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A Simple Exact Error Rate Analysis for DS-CDMA with Arbitrary Pulse Shape in Flat Nakagami Fading

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SUMMARY A simple exact error rate analysis is presented for random binary direct sequence code division multiple access (DS-CDMA) considering a general pulse shape and flat Nakagami fading channel. First of all, a simple model is developed for the multiple access interference (MAI). Based on this, a simple exact expression of the characteristic function (CF) of MAI is developed in a straight forward manner. Finally, an exact expression of error rate is obtained following the CF method of error rate analysis. The exact error rate so obtained can be much easily evaluated as compared to the only reliable approximate error rate expression currently available, which is based on the Improved Gaussian Approximation (IGA).

key words: *DS-CDMA, multi-access communication, error rate analysis, Nakagami fading*

1. Introduction

Error analysis for direct sequence code division multiple access (DS-CDMA) has been a topic of great interest for the last three decades. Numerous studies have been performed on the topic covering both exact and approximate type analysis (see [1]–[12] and the references there in). Among those, papers contributing the most well-known approximate results for DS-CDMA in additive white Gaussian noise (AWGN) channel can be found in [1], [2] that are based on standard, improved and simplified improved Gaussian approximations: SGA, IGA and SIGA respectively. For fading channel, studies have mainly been performed based on the SGA for the sake of simplicity and can be found in [3]–[5]. Recently, the SIGA and IGA were also extended for Nakagami fading channel in [6].

Since deriving the exact expression of bit error probability (BEP) for DS-CDMA is believed to be far more complex, it has been the subject of very few analyses. [7] presents the most well-known exact analysis so far for DS-CDMA in AWGN channel. Since, the complex analysis becomes much more complex in a fading channel, [8] presented an exact analysis and [9] presented a precise (but not exact) analysis for just a binary phase shift keying (BPSK) link in the presence of co-channel interference (CCI) without considering any spreading (hence not CDMA) for mathematical tractability. The authors interestingly discovered that at this moment, the exact BEP for binary DS-CDMA employing an arbitrary pulse shape in flat Nakagami fading channel doesn't exist in literature (see [1]–[12] and the references there in).

Recently, an exact error analysis for DS-CDMA was given in [10] considering a flat Nakagami fading channel and rectangular pulse shape by extending a previous work for Rayleigh channel. However, the computational complexity of the method is so high that even the authors presented numerical examples in the paper using an approximate model. Hence, a simpler exact error analysis for such systems considering a general pulse shape and fading channel is highly demanding now. In this letter, we will present such an analysis that results in a simple-to-evaluate exact error rate.

2. System Model

We consider a conventional DS-CDMA system that uses BPSK modulation. A typical representation of the signal of an arbitrary user $k \in (1, 2, \dots, K)$ at the transmission side has the form

$$s_k(t) = \sqrt{\frac{E_k}{N}} \sum_{n=-\infty}^{+\infty} b_{\lfloor n/N \rfloor}^{(k)} a_n^{(k)} \psi(t - nT_c) \cos(2\pi f_c t + \theta_k) \tag{1}$$

where t is time and $\psi(t)$ is the baseband chip waveform with unit energy. Here, f_c is the carrier frequency and θ_k is the phase at the time of transmission, which is uniform in $[0, 2\pi]$. The rest of the signal structure is described as follows:

- E_k is the bit energy of user k given by $E_k = P_k T_b$, where P_k is the average power of user k and T_b is the bit duration.
- N is the number of chips or pulses representing one information bit.
- T_c is the chip duration that is also equal to duration of the pulse $\psi(t)$. So, the bit duration is $T_b = NT_c$.
- $\{b_i^{(k)}\}$ is the i-th bit of user k which is a random variable (RV) uniform on $\{+1, -1\}$. Here, $i = \lfloor n/N \rfloor$ and $\lfloor . \rfloor$ represents floor function.
- $\{a_n^{(k)}\}$ is the random polarity code for user k, which is also an RV uniform on $\{+1, -1\}$ and is periodic with period N.

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$$\Phi_{I_k|\alpha_k,\beta_k,\phi_k,L_n^{(k)},M_n^{(k)}}(\omega) = \exp\left(j\omega \sum_{n=iN}^{(i+1)N-1} \sqrt{\frac{E_k}{N}}\beta_k \cos\phi_k \left(L_n^{(k)}\hat{R}_\psi(\alpha_k) + M_n^{(k)} R_\psi(\alpha_k)\right)\right) \quad (7)$$

$$\Phi_{I_k|\alpha_k,\beta_k,\phi_k}(\omega) = \left[\frac{1}{2}\left\{\cos\left(\omega\sqrt{\frac{E_k}{N}}\beta_k\cos\phi_k\left(\hat{R}_\psi(\alpha_k)+R_\psi(\alpha_k)\right)\right)\cos\left(\omega\sqrt{\frac{E_k}{N}}\beta_k\cos\phi_k\left(\hat{R}_\psi(\alpha_k)-R_\psi(\alpha_k)\right)\right)\right\}\right]^N \quad (8)$$

3. Multiple Access Interference Modeling

Considering a time-invariant slow flat Nakagami faded channel, the received signal can be given by

$$r(t) = \sum_{k=1}^{K} \sqrt{\frac{E_k}{N}} \sum_{n=-\infty}^{+\infty} b_{\lfloor n/N \rfloor}^{(k)} a_n^{(k)} \beta_k \times \psi_k(t - nT_c - \tau_k)\cos(2\pi f_c t + \vartheta_k) + n(t) \quad (2)$$

where τ_k is the delay of the k-th user signal with respect to the desired user 1's signal ($\tau_1 = 0$) and $n(t)$ is AWGN noise with two sided power spectral density of $N_o/2$. β_k is the fading amplitude having a Nakagami distributed probability density function (pdf) given by $f_{\beta_k}(\beta_k) = \frac{2m_k^{m_k}\beta_k^{2m_k-1}}{\Omega_k^{m_k}\Gamma(m_k)}\exp\left(-\frac{m_k\beta_k^2}{\Omega_k}\right)$, $\beta_k \geq 0$ where m_k $(0.5 \leq m_k \leq \infty)$ is the Nakagami fading parameter for user k, $\Gamma(.)$ is gamma function and $\Omega_k = E[\beta_k^2]$, $E[.]$ representing mean value. ϑ_k in (2) is the phase of the k-th user's received signal that includes the effects of phase of transmission θ_k and phase alteration due to delay and fading effect and is uniformly distributed over $[0, 2\pi]$. Let $\tau_k = \alpha_k + \gamma_k T_c$ where, γ_k is an RV uniform on $\{0, 1, \ldots, N-1\}$ and $0 \leq \alpha_k < T_c$.

The received signal is correlated with a template of the form $s_{temp}^{(1)}(t) = \sum_{n=iN}^{(i+1)N-1} a_n^{(1)}\psi(t - nT_c)\cos(2\pi f_c t + \vartheta_1)$. Hence, the decision statistics for a coherent correlation receiver while detecting the the i-th bit of the desired user 1 can be given by

$$y^{(1)} = \sqrt{E_1 N} b_i^{(1)} \beta_1 + \sum_{k=2}^{K} I_k + \eta, \quad (3)$$

$$I_k = \sum_{n=iN}^{(i+1)N-1} \sqrt{\frac{E_k}{N}} \beta_k \cos\phi_k W_n^{(k)}. \quad (4)$$

The right side of (3) has three parts, of which the first part is the desired signal component, the second part is the multiple access interference (MAI) component further given in (4) and the third part η is the AWGN component having variance of $\sigma_\eta^2 = N_o N/2$. In (4), $\phi_k = \vartheta_k - \vartheta_1$ and $W_n^{(k)}$, which is the MAI component on the n-th chip of user 1 from user k, is given by

$$W_n^{(k)} = L_n^{(k)}\hat{R}_\psi(\alpha_k) + M_n^{(k)} R_\psi(\alpha_k) \quad (5)$$

where $L_n^{(k)}$ and $M_n^{(k)}$ are RVs uniform on $\{+1, -1\}$ and $\hat{R}_\psi(\alpha_k)$, $R_\psi(\alpha_k)$ are the partial autocorrelation functions of the baseband chip waveform given by $\hat{R}_\psi(\alpha_k) = \int_{\alpha_k}^{T_c} \psi(t)\psi(t-\alpha_k)dt$ and $R_\psi(\alpha_k) = \hat{R}_\psi(T_c - \alpha_k)$.

4. The Exact Characteristic Function of MAI

For an asynchronous system α_k is uniformly distributed over $[0, T_c]$. The conditional CF of MAI from an arbitrary user k can be given by

$$\Phi_{I_k|\alpha_k,\beta_k,\phi_k,L_n^{(k)},M_n^{(k)}}(\omega) = E\left[\exp(j\omega I_k)\,|\alpha_k,\beta_k,\phi_k,L_n^{(k)},M_n^{(k)}\right] \quad (6)$$

where $j = \sqrt{-1}$. Here note that (6) presents the CF conditioned on the independent RVs $\alpha_k, \beta_k, \phi_k, L_n^{(k)}, M_n^{(k)}$. Putting I_k from (4) in (6), we get (7) as shown above. Now there are two possibilities: either $L_n^{(k)} = M_n^{(k)}$ or $L_n^{(k)} = -M_n^{(k)}$. Counting for both the possibilities and using the identity $\cos z = [\exp(jz) + \exp(-jz)]/2$ we obtain the expression shown in (8) from (7). Using simple trigonometric identities, (8) simplifies to

$$\Phi_{I_k|\alpha_k,\beta_k,\phi_k}(\omega) = \cos^N\left(\omega\sqrt{E_k/N}\beta_k\cos\phi_k\hat{R}_\psi(\alpha_k)\right) \times \cos^N\left(\omega\sqrt{E_k/N}\beta_k\cos\phi_k R_\psi(\alpha_k)\right). \quad (9)$$

The expression in (9) at a first sight may seen not to be of a convenient form, since it has two cosine functions multiplied each with power N. However, this is not the fact. Since, a cosine with either even or odd integer power can be expressed as a weighted summation of cosines having unity power as shown in the appendix, multiplication of two cosine functions each having power N can also be expressed in a similar way (see (17), (18)). Now after using the identities (17), (18) derived in appendix into (9), we take the following two actions in sequel: first, we integrate the expressions obtained over the density of ϕ_k, which is uniform over $[0, 2\pi]$ and use the identity $J_0(z) = \frac{1}{2\pi}\int_0^{2\pi}\cos(z\cos\phi)d\phi$ where J_0 is the zeroth order Bessel function of the first kind. This provides $\Phi_{I_k|\alpha_k,\beta_k}(\omega)$ expressed in terms of J_0. Next, we integrate $\Phi_{I_k|\alpha_k,\beta_k}(\omega)$ over the Nakagami density of β_k and employ the identity

$$\int_0^\infty J_0(z\beta_k) f_{\beta_k}(\beta_k) d\beta_k = {}_1F_1\left(m_k; 1; -\frac{\Omega_k z^2}{4m_k}\right) \triangleq \mathcal{F}_k\left(z^2\right) \quad (10)$$

$$\Phi_{I_k|\alpha_k}(\omega) = \frac{1}{2^{2N}}\left[\sum_{q_1=0}^{\frac{N}{2}-1}\sum_{q_2=0}^{\frac{N}{2}-1} 2\binom{N}{q_1}\binom{N}{q_2}\left\{\mathcal{F}_k\left(\omega^2[\hat{x}(q_1)+x(q_2)]^2\right)+\mathcal{F}_k\left(\omega^2[\hat{x}(q_1)-x(q_2)]^2\right)\right\}\right.$$
$$\left.+\binom{N}{\frac{N}{2}}\sum_{q=0}^{\frac{N}{2}-1}2\binom{N}{q}\left\{\mathcal{F}_k\left(\omega^2\hat{x}^2(q)\right)+\mathcal{F}_k\left(\omega^2x^2(q)\right)\right\}+\binom{N}{\frac{N}{2}}^2\right], N \text{ even} \quad (11)$$

$$\Phi_{I_k|\alpha_k}(\omega) = \frac{1}{2^{2N-1}}\left[\sum_{q_1=0}^{\frac{N-1}{2}}\sum_{q_2=0}^{\frac{N-1}{2}}\binom{N}{q_1}\binom{N}{q_2}\left\{\mathcal{F}_k\left(\omega^2[\hat{x}(q_1)+x(q_2)]^2\right)+\mathcal{F}_k\left(\omega^2[\hat{x}(q_1)-x(q_2)]^2\right)\right\}\right], N \text{ odd} \quad (12)$$

from [9, (41), (42)] where $f_{\beta_k}(\beta_k)$ is the Nakagami density of β_k and ${}_1F_1(:;:;:)$ is the confluent hypergeometric function. After these two operations, $\Phi_{I_k|\alpha_k}(\omega)$ is given by (11) for N even and (12) for N odd where $\mathcal{F}_k(.)$ is defined in (10) and $\hat{x}(q) = (N-2q)\sqrt{E_k/N}\hat{R}_\psi(\alpha_k)$ $x(q) = (N-2q)\sqrt{E_k/N}R_\psi(\alpha_k)$. Finally, to obtain the unconditional exact CF we have to integrate (11), (12) over the density of α_k, which is uniformly distributed in $[0, T_c]$ giving $\Phi_{I_k}(\omega) = \frac{1}{T_c}\int_0^{T_c}\Phi_{I_k|\alpha_k}(\omega)d\alpha_k$. Since, the MAI coming from different users $k = 2, 3, \ldots, K$ are independent, the exact CF of total MAI takes the form $\Phi_I(\omega) = \prod_{k=2}^K \Phi_{I_k}(\omega)$.

Note that the MAI from other users depends on the cross-correlation properties of the spreading sequences. In this work, the effect is taken into account in terms of CF without deriving the cross-correlation function explicitly. We also didn't need the density of collision statistics of the spreading sequences since those are random. This facilitates obtaining the simpler expressions of exact CF for a general pulse shape involving confluent hypergeometric functions that are readily computed by Matlab. However, [10] followed a method where the density of collision statistics of the spreading sequences were required that eventually led to a complex expression for CF exact for only a rectangular pulse shape.

5. The Exact Error Probabilities

The exact BEP conditioned on the fading amplitude of the desired user user 1, β_1, can now be given by

$$P_{e|\beta_1} = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\sin(\sqrt{E_1N}\beta_1\omega)}{\omega}\Phi_I(\omega)\Phi_\eta(\omega)d\omega \quad (13)$$

where $\Phi_\eta(\omega) = \exp(-\sigma_\eta^2\omega^2/2)$ is the CF of AWGN. Integrating (13) over the Nakagami density of β_1, and using two identities given in [13, section 8.3, Eqs. (17), (35)] we obtain the unconditional exact BEP expression as

$$P_e = \frac{1}{2} - \frac{1}{\pi}\frac{\Gamma(m_1+\frac{1}{2})}{\Gamma(m_1)}\sqrt{\frac{E_1N\Omega_1}{m_1}}\int_0^{+\infty}\Phi_I(\omega)$$
$$\times\Phi_\eta(\omega){}_1F_1\left(m_1+\frac{1}{2};\frac{3}{2};-\frac{E_1N\Omega_1\omega^2}{4m_1}\right)d\omega. \quad (14)$$

6. Error Probabilities Based on Gaussian Approximation

The SGA: Under the SGA of MAI, the BEP conditioned on the fading amplitude of the desired user 1, β_1 can be given by $P_{e|\beta_1}^{SGA} = 0.5\text{erfc}\left(\beta_1\sqrt{0.5E_1N/\left(\sigma_\eta^2+\mu_I\right)}\right)$, where μ_I is the mean of total MAI variance given by $\mu_I = \sum_{k=2}^K \Omega_k E_k \rho_\psi$. Here, $\rho_\psi = \frac{1}{T_c}\int_0^{T_c}\hat{R}_\psi^2(\alpha_k)d\alpha_k = \frac{1}{T_c}\int_0^{T_c}R_\psi^2(\alpha_k)d\alpha_k$. Now integrating $P_{e|\beta_1}^{SGA}$ over the density of β_1 and employing the identity from [13, p.20, Eq. (45)], the unconditional BEP under SGA can be given by

$$P_e^{SGA} = \frac{\Lambda^{m_1}}{2\sqrt{\pi}}\frac{\Gamma(m_1+\frac{1}{2})}{m_1\Gamma(m_1)}\,{}_2F_1\left(m_1,\frac{1}{2};m_1+1;\Lambda\right) \quad (15)$$

where $\Lambda = \left(1+\frac{E_1N\Omega_1}{2m_1(\sigma_\eta^2+\mu_I)}\right)^{-1}$ and ${}_2F_1(:;:;:)$ is Gauss hypergeometric function. Note that this is a new expression of BEP for DS-CDMA in flat Nakagami fading based on the SGA, which is equivalent to, however, different from the currently available three different expressions that can be found in [3, Eq. (16a)], [5, Eq. (5.16)], [6, Eq. (20)].

The SIGA and IGA: Though the SGA results into a simple expression for error rate, it has been speculated in several papers [1], [2], [6], [9]–[12] that the BEP thus obtained may be substantially inaccurate. Another method the SIGA is known to be simple and accurate, however, has limited region of validity [11]. Rather the IGA can be a reliable way to obtain approximate yet accurate results [1], [12] without any limitation on the validity. Though it has been possible to develop closed-form BEP for IGA considering rectangular pulse in AWGN channel [1], [12], such BEP for a general pulse shape involves total $2(K-1)$ in-folded integrations over delays and phases of all interfering users [12]. For a Nakagami fading channel, we find that the BEP that can be given by as shown in (16), involves $(3K-2)$ in-folded integrations that are very cumbersome. In (16) Ψ_{I_k} is the conditional variance of the MAI from user k given by $\Psi_{I_k} = E_k\beta_k^2\cos^2\phi_k\left(\hat{R}^2(\alpha_k)+R^2(\alpha_k)\right)$, $f_B(\beta) = f_{\beta_1}(\beta_1)\ldots f_{\beta_K}(\beta_K)$ and $d\Delta = d\phi_2\ldots d\phi_K d\alpha_2\ldots d\alpha_K$ $d\beta_1\ldots d\beta_K$.

$$P_e^{IGA} = \underbrace{\int_0^\infty \cdots \int_0^\infty}_{K} \underbrace{\int_0^{T_c} \cdots \int_0^{T_c}}_{K-1} \underbrace{\int_0^{2\pi} \cdots \int_0^{2\pi}}_{K-1} \frac{1}{2}\text{erfc}\left(\beta_1 \sqrt{\frac{0.5E_1N}{\left(\sigma_\eta^2 + \sum_{k=2}^K \Psi_{I_k}\right)}}\right) \frac{f_B(\beta)d\Delta}{(2\pi T_c)^{(K-1)}} \quad (16)$$

$$\cos^{2n} z_1 \cos^{2n} z_2 = \frac{1}{2^{4n}}\left[\sum_{q_1=0}^{n-1}\sum_{q_2=0}^{n-1} 2\binom{2n}{q_1}\binom{2n}{q_2}\{\cos[\hat{z}_1(q_1) + \hat{z}_2(q_2)] + \cos[\hat{z}_1(q_1) - \hat{z}_2(q_2)]\}\right.$$
$$\left. + \binom{2n}{n}\sum_{q=0}^{n-1} 2\binom{2n}{q}\{\cos[\hat{z}_1(q)] + \cos[\hat{z}_2(q)]\} + \binom{2n}{n}^2\right] \quad (17)$$

$$\cos^{2n-1} z_1 \cos^{2n-1} z_2 = \frac{1}{2^{4n-3}}\sum_{q_1=0}^{n-1}\sum_{q_2=0}^{n-1}\binom{2n-1}{q_1}\binom{2n-1}{q_2}\{\cos[\hat{z}_1(q_1) - z_1 + \hat{z}_2(q_2) - z_2]$$
$$+ \cos[\hat{z}_1(q_1) - z_1 - \hat{z}_2(q_2) + z_2]\} \quad (18)$$

7. Numerical Examples and Conclusion

In this section, the results presented in the previous sections are evaluated by illustrative examples. Theoretical results are verified by Monte Carlo simulations. A 1) rectangular pulse shape and a 2) Gaussian pulse shape given by $\psi(t) = \frac{v(t)}{\sqrt{\mathcal{E}_v}}$ where, 1) $v(t) = 1$, $0 < t \leq T_c$, $v(t) = 0$, otherwise and 2) $v(t + T_c/2) = e^{-\pi\left(\frac{t}{t_m}\right)^2}$ with $t_m = 0.4T_c$ respectively are considered. The energy of $v(t)$ is $\mathcal{E}_v = \int_{-\infty}^{+\infty} v^2(t)dt$. The partial autocorrelations of $\psi(t)$ are given by 1) $\hat{R}_\psi(\alpha) = (T_c - \alpha)/\mathcal{E}_v$ and 2) $\hat{R}_\psi(\alpha) = e^{-0.5\pi(\alpha/t_m)^2}$ respectively and $R_\psi(\alpha) = \hat{R}_\psi(T_c - \alpha)$ in both cases. Asynchronous system with all users having the same power level and similar fading level (let, $\Omega_k E_k = E_b$, $m_k = m$, $k = 1, 2, \ldots, K$) is considered. Figure 1 shows the exact CF for asynchronous DS-CDMA with $K = 2$, implying one interfering users. The other parameters are $N = 15$, $E_b/N_o = 20$ dB and $m = 1$ and 5.

Figure 2 shows the BEP versus total users K of the system with $N = 63$, $E_b/N_o = 20$ dB and $m = 1$ and 5. BEP from our exact method absolutely agrees with those from simulations. Though the SGA is found to be an accurate approximation at $m = 1$ (Rayleigh fading) irrespective of the pulse shape and simultaneous users, as the fading severity decreases to $m = 5$, it can no longer provide accurate approximation for any of the pulses, especially for small number of users. Figure 3 shows more detail results on accuracy of the SGA for various Nakagami parameters, m. We set $K = 3$, $N = 20$ and $E_b/N_o = 10$, 20 dB. As seen, at high SNR value of $E_b/N_o = 20$ dB, the SGA provides reasonably accurate approximation of the exact BEP for $0.5 \leq m \leq 1.5$. However, the range of m where the SGA is accurate is a strong function of SNR. At low SNR like 10 dB, the SGA can be reasonably accurate upto much higher values of m.

Though bit error analysis of DS-CDMA has been widely studied, we discovered that the exact error analysis for DS-CDMA with a general pulse shape and Nakagami fading channel has not been studied yet. This letter provides such a study, the strongest point of which is the simplicity. Though [10] studied the same for a rectangular pulse shape,

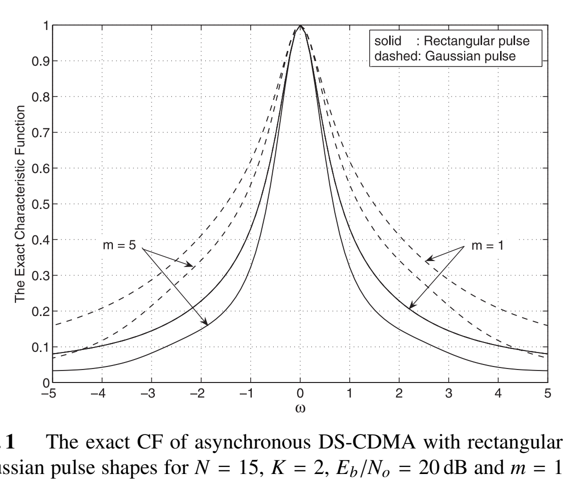

Fig. 1 The exact CF of asynchronous DS-CDMA with rectangular and Gaussian pulse shapes for $N = 15$, $K = 2$, $E_b/N_o = 20$ dB and $m = 1$ and 5.

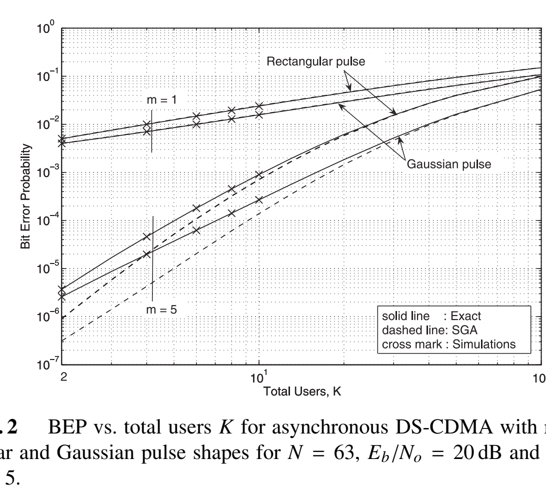

Fig. 2 BEP vs. total users K for asynchronous DS-CDMA with rectangular and Gaussian pulse shapes for $N = 63$, $E_b/N_o = 20$ dB and $m = 1$ and 5.

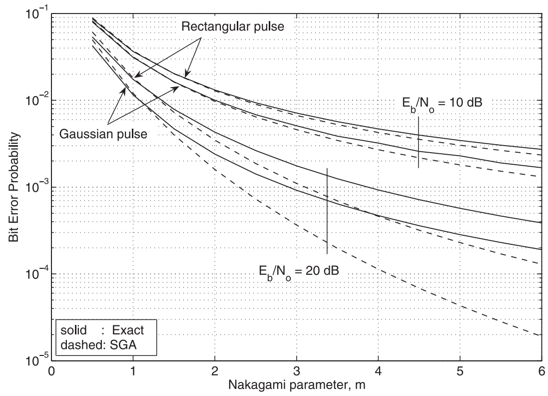

Fig. 3 BEP vs. Nakagami parameter m for asynchronous DS-CDMA with rectangular and Gaussian pulse shapes for $K = 3$ with $N = 20$ and $E_b/N_o = 10, 20$ dB.

simplicity wasn't guaranteed and the work is not easily extensible for a general pulse shape. We agree that the computation complexity of our method increases with chip length N and total number of users K, however, it's still simpler than the widely used IGA method [1]. DS-CDMA has been applied for numerous practical applications including third generation (3G) mobile communications. This is true that many of those applications experience frequency selective fading channel. Our this work on flat fading can be considered as a first but very important step toward developing such analysis for frequency selective fading channel, which we keep for a future study.

Appendix. From [14, appendix G] we obtain the trigonometric identities for integer $n \geq 1$, $\cos^{2n} z = \frac{1}{2^{2n}} \left\{ \sum_{q=0}^{n-1} 2 \begin{pmatrix} 2n \\ q \end{pmatrix} \cos[2(n-q)z] + \begin{pmatrix} 2n \\ n \end{pmatrix} \right\}$ and

$\cos^{2n-1} z = \frac{1}{2^{2n-2}} \left\{ \sum_{q=0}^{n-1} \begin{pmatrix} 2n-1 \\ q \end{pmatrix} \cos[(2n-2q-1)z] \right\}$.

Using these two identities and with simple mathematical manipulation we easily obtain the following two expressions in (17) and (18), where $\hat{z}_1(q) = 2(n-q)z_1$ and $\hat{z}_2(q) = 2(n-q)z_2$.

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