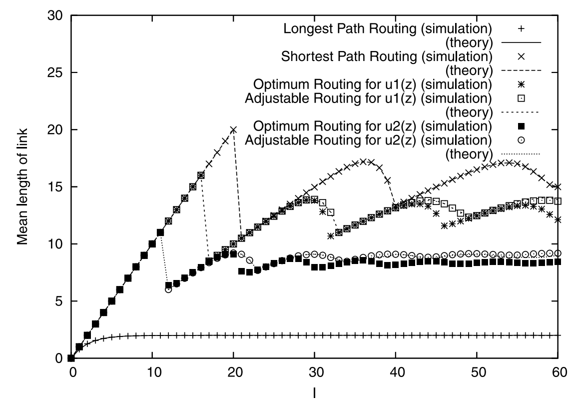
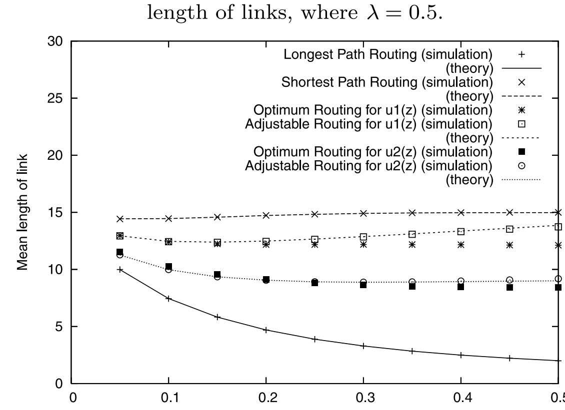


Fig. 3 Functions of link ETX $u_1(z)$, $u_2(z)$.



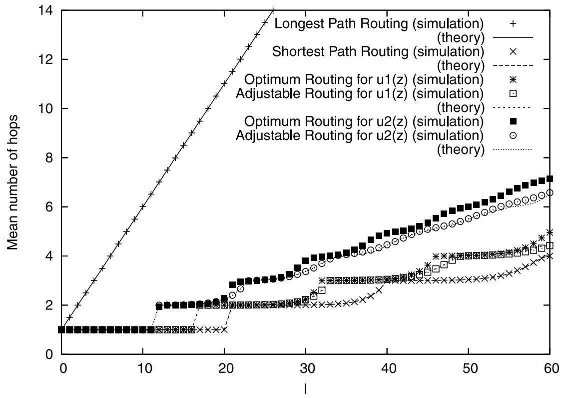
(a) Relation between source-destination distance ℓ and mean length of links, where $\lambda = 0.5$.



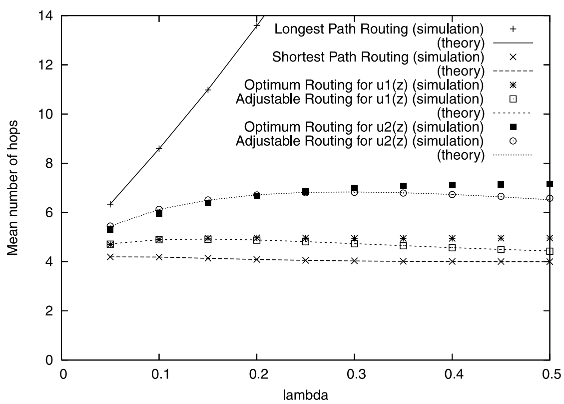
(b) Relation between node density λ and mean length of links, where $\ell = 60$.

Fig. 4 Mean length of links.

Figure 4 shows the numerical results of $E(L(r_L))$, $E(L(r_S))$, and $E(L(r_A))$, and the simulation results of $E(L(r_L))$, $E(L(r_S))$, $E(L(r_O))$, and $E(L(r_A))$. Figure 5 shows the numerical results of $E(H(r_L))$, $E(H(r_S))$, and $E(H(r_A))$, and the simulation results of $E(H(r_L))$, $E(H(r_S))$, $E(H(r_O))$, and $E(H(r_A))$. Figures 6 and 7 show the numerical results of

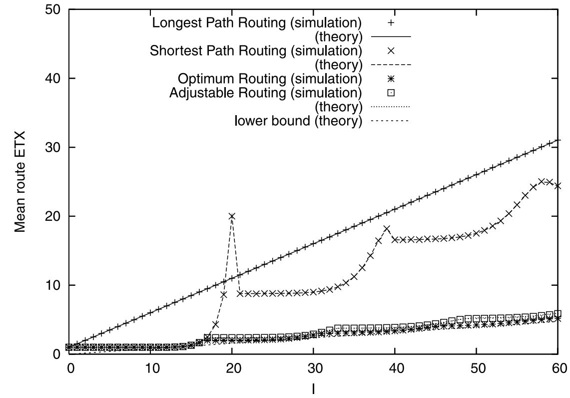


(a) Relation between source-destination distance ℓ and mean number of hops, where $\lambda = 0.5$.

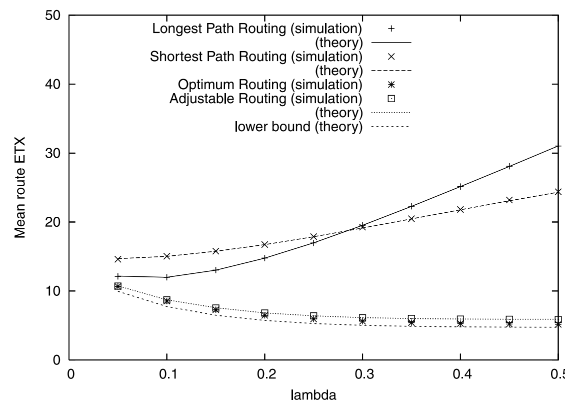


(b) Relation between node density λ and mean number of hops, where $\ell = 60$.

Fig. 5 Mean number of hops.



(a) Relation between source-destination distance ℓ and mean route ETX, where $\lambda = 0.5$.



(b) Relation between node density λ and mean route ETX, where $\ell = 60$.

Fig. 6 Mean route ETX for $u_1(z)$.

$E(U(r_L))$, $E(U(r_S))$, $E(U(r_A))$, and $E(U_{low})$, and the simulation results of $E(U(r_L))$, $E(U(r_S))$, $E(U(r_O))$, $E(U(r_A))$. In Figs. 4(a), 5(a), 6(a), and 7(a), the horizontal axis is ℓ, and $\lambda = 0.5$. In Figs. 4(b), 5(b), 6(b), and 7(b), the horizontal axis is λ, and $\ell = 60$. Note that Fig. 4 includes two curves of $E(L(r_O))$, two curves of $E(L(r_A))$ that are different in $u(z)$, just one curve of $E(L(r_L))$, and one curve of $E(L(r_S))$ because $E(L(r_L))$ and $E(L(r_S))$ do not depend on $u(z)$. In the same manner, Fig. 5 includes two curves of $E(H(r_O))$, two curves of $E(H(r_A))$, one curve of $E(H(r_L))$, and one curve of $E(H(r_S))$. Figure 6 shows $E(U(r_L))$, $E(U(r_S))$, $E(U(r_O))$, $E(U(r_A))$, and $E(U_{low})$ for $u_1(z)$, and Fig. 7 shows those for $u_2(z)$.

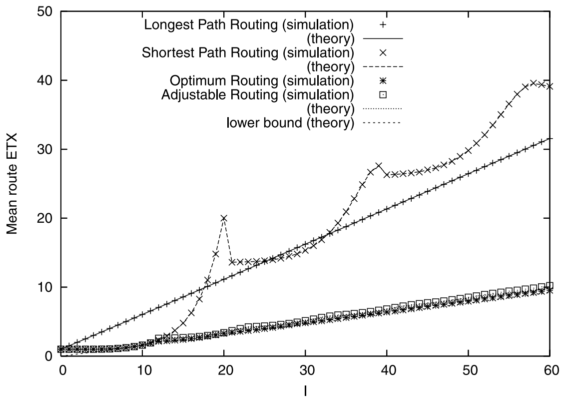
In Figs. 4(a), 5(a), 6(a), and 7(a), curves of SPR discontinuously change at $\ell = 20$. This is because $H(r_S)$ is always equal to 1 if $\ell \leq 20$, and $H(r_S) \geq 2$ if $\ell > 20$ because $d = 20$. Also, curves of OR and AR discontinuously change at $\ell = d_{s,opt}$ because $H(r_O)$ and $H(r_A)$ are always equal to 1 if $\ell \leq d_{s,opt}$ while $H(r_O)$ and $H(r_A)$ may be equal to more than 2 if $\ell > d_{s,opt}$.

In these figures, the numerical results of $E(L(r_L))$, $E(H(r_L))$, $E(U(r_L))$, $E(L(r_S))$, $E(H(r_S))$, $E(U(r_S))$, $E(L(r_A))$, $E(H(r_A))$, and $E(U(r_A))$ agree well with the simulation results. Then we can confirm that the theoretical analyses of LPR, SPR, and AR are valid. We can also confirm that $E(L(r_L)) \leq E(L(r_O)) \leq E(L(r_S))$ in Fig. 4 and that $E(H(r_S)) \leq E(H(r_O)) \leq E(H(r_L))$ in Fig. 5. Figures 6 and 7 show that OR significantly reduces route ETX compared with LPR and SPR. From these results, we can confirm that selecting a path with shorter links than SPR and with fewer hops than LPR reduces route ETX as can be expected intuitively.

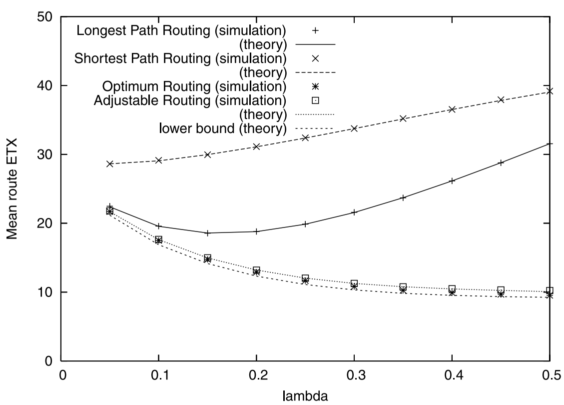
In Figs. 4 to 7, we can confirm that $E(L(r_A))$, $E(H(r_A))$, and $E(U(r_A))$ are close to $E(L(r_O))$, $E(H(r_O))$, and $E(U(r_O))$, respectively, for both $u_1(z)$ and $u_2(z)$. These results show that we can use $E(L(r_A))$, $E(H(r_A))$, and $E(U(r_A))$ as approximations for $E(L(r_O))$, $E(H(r_O))$, and $E(U(r_O))$. From Figs. 6 and 7, we can confirm that $E(U_{low})$ is close to $E(U(r_O))$ for both $u_1(z)$ and $u_2(z)$ while it is easier to compute $E(U_{low})$ than $E(U(r_A))$. This means that $E(U_{low})$ is a good lower bound for $E(U(r_O))$.

8. Conclusion

We theoretically computed the mean length of links, the mean number of hops, and the mean route ETX of the Longest Path Routing and Shortest Path Routing. Also, we proposed Adjustable Routing, which is an approximation to Optimum Routing, and analyzed the above three characteristic values of Adjustable Routing. We showed that



(a) Relation between source-destination distance ℓ and mean route ETX, where $\lambda = 0.5$.



(b) Relation between node density λ and mean route ETX, where $\ell = 60$.

Fig. 7 Mean route ETX for $u_2(z)$.

Adjustable Routing well describes Optimum Routing. We also computed a lower bound of the mean route ETX of Optimum Routing and showed that it was close to the mean route ETX of Optimum Routing. We compared Longest Path Routing, Shortest Path Routing, and Optimum Routing through analyses and showed that Optimum Routing significantly reduces the mean route ETX compared with Longest Path Routing and Shortest Path Routing. Future problems include route ETX analyses considering multiple flows of data, that cause interferences, and extensions of analyses to two-dimensional networks.

Acknowledgments

This research was partially supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid. The authors would like to thank anonymous reviewers for their valuable comments.

References

[1] C.E. Perkins, Ad Hoc Networking, Addison-Wesley, 2001.
[2] I.F. Akyildiz, X. Wang, and W. Wang, "Wireless mesh networks: A survey," Comput. Netw., vol.47, no.4, pp.445–487, March 2005.
[3] A. Adya, P. Bahl, J. Padhye, A. Wolman, and L. Zhou, "A multi-radio unification protocol for IEEE 802.11 wireless networks," IEEE International Conference on Broadband Networks (Broadnets) 2004, pp.344–354, 2004.
[4] D.S.J. De Couto, D. Aguayo, J. Bicket, and R. Morris, "A high-throughput path metric for multi-hop wireless routing," ACM MOBICOM 2003, pp.134–146, Sept. 2003.
[5] B. Awerbuch, D. Holmer, and H. Rubens, "The medium time metric: High throughput route selection in multi-rate ad hoc wireless networks," Technical Report, Johns Hopkins University, Department of Computer Science, Oct. 2004.
[6] R. Draves, J. Padhye, and B. Zill, "Routing in multi-radio, multi-hop wireless mesh networks," ACM MOBICOM 2004, pp.114–128, 2004.
[7] P. Hall, Introduction to theory of coverage processes, John Wiley & Sons, 1988.
[8] W. Feller, An introduction to probability theory and its applications, vol.2, John Wiley & Sons, 1966.
[9] M. Zuniga and B. Krishnamachari, "Analyzing the transitional region in low power wireless links," IEEE SECON 2004, pp.517–526, Oct. 2004.
[10] K. Sohrabi, B. Manriquez, and G. Pottie, "Near ground wideband channel measurement in 800–1000 MHz," IEEE Vehicular Technology Conference, vol.1, pp.571–574, July 1999.

Appendix A: Computations of $P(R \neq \emptyset | N = k-1)$ and $f_{Y_1}(y|N = k-1)$

In [7], it is assumed that η points are distributed independently and uniformly in the interval $(0, 1)$, and the probability that the largest division formed by these η points does not exceed ν, where ν is any real number, is computed. Denote this probability by $p_\eta(\nu)$. Then $p_\eta(\nu)$ is given as follows [7]:

$$p_\eta(\nu) = 1 + \sum_{i=1}^{\lceil \frac{1}{\nu} \rceil - 1} (-1)^i \binom{\eta+1}{i} (1 - i\nu)^\eta . \qquad \text{(A· 1)}$$

In our model, N means the number of nodes in $[0, \ell]$ and is a Poisson random variable. We have a well-known property of a Poisson distribution as follows: If we observe the positions of nodes in the interval $[0, \ell]$ when $N = \eta$, where η is a non-negative integer, then these η nodes are distributed independently and uniformly in the interval $[0, \ell]$. Then $P(R \neq \emptyset | N = k - 1)$ is the probability that the largest division formed by $k - 1$ nodes which are independently and uniformly distributed in $[0, \ell]$ does not exceed d. We can derive $P(R \neq \emptyset | N = k-1)$ in the same manner as the derivation of $p_\eta(\nu)$ as follows:

$$\begin{aligned} P(R \neq \emptyset | N = k - 1) &= p_{k-1}\left(\frac{d}{\ell}\right) \\ &= 1 + \sum_{i=1}^{\lceil \frac{\ell}{d} \rceil - 1} (-1)^i \binom{k}{i} \left(1 - i\frac{d}{\ell}\right)^{k-1} . \end{aligned} \qquad \text{(A· 2)}$$

Next, we compute $f_{Y_1}(y|N = k-1)$. In [7], the probability that the smallest of η points which are independently and uniformly distributed in $(0, 1)$ takes a value lying between ν and $\nu + d\nu$, where $0 < \nu < 1$, is exactly computed. Denote this probability by $q_\eta(\nu)$. Then $q_\eta(\nu)$ is given as follows [7]:

$$q_\eta(\nu) = \eta(1 - \nu)^{\eta-1} d\nu. \qquad \text{(A· 3)}$$

In our model, $f_{Y_1}(y|N = k-1)dy$ is the probability that the smallest of $k-1$ points which are independently and uniformly distributed in $[0, \ell]$ takes a value lying between y and $y+dy$. We can derive $f_{Y_1}(y|N = k-1)dy$ in the same manner as the derivation of $q_\eta(\nu)$ as follows:

$$f_{Y_1}(y|N = k-1)dy = q_{k-1}\left(\frac{y}{\ell}\right) = (k-1)\left(1-\frac{y}{\ell}\right)^{k-2}\frac{dy}{\ell}. \tag{A·4}$$

Then

$$f_{Y_1}(y|N = k-1) = \frac{k-1}{\ell}\left(1-\frac{y}{\ell}\right)^{k-2}. \tag{A·5}$$

Appendix B: Proof of Relation $H(r_S) \le 2\left\lceil \frac{\ell}{d} \right\rceil - 1$

In SPR, $X_{S,i+1} > X_{S,i-1} + d$ for $i = 1, 2, \ldots, H(r_S) - 1$. Then $Y_{S,i+1} + Y_{S,i} = X_{S,i+1} - X_{S,i-1} > d$ for $i = 1, 2, \ldots, H(r_S) - 1$. Assume that $H(r_S) \ge 2\left\lceil \frac{\ell}{d} \right\rceil$. Then

$$\ell = \sum_{i=1}^{H(r_S)} Y_{S,i} \ge \sum_{i=1}^{\lceil \frac{\ell}{d} \rceil}(Y_{S,2i-1} + Y_{S,2i}) > \sum_{i=1}^{\lceil \frac{\ell}{d} \rceil} d = \left\lceil \frac{\ell}{d} \right\rceil d \ge \ell, \tag{A·6}$$

which is a contradiction. Therefore, $H(r_S) \le 2\left\lceil \frac{\ell}{d} \right\rceil - 1$.

Appendix C: Proof of Property 1

Define that $u_{dif}(z) = u(z) - 2u(\frac{z}{2})$ and $u'(z) = \frac{du(z)}{dz}$. Then $\frac{du_{dif}(z)}{dz} = u'(z) - u'(\frac{z}{2})$. Because $u(z)$ is a convex monotonically increasing function, $\frac{du_{dif}(z)}{dz} > 0$ for $z > 0$. Also, from the assumptions, $u_{dif}(0) = -u(0) < 0$, and $u_{dif}(d) = u(d) - 2u(\frac{d}{2}) \ge 0$. Consequently, a unique solution of $u_{dif}(z) = 0$ exists in the interval $(0, d]$.

Appendix D: Proof of Property 2

From the definitions in Appendix C, we have:

Lemma 1: If z_1 and z_2 are positive real numbers and $z_1 + z_2 \le d_{s,opt}$, then

$$u(z_1 + z_2) \le u(z_1) + u(z_2). \tag{A·7}$$

Proof. Because $u_{dif}(d_{s,opt}) = 0$ and $u_{dif}(z)$ is a monotonically increasing function, $u_{dif}(z_1+z_2) \le 0$. Then $u(z_1+z_2) \le 2u(\frac{z_1+z_2}{2})$. Also, $2u(\frac{z_1+z_2}{2}) \le u(z_1) + u(z_2)$ because $u(z)$ is a convex monotonically increasing function. From these inequalities, Lemma 1 holds. □

From this lemma,

Lemma 2: Let $M(v_i, v_{i+k})$ be the path that connects two nodes v_i and v_{i+k} and includes v_{i+1}, v_{i+2}, …, v_{i+k-1} as relay nodes, where k is a positive integer. Suppose again that AR constructs a path consisting of $v_{A,0}$, $v_{A,1}$, …, $v_{A,H(r_A)}$. If $d_s = d_{s,opt}$, then

$$u(Y_{A,i}) \le U(M(v_{A,i-1}, v_{A,i})), \tag{A·8}$$

where $i = 1, 2, \ldots, H(r_A)$.

Proof. Assume that $v_{A,i-1}$ corresponds to v_k. First, suppose that $t'_k \le 1$. Then $v_{A,i} = v_{k+1}$ due to the nature of AR. Hence $M(v_{A,i-1}, v_{A,i})$ is a direct link between $v_{A,i-1}$ and $v_{A,i}$ if $t'_k \le 1$. As a result, Lemma 2 holds if $t'_k \le 1$. Conversely, suppose that $t'_k \ge 2$. Then $v_{A,i} = v_{k+t'_k}$ and $Y_{A,i} \le d_{s,opt}$ due to the nature of AR. Hence,

$$U(M(v_{A,i-1}, v_{A,i})) = u(Y_{k+1}) + u(Y_{k+2}) + \ldots + u(Y_{k+t'_k}). \tag{A·9}$$

By repeatedly applying Lemma 1 to $u(Y_{k+1}) + u(Y_{k+2}) + \ldots + u(Y_{k+t'_k})$, we can obtain

$$\begin{aligned} u(Y_{A,i}) &= u(Y_{k+1} + Y_{k+2} + \ldots + Y_{k+t'_k}) \\ &\le u(Y_{k+1}) + u(Y_{k+2}) + \ldots + u(Y_{k+t'_k}) \\ &= U(M(v_{A,i-1}, v_{A,i})). \end{aligned} \tag{A·10}$$

□

From this lemma, we have $U(r_A) \le U(r_L)$.

Appendix E: Proof of Property 3

Property 3 can be proved from the following lemmas.

Lemma 3:

$$\begin{cases} \dfrac{u(z)}{z} \le \dfrac{u(d_{s,opt})}{d_{s,opt}}, & \frac{d_{s,opt}}{2} \le z \le d_{s,opt}, \\ \dfrac{u(z)}{z} > \dfrac{u(d_{s,opt})}{d_{s,opt}}, & \text{otherwise.} \end{cases} \tag{A·11}$$

Proof. First, suppose that $\frac{d_{s,opt}}{2} \le z \le d_{s,opt}$. Then

$$\frac{u(z) - u\left(\frac{d_{s,opt}}{2}\right)}{z - \frac{d_{s,opt}}{2}} \le \frac{u(d_{s,opt}) - u\left(\frac{d_{s,opt}}{2}\right)}{d_{s,opt} - \frac{d_{s,opt}}{2}} \tag{A·12}$$

because $u(z)$ is a convex monotonically increasing function. From Eqs. (29) and (A·12), $\frac{u(z)}{z} \le \frac{u(d_{s,opt})}{d_{s,opt}}$. Also, we have $\frac{u(z)}{z} > \frac{u(d_{s,opt})}{d_{s,opt}}$ if $z > d_{s,opt}$ or $z < \frac{d_{s,opt}}{2}$ in the same manner as the case where $\frac{d_{s,opt}}{2} \le z \le d_{s,opt}$. □

Lemma 4: If $\frac{d_{s,opt}}{2} \le Y_{A,i} \le d_{s,opt}$ for $i = 1, 2, \ldots, H(r_A)$, then $U(r_A) \le \frac{u(d_{s,opt})}{d_{s,opt}}\ell$.

Proof. From Lemma 3,

$$\begin{aligned} U(r_A) &= \sum_{i=1}^{H(r_A)} u(Y_{A,i}) = \sum_{i=1}^{H(r_A)} \frac{u(Y_{A,i})}{Y_{A,i}} Y_{A,i} \\ &\le \sum_{i=1}^{H(r_A)} \frac{u(d_{s,opt})}{d_{s,opt}} Y_{A,i} = \frac{u(d_{s,opt})}{d_{s,opt}}\ell. \end{aligned} \tag{A·13}$$

□

In the same manner, we have:

Lemma 5: If $Y_{S,i} > d_{s,opt}$ for $i = 1, 2, \ldots, H(r_S)$, then $U(r_S) > \frac{u(d_{s,opt})}{d_{s,opt}}\ell$.

Appendix F: Proof of Property 4

Let d_0 be a value of z such that $\frac{u(z)}{z}$ is the smallest. Then Property 4 is proved by Lemmas 4, 6, and 7:

Lemma 6: $U(r_O) \geq \frac{u(d_0)}{d_0}\ell$.

Proof. From the definition of d_0, $\frac{u(Y_{O,i})}{Y_{O,i}} \geq \frac{u(d_0)}{d_0}$. Then

$$U(r_O) = \sum_{i=1}^{H(r_O)} u(Y_{O,i}) = \sum_{i=1}^{H(r_O)} \frac{u(Y_{O,i})}{Y_{O,i}} Y_{O,i} \geq \sum_{i=1}^{H(r_O)} \frac{u(d_0)}{d_0} Y_{O,i} = \frac{u(d_0)}{d_0}\ell. \quad \text{(A· 14)}$$

□

Lemma 7: $\frac{u(d_{s,opt})}{d_{s,opt}} \leq 2\frac{u(d_0)}{d_0}$.

Proof. From Lemma 3 and the relation $\frac{u(d_0)}{d_0} \leq \frac{u(d_{s,opt})}{d_{s,opt}}$, $\frac{d_{s,opt}}{2} \leq d_0 \leq d_{s,opt}$. Also, $u(\frac{d_{s,opt}}{2}) \leq u(d_0) \leq u(d_{s,opt})$ because $u(z)$ monotonically increases. From these relations, $d_0 u\left(\frac{d_{s,opt}}{2}\right) \leq d_{s,opt}u(d_0)$. From this relation and Eq. (29), we have $\frac{u(d_{s,opt})}{d_{s,opt}} \leq 2\frac{u(d_0)}{d_0}$. □

Appendix G: Proof of relation $U_{low} \leq U(r_O)$

Because $u(z) \geq \hat{u}(z)$ for any z,

$$\begin{aligned} U(r_O) &= u(Y_1 + Y_2 + \ldots + Y_{i_1}) \\ &+ u(Y_{i_1+1} + Y_{i_1+2} + \ldots + Y_{i_2}) + \ldots \\ &+ u(Y_{i_{k-1}+1} + Y_{i_{k-1}+2} + \ldots + Y_{i_k}) \\ &\geq \hat{u}(Y_1 + Y_2 + \ldots + Y_{i_1}) \\ &+ \hat{u}(Y_{i_1+1} + Y_{i_1+2} + \ldots + Y_{i_2}) + \ldots \\ &+ \hat{u}(Y_{i_{k-1}+1} + Y_{i_{k-1}+2} + \ldots + Y_{i_k}), \end{aligned} \quad \text{(A· 15)}$$

where $i_k = N+1$. Also, $\hat{u}(z_1+z_2)-\hat{u}(z_1) \geq \hat{u}(z_2)-\hat{u}(0) = \hat{u}(z_2)$ for any z_1 and z_2 because $\hat{u}(0) = 0$ and $\hat{u}(z)$ is a convex monotonically increasing function. Namely, $\hat{u}(z_1 + z_2) \geq \hat{u}(z_1) + \hat{u}(z_2)$. By repeatedly using this relation, we have

$$\begin{aligned} &\hat{u}(Y_{i_j+1} + Y_{i_j+2} + \ldots + Y_{i_{j+1}}) \\ &\geq \hat{u}(Y_{i_j+1}) + \hat{u}(Y_{i_j+2}) + \ldots + \hat{u}(Y_{i_{j+1}}). \end{aligned} \quad \text{(A· 16)}$$

From Eqs. (A· 15) and (A· 16), $U(r_O) \geq \sum_{i=1}^{N+1} \hat{u}(Y_i) = U_{low}$.

Appendix H: Computation of Link ETX

Assume that link ETX function $u(z)$ is a function of $p(z)$, which is the probability that a packet is successfully transmitted over a wireless link of length z, and that $u(z) = \frac{1}{p(z)^2}$ as assumed in [4]. We use the following model for $p(z)$, which is derived in [9] with assumptions that non-coherent FSK modulation and NRZ encoding are used.

$$p(z) = \int_{-\infty}^{\infty} f_{X_\sigma}(x) \left(1 - \frac{1}{2} e^{-\frac{\gamma(z)}{2} \cdot \frac{1}{0.64}}\right)^{8\alpha} dx. \quad \text{(A· 17)}$$

In this equation, $X_{\sigma\ dB}$ is a zero-mean Gaussian RV (in dB) with standard deviation σ, $f_{X_\sigma}(x)$ is the probability density function of X_σ, α is the packet size (in bytes), and $\gamma(z)_{dB} = P_{t\ dB} - PL(z)_{dB} - P_{n\ dB}$, where $P_{t\ dB}$ is the transmitting power, $PL(z)_{dB}$ is the propagation loss at a length z, and $P_{n\ dB}$ is the noise floor. $PL(z)_{dB}$ can be computed as $PL(z)_{dB} = PL(z_0)_{dB} + 10n\log_{10}\left(\frac{z}{z_0}\right) + X_{\sigma\ dB}$, where $PL(z_0)_{dB}$ is the propagation loss at a reference length z_0 and n is the path loss exponent. In this paper, we use two examples of link ETX functions $u_1(z)$ and $u_2(z)$, which are calculated with the following parameters obtained by [10]. $u_1(z)$ is assumed at a one-sided patio, and $n = 3.2$, $\sigma = 1.85$, $z_0 = 1$ m, $PL(z_0)_{dB} = -36.6$ dB, and $P_{t\ dB} = 0.3353$ dBm. $u_2(z)$ is assumed at a concrete canyon, and $n = 2.7$, $\sigma = 5.1$, $z_0 = 1$ m, $PL(z_0)_{dB} = -46.35$ dB, and $P_{t\ dB} = 1.2097$ dBm.

Kazuyuki Miyakita graduated from the advanced engineering course of Nagaoka National College of Technology, and received the B.E. and M.E. degrees from National Institution of Academic Degrees in 2005 and Niigata University in 2007, respectively. He is a Ph.D. candidate of Graduate School of Science and Technology, Niigata University.

Keisuke Nakano received B.E., M.E. and Ph.D. degrees from Niigata University in 1989, 1991 and 1994, respectively. He is currently an Associate Professor of the Department of Information Engineering at Niigata University. He received the Best Paper Award of IEEE IC-NNSP'95. He also received the Best Paper Award of IEICE in 1997. He was a visiting scholar at the University of Illinois in 1999. His research interests include computer and mobile networks. He is a member of IEEE.

Masakazu Sengoku received a B.E. degree from Niigata University in 1967 and M.E. and Ph.D. degrees from Hokkaido University in 1969 and 1972, respectively. He is a Professor in the Department of Information Engineering at Niigata University. His research interests include network theory, graph theory, and mobile communications. He received the Best Paper Awards of IEICE in 1992, 1996, 1997 and 1998, and the Achievement Award of IEICE in 2005. He also received the Best Paper Award of the 1995 IEEE ICNNSP. He is a fellow of the IEEE.

Shoji Shinoda received B.E., M.E. and D.E. degrees, all in electrical engineering, from Chuo University, Tokyo, Japan, in 1964, 1966, and 1973, respectively. In April, 1965, he joined the Faculty of Science and Engineering at Chuo University, Tokyo, Japan. Since then, he has engaged in education and research in the fields of electrical circuit theory, network flow and tension theory, discrete systems, and mobile communication systems. He is now a Professor in the Department of Electrical, Electronic and Communication Engineering at Chuo University. He has published more than one hundred technical papers in these fields. He is also the recipient of the 1992 IEICE Best Paper Award, the 1997 IEICE Best Paper Award, the 1998 IEICE Best Paper Award, the Best Paper Award of the 1995 IEEE International Conference on Neural Networks and Signal Processing, and the 2005 IEICE Achievement Award. He is now a fellow of the IEEE and a member of the Society of Instrument and Control Engineers, the Japan Society of Simulation Technology, and the Korean Institute of Telematics and Electronics.