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# A Novel DOA Estimation Error Reduction Preprocessing Scheme of Correlated Waves for Khatri-Rao Product Extended-Array

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**SUMMARY** In this paper, we study on direction-of-arrival (DOA) estimation error reduction by Khatri-Rao (KR) product extended array in the presence of correlated waves. Recently, a simple array signal processing technique called KR product extended array has been proposed. By using the technique, degrees-of-freedom of an array can be easily increased. However, DOA estimation accuracy deteriorates when correlated or coherent waves arrive. Such highly correlated waves often arrive for radar application, hence error reduction technique has been desired. Therefore, in this paper, we propose a new method for error reduction preprocessing scheme by using $N$-th root of matrix. The $N$-th root of matrix has a similar effect to the spatial smoothing preprocessing for highly correlated signals. As a result, DOA estimation error due to signal correlation will be reduced. The optimal order of $N$ depends on the data itself. In this paper, a simple iterative method to obtain adaptive $N$ is also proposed. Computer simulation results are provided to show performance of the proposed method.

***key words:*** *DOA estimation, Khatri-Rao product, nested array, degrees of freedom*

## 1. Introduction

The direction-of-arrival (DOA) estimation is one of the important techniques in wireless communications, radar, radio surveillance, and other applications which utilize array antennas. In such applications, small number of array elements would be desirable as long as required specification is satisfied. Degrees-of-freedom (DOF) will be one of severe requirements in array because it is basically limited by the number of array elements. Recently, a new simple array signal processing technique called Khatri-Rao (KR) product array [1] and nested array [2] have been attracting attention. By using the technique, DOF of the array can be easily increased when the incident waves are uncorrelated. The technique is very attractive although uncorrelated wave incidence is a hard assumption. When incident waves are correlated, accuracy of DOA estimation deteriorates by the emerging correlation term(s) among the waves [3]. For applications such as radar, indoor/urban communication and sensing, highly correlated and/or coherent waves will arrive. When applying the technique to such applications, this problem becomes very important.

In this report, we propose a new simple preprocessing method to decrease DOA estimation error for the KR product extended array by using $N$-th root of the data correlation matrix. The preprocessing by taking the $N$-th root of data correlation matrix has a similar effect of decorrelation preprocessing scheme by the Spatial Smoothing Preprocessing (SSP) [4]. The optimal order of $N$ depends on the data itself. Therefore, a simple iterative method to obtain adaptive $N$ is also proposed. Computer simulation results are provided to show performance of the proposed method.

This paper is organized as follows. Section 2 shows the received data model of the antenna array in this study. Definitions and theoretical property of the Khatri-Rao product extended array and 2-level nested array are explained in Sect. 3 briefly. In Sect. 4, we'd like to describe the proposed preprocessing scheme. In Sect. 5, we will show DOA estimation error of Khatri-Rao product extended array in the presence of correlated waves by computer simulations. Section 6 shows performance of the proposed preprocessing scheme by computer simulation. Finally, we'll provide conclusions in Sect. 7.

## 2. Data Model

Here we consider that $K$ waves impinge on an array having $L$ elements. The received data vector can be written by

$$\begin{aligned} \boldsymbol{x}(t) &= \sum_{k=1}^{K} \boldsymbol{a}(\theta_k) s_k(t) + \boldsymbol{n}(t) \\ &= \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \end{aligned} \tag{1}$$

where $\boldsymbol{a}(\theta_k)$ and $s_k(t)$ are the mode vector and complex amplitude of the $k$-th incident wave, respectively, and $\boldsymbol{n}(t)$ is the additive Gaussian noise vector. In addition, $\boldsymbol{A}$ is the $L \times K$ mode matrix. The received data correlation matrix can be estimated by

$$\boldsymbol{R}_{xx} = \mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}^H(t)] \tag{2}$$

$$= \boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H + \boldsymbol{R}_N, \tag{3}$$

where $\mathrm{E}[\cdot]$ and the superscript $H$ denote the ensemble averaging, and complex conjugate transpose, respectively. $\boldsymbol{S}$ is source correlation matrix, and $\boldsymbol{R}_N$ is noise correlation matrix defined below,

$$\boldsymbol{S} = \mathrm{E}[\boldsymbol{s}(t)\boldsymbol{s}^H(t)], \tag{4}$$

$$\boldsymbol{R}_N = \mathrm{E}[\boldsymbol{n}(t)\boldsymbol{n}^H(t)]. \tag{5}$$

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## 3. Concept of Khatri-Rao Product Extended-Array

### 3.1 The Khatri-Rao Product Extended-Array

The KR product is defined by two matrices having the same number of columns. By using the KR product extension to the data correlation matrix shown in (2), the DOF of the array can be increased [1]. When we transform the data correlation matrix in (2) into a vector by stacking each column of the matrix, we obtain

$$\begin{aligned} z &= \mathrm{vec}[\boldsymbol{R}_{xx}] \\ &= \mathrm{vec}[\boldsymbol{ASA}^H] + \mathrm{vec}[\boldsymbol{R}_N] \\ &= (\boldsymbol{A}^* \square \boldsymbol{A})\bar{\boldsymbol{s}} + \mathrm{vec}[\boldsymbol{R}_N], \end{aligned} \tag{6}$$

where the superscript $*$ is the complex conjugate, $\square$ denotes the KR product operator, and vec[$\cdot$] is the operator to transform a matrix into a vector. Also, $\bar{\boldsymbol{s}}$ denotes the $K$-dimensional column vector consisting of the diagonal elements of $\boldsymbol{S}$.

The equation in (6) can be written as follows

$$z = \boldsymbol{A}'\bar{\boldsymbol{s}} + \boldsymbol{c}. \tag{7}$$

where $\boldsymbol{A}'$ is expansion mode matrix and $\boldsymbol{c}$ is transformation noise vector, and they are defined by

$$\boldsymbol{A}' = (\boldsymbol{A}^* \square \boldsymbol{A}), \tag{8}$$

$$\boldsymbol{c} = \mathrm{vec}[\boldsymbol{R}_N]. \tag{9}$$

This equation has the same form as that in (1). The DOF of this data depends on rank of the extended mode vector $\boldsymbol{A}'$. If columns of $\boldsymbol{A}'$ have full rank, DOF becomes $L^2 - 1$. However, when there are repeated elements in each column, the DOF will decrease. For a uniform linear array (ULA) with uncorrelated wave incidence, the received data correlation matrix $\boldsymbol{R}_{xx}$ becomes a Toeplitz matrix, then the number of independent elements becomes $2L - 1$. Therefore the DOF of the ULA becomes $2(L - 1)$. More detailed proof can be found in [1].

### 3.2 Two-Level Nested Array

In this section, we consider that KR expansion apply to non ULA to increase DOF. The 2-Level Nested Array (2L-NA) is proposed in [2]. It is defined by concatenation of two different uniform linear arrays having some optimal different element spacing. Figure 1 shows an example of the 4-element 2L-NA. The first level ULA has $L_1$ elements with element spacing of $\Delta d_1$, and the second level ULA has $L_2$ elements with spacing of $\Delta d_2$. Here we consider the array of 2L-NA whose total number of elements is $L(= L_1 + L_2)$. The optimal 2L-NA is the 2L-NA having the maximum DOF after the KR transform, or minimum repeated elements in the data correlation matrix. Element spacing $\Delta d_2$ is arranged so as to satisfy $\Delta d_2 = (L_1 + 1)\Delta d_1$. When we apply the KR product array processing to the 2L-NA, the DOF becomes $2L_2(L_1 + 1) - 2$ for plane wave incidence. See [2] for the detail derivation of the DOF. Optimal $L_1$ and $L_2$ are listed in Table 1 for the case where the total number of the element $L$ is constant ($L = L_1 + L_2$)

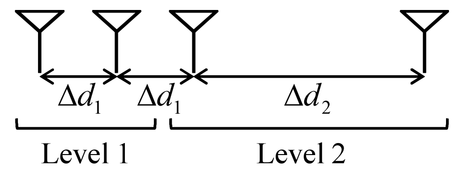


**Fig. 1** 4-element 2-Level Nested Array.

**Table 1** Optimal $L_1$ and $L_2$.

| $L$ | optimal $L_1, L_2$ | DOF |
|---|---|---|
| even | $L_1 = L_2 = \frac{1}{2}L$ | $\frac{L^2-2}{2} + L - 1$ |
| odd | $L_1 = \frac{L-1}{2}, L_2 = \frac{L+1}{2}$ | $\frac{L^2-1}{2} + L - 1$ |

## 4. Proposed Method

In this paper, we propose to apply $N$-th root of the received data correlation matrix as a preprocessor to decrease effect of signal correlation in the KR processing. Firstly, we will describe the effect of $N$-th root of the received data correlation matrix briefly. Applying eigendecomposition to the correlation matrix, $\boldsymbol{R}_{xx}$ can be decomposed by

$$\boldsymbol{R}_{xx} = \boldsymbol{ASA}^H + \boldsymbol{R}_N \tag{10}$$

$$= \sum_{i=1}^{L} \lambda_i e_i e_i^H = \boldsymbol{E\Lambda E}^H, \tag{11}$$

where $\lambda_i$ and $e_i$ denote the $i$-th eigenvalue and corresponding eigenvector, respectively. And, $\Lambda$ is the diagonal matrix consisting of eigenvalues $\lambda_i$, assuming that $\lambda_i \geq \lambda_{i+1}$, and $\boldsymbol{E}$ is the matrix whose column is the eigenvector corresponding to $\lambda_i$. The $N$-th root of the received data correlation matrix can be written as follows:

$$\boldsymbol{R}_{xx}^{\frac{1}{N}} = \boldsymbol{E\Lambda}^{\frac{1}{N}}\boldsymbol{E}^H = \boldsymbol{E}\begin{bmatrix} \lambda_1^{\frac{1}{N}} & & 0 \\ & \ddots & \\ 0 & & \lambda_L^{\frac{1}{N}} \end{bmatrix}\boldsymbol{E}^H. \tag{12}$$

As can be seen in this equation, ratios of the maximum eigenvalue to the others become smaller as the $N$ increases, although the eigenvectors are unchanged. This property is similar to the effect seen in decorrelation preprocessing scheme by the SSP [4]. Namely, we can say that almost the same effect can be obtained by the $N$-th root preprocessing scheme. Note that the correlation component will be suppressed effectively expect for the coherent waves, although the original power of the signal will be changed by the preprocessing. As a result, DOA estimation error due to signal correlation will be reduced.

## 5. Khatri-Rao Product Extended-Array in Presence of Correlated Waves

In this section, we will consider the error caused by the KR product array in the case of correlated wave incidence.

### 5.1 Theoretical Examination in Coherent 2 Waves

For simplicity, the data model of 2 coherent waves impinge on the 2L-NA having $L$-elementis considered as an example. Also, we neglect the noise term, and define the mode matrix $\boldsymbol{A}$ and source correlation matrix $\boldsymbol{S}$ by

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2)], \tag{13}$$

$$\boldsymbol{S} = \begin{bmatrix} \mathrm{E}[|s_1(t)|^2] & \mathrm{E}[s_1(t)s_2^*(t)] \\ \mathrm{E}[s_2(t)s_1^*(t)] & \mathrm{E}[|s_2(t)|^2] \end{bmatrix} \tag{14}$$

$$= \begin{bmatrix} P_1 & \rho_{12}\sqrt{P_1P_2} \\ \rho_{21}\sqrt{P_1P_2} & P_2 \end{bmatrix}, \tag{15}$$

where $\rho_{ij}$ is the correlation coefficient between the $i$-th and $j$-th signal, and $P_i$ is the power of the $i$-th signal. The data correlation matrix $\boldsymbol{R}_{xx}$ of the above data can be written by

$$\begin{aligned}\boldsymbol{R}_{xx} =& P_1\boldsymbol{a}(\theta_1)\boldsymbol{a}(\theta_1)^H + \rho_{21}\sqrt{P_1P_2}\boldsymbol{a}(\theta_2)\boldsymbol{a}(\theta_1)^H \\ &+ \rho_{12}\sqrt{P_1P_2}\boldsymbol{a}(\theta_1)\boldsymbol{a}(\theta_2)^H + P_2\boldsymbol{a}(\theta_2)\boldsymbol{a}(\theta_2)^H. \end{aligned} \tag{16}$$

First and fourth terms in the equation are autocorrelation terms of the incident wave itself. The other terms are the (mutual) correlation term between the signals, where correlation coefficient are written by $\rho_{ij}$. If the incident waves are uncorrelated, $\rho_{ij}$ becomes zero and corresponding signal correlation terms disappear. However, these terms remain when the incident waves are correlated or coherent.

Here we will apply the KR product extended processing to $\boldsymbol{R}_{xx}$.

$$\begin{aligned} z =& \mathrm{vec}[\boldsymbol{R}_{xx}] \\ =& \mathrm{vec}[\boldsymbol{a}(\theta_1)\boldsymbol{a}(\theta_1)^H]P_1 + \mathrm{vec}[\boldsymbol{a}(\theta_2)\boldsymbol{a}(\theta_2)^H]P_2 \\ &+ \mathrm{vec}[\boldsymbol{a}(\theta_1)\boldsymbol{a}(\theta_2)^H]\rho_{12}\sqrt{P_1P_2} \\ &+ \mathrm{vec}(\boldsymbol{a}(\theta_2)\boldsymbol{a}(\theta_1)^H)\rho_{21}\sqrt{P_1P_2}. \end{aligned} \tag{17}$$

As can be seen in (17), the signal correlation terms remain like signals by the KR product extended data. Performance of DOA estimation deteriorates by the these correlation terms.

### 5.2 DOA Estimation by Using the Khatri-Rao Product Extended Array

In this subsection, we will show errors caused by signal correlation in the KR product extended array by computer simulations. The algorithm employed here is the MUSIC algorithm. Table 2 shows the DOA estimation technique by using KR product extended array proposed in [2].

Here we'd like to explain it briefly. (7) has the same form as that in (1). Therefore, superresolution technique is applicable for DOA estimation to the data. Note that elements of $\bar{\boldsymbol{s}}$ in the KR product extended data correspond to the signal power of each incident wave, hence $\bar{\boldsymbol{s}}$ has stationary signals or coherent signals when power of incident waves is time invariant.

**Table 2** DOA estimation technique with the KR processing in [2].

| | |
|---|---|
| **Step 1.** | Calculate The received data correlation matrix $\boldsymbol{R}_{xx} = \mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}^H(t)]$ |
| **Step 2.** | Apply KR product extended array processing to obtain $\bar{z}$ from $\boldsymbol{R}_{xx}$. |
| **Step 3.** | Calculate KR extended correlation matrix $\bar{\boldsymbol{R}} = \bar{z}\bar{z}^H$ |
| **Step 4.** | Apply SSP to obtain $\hat{\boldsymbol{R}}$. |
| **Step 5.** | Apply DOA estimation method (MUSIC, etc.) |

In this case, regardless of correlation of original incident waves, all incident waves behave like coherent waves by this preprocessing. Namely, rank of the received data correlation matrix becomes one, hence the superresolution technique such as the MUSIC cannot be applied directly. To solve this problem, the SSP is employed in [2]. In this paper, we employ the 2L-NA having $L$ elements where $L$ is even for simplicity. The number of elements for the level-1 and level-2 array is the same as $L/2$ as listed in Table 1. Number of the extended array elements by the KR-transform is $(L^2-2)/2+L$. Here we apply the SSP to destroy the signal coherence. Since all of the incident waves becomes coherent by the KR-transform, the maximum number of resolvable waves are half of the number of the total elements [4]. Therefore, the number of subarrays $M$ required in this processing is derived by $(L^2/4+L/2)$. The number of elements in each subarrays, $L_e$, becomes $L_e = ((L^2-2)/2+L) - (L^2/4+L/2) + 1 = L^2/4 + L/2$.

The original extended data vector array obtained by (7) becomes $L^2$ and its mode matrix of $(\boldsymbol{A}^*\Box\boldsymbol{A})$ is $L^2 \times K$, but we remove the repeated elements because they do not basically contribute DOF and resolution improvement. Therefore, DOF becomes $K \leq (L^2-2)/2 + L - 1$ even if incident waves are uncorrelated. We defined the $\bar{\boldsymbol{A}}$ by $(\boldsymbol{A}^*\Box\boldsymbol{A})$ with removing the repeated elements. The $i$-th sensor location $d_i$ of $\bar{\boldsymbol{A}}$ is

$$d_i = (-L^2/4 - L/2 + i)\Delta d,\ i = 1, \cdots, (L^2-1)/2 + L, \tag{18}$$

where $\Delta d$ is element spacing of the ULA. The corresponding elements removed in $(\boldsymbol{A}^*\Box\boldsymbol{A})$ are also removed in the noise vector $\boldsymbol{c}$, thereby it has $\sigma^2$ that is the noise power at only the $(L^2/4+L/2)$-th element. Therefore, corresponding new extended data vector data is given by

$$\bar{z} = \bar{\boldsymbol{A}}\bar{\boldsymbol{s}} + \sigma^2\boldsymbol{e}, \tag{19}$$

where $\boldsymbol{e}$ is the vector whose $(L^2/4+L/2)$-th element is 1 and the others are 0. The $i$-th subarray which includes elements from the $((L^2/4+L/2)-i+1)$-th to $((L^2-2)/2+L-i+1)$-th data can be denoted by

$$\bar{z}_i = \bar{\boldsymbol{A}}_i\Phi^{(i-1)}\bar{\boldsymbol{s}} + \sigma^2\boldsymbol{e}_i, \tag{20}$$

$$\Phi = \mathrm{diag}(e^{-j\frac{2\pi}{\lambda}\Delta d \sin\theta_1}, e^{-j\frac{2\pi}{\lambda}\Delta d \sin\theta_2}, \cdots, e^{-j\frac{2\pi}{\lambda}\Delta d \sin\theta_L}), \tag{21}$$

where $\bar{\boldsymbol{A}}_i$ is a matrix consisting of $((L^2/4+L/2)-i+1)$-th to $((L^2-2)/2+L-i+1)$-th rows of $\bar{\boldsymbol{A}}$, diag($\cdot$) denotes a diagonal matrix, and $\boldsymbol{e_i}$ is a vector which has 1 at the $i$-th position, others are 0. Taking the average of all overlapping subarrays we obtain the following spatial smoothed correlation matrix [2],

$$\bar{\boldsymbol{R}} = \frac{1}{(\frac{L^2}{4}+\frac{L}{2})}\sum_{i=1}^{L^2/4+L/2}\bar{\boldsymbol{R}}_i, \quad \bar{\boldsymbol{R}}_i = \bar{z}_i\bar{z}_i^H. \tag{22}$$

The matrix $\bar{\boldsymbol{R}}$ can be expressed as $\bar{\boldsymbol{R}} = \hat{\boldsymbol{R}}^2$ where

$$\hat{\boldsymbol{R}} = \frac{1}{\sqrt{\frac{L^2}{4}+\frac{L}{2}}}(\bar{\boldsymbol{A}}_1\boldsymbol{\Gamma}\bar{\boldsymbol{A}}_1^H + \sigma^2\boldsymbol{I}). \tag{23}$$

where $\Gamma$ is a diagonal matrix whose element corresponds to the power of each signal. It has the same form as the received data correlation matrix of ULA without $\frac{1}{\sqrt{\frac{L^2}{4}+\frac{L}{2}}}$. Therefore, we can apply the superresolution techniques for DOA estimation to the $\bar{\boldsymbol{R}}^{\frac{1}{2}}$. The DOF in this case becomes $L^2/4 + L/2 - 1$ due to the SSP.

For DOA estimation error discussed in the following section, we evaluate it by using Root Mean Square Error (RMSE) defined by

$$RMSE = \sqrt{\frac{1}{N_t}\sum_{l=1}^{N_t}\frac{1}{K}\sum_{k=1}^{K}(\hat{\theta}_k^{(l)} - \theta_k^{(l)})^2}. \tag{24}$$

where $N_t$ is number of trial, $\theta_k^{(l)}$ is true DOA value of the $l$-th trials, and $\hat{\theta}_k^{(l)}$ is estimated DOA value of the $l$-th trials. In the simulation of this paper, estimated DOA value is assumed to correspond to the nearest DOA of the true value.

### 5.3 Examination of Correlation Effect on Simulation

Computer simulation results of the 6-element 2L-NA with KR product extended array preprocessing have demonstrated to show effectiveness of the proposed method. Number of elements in the extended data array becomes 23 which is divided by 11 overlapped subarrays having 12 elements to apply the SSP. We employed MUSIC algorithm [5] for the DOA estimation. We assumed 11 incident waves whose DOAs are −67, −45, −36, −25, −15, 0, 8, 20, 40, 53, 75 degree, respectively. All incoming waves were assumed to have the same signal power, and each SNR was set as 20 [dB]. 1000 snapshots were taken, and averaged RMSE of the DOAs were evaluated by 300 trials. Note that in this simulation, we cannot estimate all the waves without using KR product extended array because number of the actual array element is 6.

Figure 2 shows the averaged RMSE of the incident waves versus their correlation coefficients. As can be seen

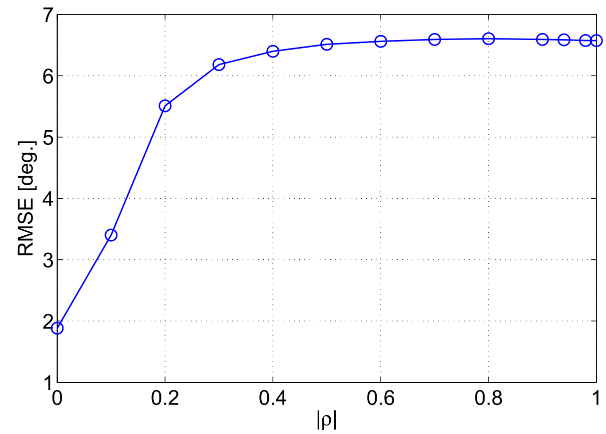


**Fig. 2** RMSE vs. correlation coefficient by conventional method (SNR = 20 [dB], Number of snapshots = 1000).

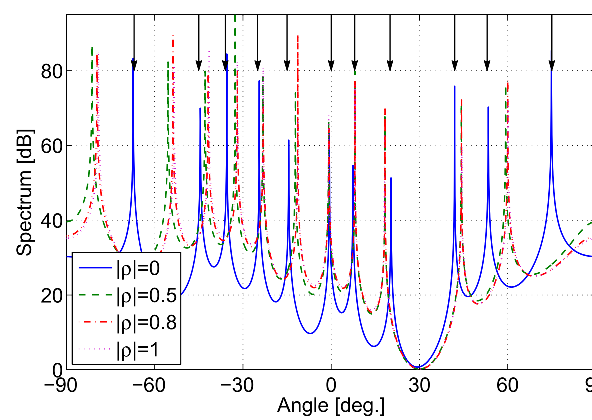


**Fig. 3** MUSIC spectrum of each correlation by conventional method (SNR = 20 [dB], Number of snapshots = 1000).

in this figure, the RMSE becomes larger as the correlation coefficient increases. Figure 3 shows examples of the estimated MUSIC spectrums for several correlation coefficients. Sharp peaks appear in all cases, but bias of the estimated DOA becomes larger for the waves having higher correlation coefficient. These results show that accuracy of DOA estimation deteriorates in the presence of correlated waves using by the KR product extended array. This error caused by the correlation discussed in Sect. 5.1. As can be seen in these results, error due to the signal correlation cannot be ignored when we employ the KR product extended array. Hence some improvements have been required.

## 6. DOA Estimation using the KR extended array with Proposed Preprocessing

In this section, we explain the DOA estimation by using the KR extended array with the proposed preprocessing.

### 6.1 Optimal N-th Root Estimation

There will be some optimal order of $N$ in the proposed preprocessing scheme. Too large $N$ may not work to decrease

**Table 3** DOA estimation technique with the KR processing with proposed preprocessing.

| | |
|---|---|
| **Step 1.** | Calculate the received data correlation matrix $\boldsymbol{R}_{xx} = \mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}^H(t)]$ |
| **Step 2.** | Apply adaptive $N$-th root preprocessing (Table 4). |
| **Step 3.** | Apply KR product extended array processing to obtain $\bar{z}$ from $\boldsymbol{R}_{xx}^{\frac{1}{N}}$. |
| **Step 4.** | Calculate KR extended correlation matrix $\bar{\boldsymbol{R}} = \bar{z}\bar{z}^H$ |
| **Step 5.** | Apply SSP to obtain $\hat{\boldsymbol{R}}$. |
| **Step 6.** | Apply DOA estimation method (MUSIC, etc.) |

**Table 4** Adaptive $N$-th root preprocessing.

| | |
|---|---|
| **Step 1.** | Initialize the iteration counter as $l = 0$, and also the order of N as $N(l) = 1$. |
| **Step 2.** | Calculate $\boldsymbol{R}_{xx}^{\frac{1}{N(l)}}$ and apply a conventional method (Table 2) to estimate $\hat{\boldsymbol{A}}(l)$. |
| **Step 3.** | Calculate the mean value, $\bar{\rho}(l)$, of the estimated correlation coefficients $\rho_{ij}(l)$ in $\boldsymbol{R}_{xx}^{\frac{1}{N(l)}}$ by using the estimated $\hat{\boldsymbol{A}}(l)$. |
| **Step 4.** | If $l = 0$ or $\bar{\rho}(l-1) > \bar{\rho}(l)$, set $\bar{\rho}(l-1) \leftarrow \bar{\rho}(l)$, and increment $N(l)$ by $\Delta N$, then go to Step 2. Otherwise go to Step 5. |
| **Step 5.** | The estimated optimal/sub-optimal value is given by $N(l)$. |

the DOA estimation error because effective SNR of each wave may often becomes low in such a larger $N$. Optimal or sub-optimal $N$ should be estimated in the DOA estimation. Therefore, in this paper, we adopt an iterative algorithm to obtain the optimal/sub-optimal order of $N$ in the DOA estimation. Intuitively, one may think that the correlation coefficient derived by the estimated DOAs will be available. However, this is not possible because of $K > L$. So, we employ the estimated signal correlation matrix defined by

$$\boldsymbol{S} = (\hat{\boldsymbol{A}}^H\hat{\boldsymbol{A}})^{+}\hat{\boldsymbol{A}}^H\boldsymbol{R}_{xx}^{\frac{1}{N}}\hat{\boldsymbol{A}}(\hat{\boldsymbol{A}}^H\hat{\boldsymbol{A}})^{+}, \tag{25}$$

where $\hat{\boldsymbol{A}}$ is the estimated mode matrix, and the superscript $+$ is the Moore-Penrose pseudo inverse matrix. Magnitude of effective correlation can be derived by

$$|\rho_{ij}| = \frac{|s_{ij}|}{\sqrt{s_{ii}}\sqrt{s_{jj}}}, \tag{26}$$

where $s_{ij}$ is the $(i, j)$-th element in $\boldsymbol{S}$. Number of magnitude of effective correlation is number of combination incident waves. The average value of all correlation coefficient as

$$\bar{\rho} = \frac{1}{K(K-1)/2}\sum_{i=1}^{K-1}\sum_{j=i+1}^{K}|\rho_{ij}|. \tag{27}$$

If $\bar{\rho}$ decreases for the $N$-th root preprocessing, then we renew the order. The proposed DOA estimation procedure with adaptive $N$-th root preprocessing is listed in Tables 3 and 4.

## 7. Computer Simulation Results

In this section, computer simulation results are provided to show performance of the proposed method.

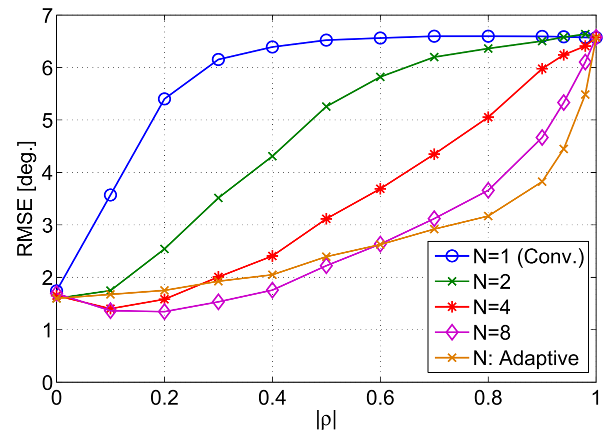


**Fig. 4** RMSE vs. correlation coefficient (Simulation 1, SNR = 20 [dB], Number of snapshots = 1000, $\Delta N = 0.1$).

### 7.1 Simulation 1: RMSE versus Correlation Coefficient

The computer simulation results of the 6-element 2L-NA with the KR product extended array are shown to demonstrate effectiveness of the proposed method. As shown in the previous section, number of elements in the extended data array becomes 23 which is divided by the overlapped 11 subarrays having 12 elements to apply the SSP. We also employed MUSIC algorithm [5] again for the DOA estimation. In this simulation, we assumed 11 incident waves whose DOAs are −67, −45, −36, −25, −15, 0, 8, 20, 40, 53, 75 degree, respectively. All incoming waves were assumed to have the same signal power, and each SNR was set to 20 [dB]. 1000 snapshots were taken in every trial, and averaged RMSE of the DOAs were evaluated by 300 trials.

Figure 4 shows the averaged RMSE of the incident waves versus their correlation coefficients. As can be seen in this figure, DOA estimation error decreases by the proposed preprocessing scheme effectively even for the highly correlated waves except for the coherent waves ($\rho = 1$). The reason why the proposed scheme cannot work properly for the coherent waves is that only one signal eigenvalue appears even for multiple incidence. RMSEs of the proposed method decrease in correlation coefficient between 0 and 0.1. When correlation coefficient is small, proposed method increases signal correlation conversely. See Appendix for the details of this correlation increase effect. In addition, the result of the proposed method is shown as “Adaptive” in the figure. Figures 5 and 6 show selected $N$ and estimate $\bar{\rho}$ respectively. From Fig. 5, we can see that smaller $N$ is selected for the lower correlated waves and higher $N$ for highly correlated waves as we expected. In the proposed technique, we adopt (25) for estimation of the approximate signal correlation matrix and signal correlation coefficients. Figure 6 shows the initial value ($N = 1$) of the averaged correlation coefficient estimated by (27), and converged correlation value (Adaptive $N$) by the iterative optimization by the proposed method. Theoretically, equation (25) is not ex-

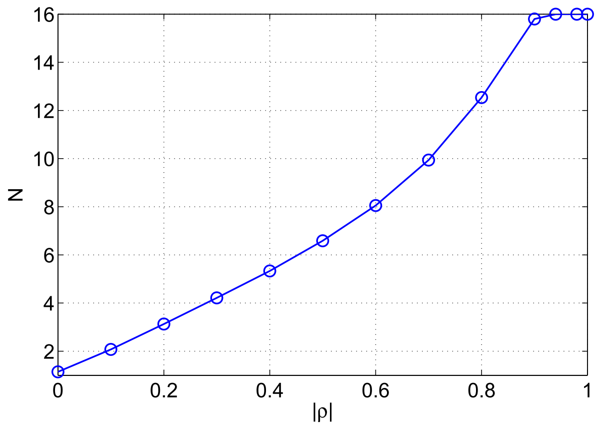


**Fig. 5** Adaptive N-th root vs. correlation coefficient (Simulation 1, SNR = 20 [dB], Number of snapshots = 1000, $\Delta N$ = 0.1).

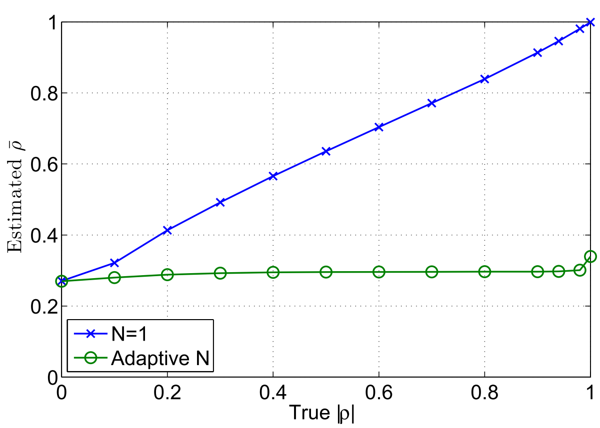


**Fig. 6** Estimated correlation coefficient $\bar{\rho}$ vs. correlation coefficient (Simulation 1, SNR = 20 [dB], Number of snapshots = 1000, $\Delta N$ = 0.1).

act for $K > L$, however, approximate value can be estimated for $N = 1$ in this example and effective averaged correlation coefficient in each $|\rho|$ becomes low by the iterative optimization. Even when $|\rho| = 0$, estimated and optimized $\bar{\rho}$ is not zero. This is due to SNR and finite snapshots. Note that the minimum $\bar{\rho}$ can be attained in almost all cases.

Figure 7 shows the estimated MUSIC spectrum with and without the proposed method. We can see clearly that the DOA estimation error is effectively reduced by the method.

### 7.2 Simulation 2: The Other Example

h In this simulation, DOAs are −57, −49, −34, −21, −14, −2, 10, 23, 36, 50, 60 degree, respectively. The other parameters are the same as used in Simulation 1. Figures 8, 9, and 10 show the averaged RMSE of the incident waves versus their correlation coefficients, the selected $N$, and the estimated MUSIC spectrum with and without the proposed method, respectively. We can see that the DOA estimation error is effectively reduced by the method also in this case. However, note that RMSE increases for $N$=2, 4, and

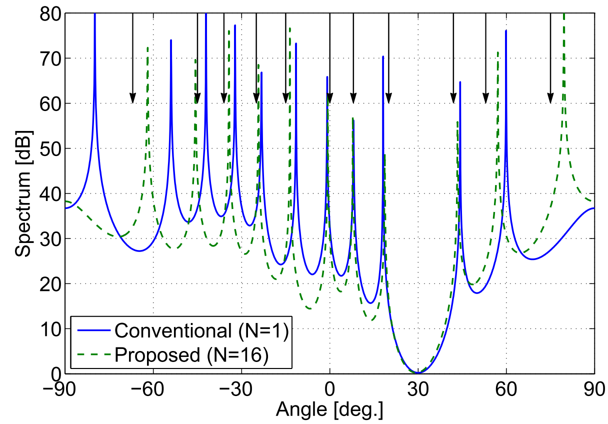


**Fig. 7** MUSIC spectrum (Simulation 1, $|\rho|$ = 0.9, SNR = 20 [dB], Number of snapshots = 1000).

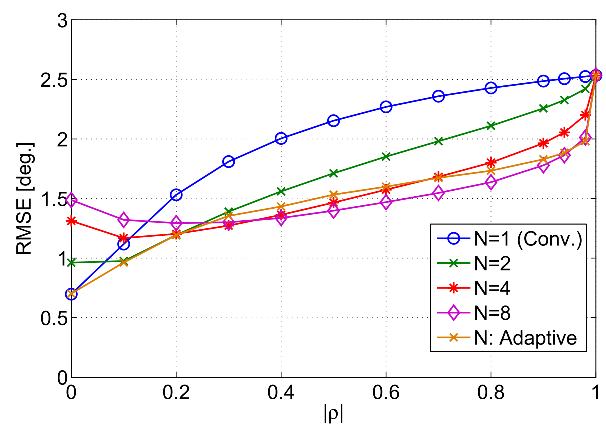


**Fig. 8** RMSE vs. correlation coefficient (Simulation 2, SNR = 20 [dB], Number of snapshots = 1000, $\Delta N$ = 0.1).

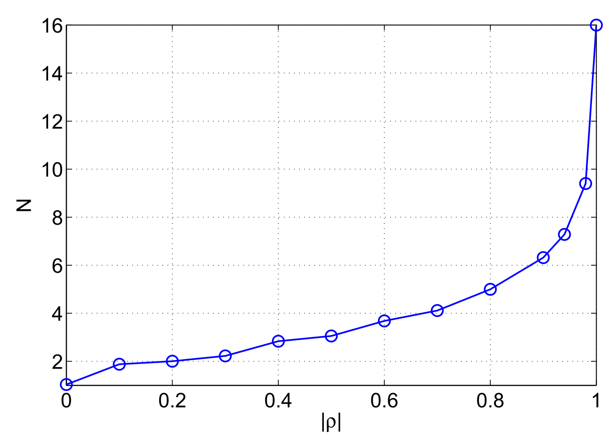


**Fig. 9** Adaptive N-th root vs. correlation coefficient (Simulation 2, SNR = 20 [dB], Number of snapshots = 1000, $\Delta N$ = 0.1).

8 at $\rho = 0$. In this case, error due to signal correlation does not appear essentially because there are no correlation terms. However correlation increased effect appear when signal correlation is low. As a result, DOA estimation error increases in this example. As can be seen in these results,

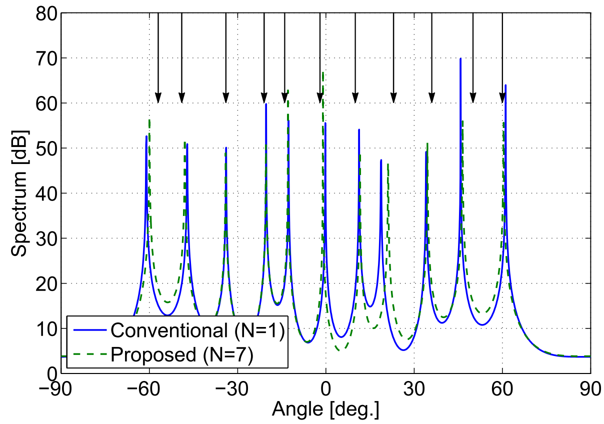


**Fig. 10** MUSIC spectrum (Simulation 2, $|\rho| = 0.9$, SNR = 20 [dB], Number of snapshots = 1000).

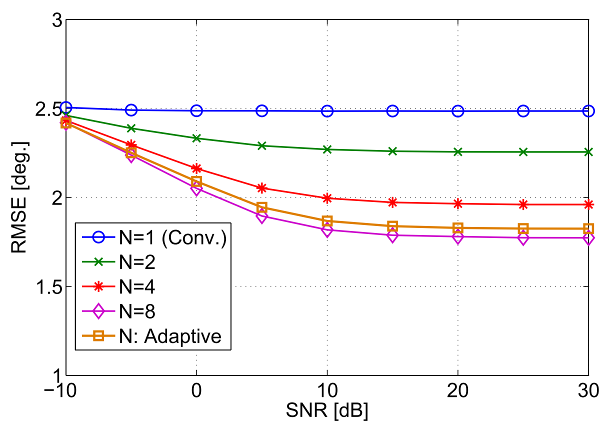


**Fig. 11** RMSE vs. SNR (Simulation 3, $|\rho| = 0.9$, Number of snapshots = 1000).

effectiveness of the proposed preprocessing depends on the signal correlation and SNR in practice. In any case, "Adaptive" result will be obtained by selecting optimal order of $N$ to prevent error increase. However, the "N=8" result is better than the "Adaptive" result. This may be mainly because bias the estimated DOAs and effective correlation coefficient due to noise and correlation terms.

### 7.3 Simulation 3: Effect of SNR and Number of Snapshots on Proposed Method

The simulation parameters in this section are basically the same as those of Simulation 2 except for SNRs. Correlation coefficient selected as $|\rho| = 0.9$ because the proposed scheme in effective especially for high SNR signals.

Figure 11 shows the averaged RMSE of the incident waves versus SNR. In the conventional method, any changes in RMSE cannot be seen almost all over the SNRs. This can be caused by the correlated term whose effect is larger than the SNR. On the other hand, when we apply the proposed preprocessing scheme, we can see error reduction at high SNRs.

Figure 12 shows the averaged RMSE of the incident

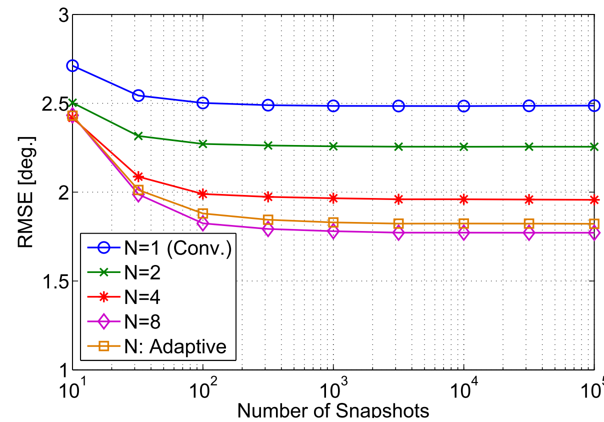


**Fig. 12** RMSE vs. Number of snapshots (Simulation 3, $|\rho| = 0.9$, SNR = 20 [dB]).

waves versus number of snapshots. Almost the same tendency shown in Fig. 11 can be seen in this figure. The effect of the proposed method appears when number of snapshots is greater than 100 in this example. From these results, we can say that the proposed method work properly when the number of snapshots and/or SNR exceeds a certain level which depends on the data itself.

## 8. Conclusion

In this paper, we have proposed a new preprocessing scheme to suppress correlation terms in the data correlation matrix for the DOA estimation by using the KR extended data array. By using the KR product extended array, DOF of the array can be easily increased when the incident waves are uncorrelated. However, when the waves are correlated, accuracy of DOA estimation deteriorates. Therefore, we have proposed the new preprocessing scheme based on the $N$-th root of data correlation matrix. The $N$-th root of matrix is similar to the effect seen in decorrelation preprocessing scheme by the SSP. The optimal order $N$ depends on the data. So, we also provide an iterative method to determine an adaptive $N$ by using estimated correlation coefficients. Simulation results show that the proposed algorithm work properly as an optimal/sub-optimal order selector in DOA estimation by the MUSIC algorithm.

The proposed method can only decrease the effect of signal correlation in the KR extended array processing, and cannot remove it perfectly. Hence the performance is limited. However algorithm is very simple that is a main feature of the scheme. In this paper, we focus on the RMSE to evaluate the scheme. However when we focus on improvement of DOA estimation accuracy of each signal, we can find that dispersion of improvement in the signals is slightly large. We only obtain small improvement in some signals. Further improvements will be needed to acquire stable performance. These problems will be studied in near future.

## Acknowledgement

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## Appendix: Signal Correlation of N-th Root of Correlation Matrix

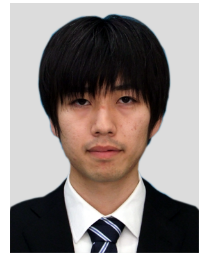
We consider signal correlation $\boldsymbol{S}$ in the N-th root of correlation matrix $\boldsymbol{R}_{xx}$. In this appendix, we ignore noise for simplicity. N-th root of correlation matrix can be written as

$$\begin{aligned}\boldsymbol{R}_{xx}^{\frac{1}{N}} &= (\boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H)^{\frac{1}{N}} \\ &= \boldsymbol{A}\bar{\boldsymbol{S}}\boldsymbol{A}^H, \qquad (\mathrm{A}\cdot 1)\end{aligned}$$

where $\bar{\boldsymbol{S}}$ is the signal correlation of N-th root of correlation matrix.

$$\begin{aligned}\bar{\boldsymbol{S}} &= (\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H\boldsymbol{R}_{xx}^{\frac{1}{N}}\boldsymbol{A}(\boldsymbol{A}^H\boldsymbol{A})^{-1} \\ &= \boldsymbol{S}\boldsymbol{A}^H(\boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H)^{\frac{1}{N}-2}\boldsymbol{A}\boldsymbol{S}. \qquad (\mathrm{A}\cdot 2)\end{aligned}$$

Hence, we may say that the signal correlation terms arise by this scheme. For simplicity, we consider uncorrelated waves having the same power, then $\boldsymbol{S} = \boldsymbol{I}$. In this case, (A· 2) becomes $\boldsymbol{A}^H(\boldsymbol{A}\boldsymbol{A}^H)^{\frac{1}{N}-2}\boldsymbol{A}$. Clearly, this matrix is not a diagonal matrix usually. As can be seen in this example, the N-th root of correlation matrix may arise weak signal correlation term in the low signal correlation environment. This is the reason why the performance of the proposed scheme sometimes deteriorates.



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