PAPER Special Section on Recent Progress in Antennas and Propagation in Conjunction with Main Topics of ISAP2012

Angular Resolution Improvement of Ocean Surface Current Radar Based on the Khatri-Rao Product Array Processing

Hiroyoshi YAMADA^{†a)}, Senior Member, Naoki OZAWA[†], Student Member, Yoshio YAMAGUCHI[†], Fellow, Keizo HIRANO^{††}, and Hiroyuki ITO^{††}, Members

SUMMARY Ocean surface current radar is a Doppler radar to observe oceanographic information using the Bragg scattering resonance mechanism. In this paper, we consider angular resolution improvement of the radar. The radar employs an antenna array with FMICW operation, then it can resolve angular distribution by Digital Beam Forming (DBF) and distance by Fourier transform of the beat signal obtained by the FMICW radar. In order to obtain sufficient angular resolution, large array length or aperture with increasing the number of elements is needed, that is often difficult to realize in the HF/VHF ocean surface current radar. In this paper we propose to apply the Khatri-Rao (KR) product array processing to the radar. To verify effectiveness of the KR product array processing in angular resolution enhancement for the ocean surface current radar, we apply the KR product array to actual experimental data set of the radar, and show that the method is available to angular resolution enhancement and Doppler spectrum improvement.

key words: ocean surface current radar, DOA estimation, Khatri-Rao product, angular resolution improvement

1. Introduction

Ocean surface current radar (OSCR) is a HF or VHF band Doppler radar which utilizes Bragg scattering to detect Doppler frequency of reflected signals and estimate ocean current velocity at Line-of-Sight [1]. If the observation can done with two different sights, direction of the current at the area of interest as well as velocity can be estimated. There are several types of the radar [2], [3], the ocean surface radar of the frequency modulated continuous-wave (FMCW) type is the one of the major radar [3]. This radar [3] employs an electrically steered phased array antenna while the CODAR in [3] uses a pulse radar technique and four element crossedloop antenna system. The phased array can enhance angular resolution easily by enlarge array length to narrow its mainbeamwidth. This is one of advantages in the FMCW-OSCR. However, large radar site will be required to enlarge array length since the OSCR is the HF/VHF radar.

In this paper, we propose to apply a novel array signal processing technique by using concept of Khatri-Rao product of matrices [4], [5]. This algorithm utilizes higher-order statistics by the Khatri-Rao product. There are several algorithms based on the signal property in the higher-order

[†]The authors are with the Graduate School of Science and Technology, Niigata University, Niigata-shi, 950-2181 Japan.

^{††}The authors are with the Nagano Japan Radio Co. Ltd., Nagano-shi, 318-2288 Japan.

a) E-mail: yamada@ie.niigata-u.ac.jp

DOI: 10.1587/transcom.E96.B.2469

statistics [6], [7], and many of them focus on enhancement of the degree-of-freedom (DOF) of the array. The method employed in this paper can be enlarge effective aperture as well as degree-of-freedom of the array, without additional elements, by using re-defined data vector estimated by the data covariance matrix when all incoming reflected signal, or radar echoes are incoherent. This is true for the OSCR. The preliminary results has reported in [8]. In this paper, we will provide experimental results in detail to verify improvement of the angular resolution and its effect in Doppler estimation. The proposed method is based on the Beamforming/Fourier transform. One may think why the authors did not apply the MUSIC algorithm [4]. There are two reasons. The first reason is that the radar echo by the ocean surface current waves will often continuously distribute in angle, and may exceed the DOF of the array even when they looks as discrete (point) targets. The second reason is that power distribution of the Doppler spectrum is required in the OSCR to estimate ocean surface status.

This paper is organized as follows. In Sect. 2, we provide problem formulation. The proposed array signal processing based on the concept of the Khatri-Rao product of matrices are provided in Sect. 3, where mathematical fundamental of the Khatri-Rao product are also explained briefly. In Sect. 4, experimental results of the actual OSCR data are presented to show availability of the propose technique. Finally, the conclusions are reported in Sect. 5.

2. Problem Formulation

The data obtained by the FMCW-OSCR is threedimensional data in (element) space, time, and frequency. When we transform the observed beat-signal of the FMCW-OSCR in each element, we can resolve radar echo in each range bin. Furthermore, by using time series of the radar echoes at a specific range bin, we can estimate Doppler frequency of the wave in the bin. Finally, when we apply the beamforming method to extract the wave in the range-bin at the specific direction. This is the brief concept of the FMCW-OSCR. Assuming, for simplicity, that we have array data in a specific range bin and Doppler frequency, and we employ a *L*-element uniform linear array (ULA) whose element spacing is Δd as shown in Fig. 1. The receiving data vector of the array can be given by

$$\mathbf{x}(t) = [x_1(t), x_2(t), \cdots, x_L(t)]^T$$

.

Manuscript received January 21, 2013.

Manuscript revised May 20, 2013.



Fig. 1 The *L*-element uniform linear array and an incident wave.

$$= \sum_{k=1}^{K} \boldsymbol{a}(\theta_k) \boldsymbol{s}_k(t) + \boldsymbol{n}(t),$$

= $\boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t),$ (1a)

$$\boldsymbol{a}(\theta_k) = [1, e^{j\frac{2\pi}{\lambda}\Delta d\sin\theta}, \cdots, e^{j\frac{2\pi}{\lambda}(L-1)\Delta d\sin\theta}]^T,$$
(1b)

$$\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_K)],$$
(1c)

$$\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^T,$$
 (1d)

$$\boldsymbol{n}(t) = [n_1(t), n_2(t), \cdots, n_L(t)]^T,$$
(1e)

where and $a(\theta_k)$ and $s_k(t)$ are the mode vector and complex amplitude of the *k*-th wave component, respectively, and n(t) is the additive noise vector. *K* is the number of wave components and ^{*T*} denotes transpose. We assume in this signal model that the incoming wave components are discrete in angle for simplicity. Note that wave component(s) may often distributed in angle in actual data. To extract angular response we often apply beamformer to the data in (1a).

3. Proposed Preprocessing by Using the Concept of Khatri-Rao Product

3.1 Mathematical Basics of Khatri-Rao product

The Khatri-Rao (KR) product is a matrix multiplication defined by the two matrices between an $N \times M$ matrix A and a $P \times M$ matrix B as

$$\boldsymbol{A}\Box\boldsymbol{B} = [\boldsymbol{a}_1 \otimes \boldsymbol{b}_1, \boldsymbol{a}_2 \otimes \boldsymbol{b}_2, \cdots, \boldsymbol{a}_M \otimes \boldsymbol{b}_M], \qquad (2)$$

where a_i and b_i denote the *i*-th column of A and B, respectively. Also, \Box and \otimes denote operators of the Khatri-Rao and Kronecker product, respectively. Then the derived matrix by the KR product becomes $NP \times M$ matrix.

The following property of the KR product will be important of the OSCR beamformer. For an $N \times K$ matrix C, a $M \times K$ matrix E and a $K \times K$ diagonal matrix D, we have

...

$$vec(CDE^{H}) = (E^{*}\Box C)d, \qquad (3)$$

where H and * denote complex conjugate transpose and complex conjugate, respectively, and *d* is the *k*-dimensional vector whose elements are the diagonal elements in *D*. The operator $vec[\cdot]$ denotes the vectorization operator stacking the columns of the argument matrix as

$$vec[\mathbf{A}] = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_K \end{bmatrix}.$$
 (4)

In the proposed scheme, above mathematical properties are used in the derivation.

3.2 Proposed Preprocessing Scheme

The angular resolution of the beamformer is limited by the array length. To enhance angular resolution, large aperture will be required. We propose to apply the array signal processing technique in [4], [5] to the FMCW-OSCR. First, we estimate data covariance matrix of (1a) by

$$\boldsymbol{R}_{xx} = E[\boldsymbol{x}(t)\boldsymbol{x}^{H}(t)] = \boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^{H} + \sigma^{2}\boldsymbol{I},$$
(5)

where $E[\cdot]$ denotes ensemble averaging, and σ^2 and I denote noise power and an identity matrix, respectively. The matrix S is the source correlation matrix defined by $E[s(t)s^H(t)]$. In the OSCR, each incoming wave is widely deployed in angle, and also moves randomly, hence all of the incoming waves can be assumed uncorrelated, then the matrix S becomes a diagonal matrix.

Applying the vectorization operation to \boldsymbol{R}_{xx} , we obtain

$$\mathbf{x} = vec[\mathbf{R}_{xx}] = (\mathbf{A}^* \Box \mathbf{A})\bar{\mathbf{s}} + vec[\sigma^2 \mathbf{I}], \tag{6}$$

where \bar{s} denotes the *K*-dimensional column vector whose elements are the diagonal elements in *S*. By using (6), the equation (6) becomes

$$z = \sum_{k=1}^{K} (\boldsymbol{a}^*(\theta_k) \otimes \boldsymbol{a}(\theta_k)) \bar{s}_k + vec[\sigma^2 \boldsymbol{I}].$$
(7)

The new vector $(\boldsymbol{a}^*(\theta_k) \otimes \boldsymbol{a}(\theta_k))$ is the new signal mode vector in the KR-product subspace. This vector has some repeated elements because ULA is assumed in this study. Essentially, these repeated elements are not effective for resolution and degree-of-freedom improvements. Omitting repeated elements and re-arrangement available elements, we can obtain an extended data vector defined by

$$\bar{\boldsymbol{z}} = [\bar{z}_1, \bar{z}_2, \cdots, \bar{z}_{2L-1}]^T
= \bar{\boldsymbol{A}}\bar{\boldsymbol{s}} + \bar{\boldsymbol{n}},$$
(8a)
$$\bar{\boldsymbol{A}} = [\bar{\boldsymbol{a}}(\theta_1), \bar{\boldsymbol{a}}(\theta_2), \cdots, \bar{\boldsymbol{a}}(\theta_K)],$$
(8b)

$$\bar{\boldsymbol{a}}(\theta_k) = [e^{-\frac{j^2\pi}{4}(L-1)\Delta d\sin\theta}, e^{-\frac{j^2\pi}{4}(L-2)\Delta d\sin\theta}, \cdots,$$

$$1, e^{j\frac{2\pi}{\lambda}\Delta d\sin\theta}, \cdots, e^{j\frac{2\pi}{\lambda}(L-1)\Delta d\sin\theta}]^T.$$
(8c)

In this paper, we assume that all of incident waves on the ULA are uncorrelated. Hence the received data correlation matrix in (5) becomes a Toeplitz matrix in which elements along a Northwest-to-Southeast diagonal are the same. In this case, the data vector \bar{z} can be easily arranged by picking up one element at the diagonals. Then the number of elements becomes 2L - 1 for the *L*-element ULA. The new mode vector in (8c) can also be easily derived by picking up

one of the diagonal elements in $vec[a(\theta_k)a^H(\theta_k)]$ which is the special case for one-signal incidence. Clearly, the array length/aperture can be almost doubled, specifically the *L*element ULA becomes equivalent 2L - 1-element ULA, by this preprocessing. Therefore, angular resolution improvements can be expected only by the signal processing. Note that when we remain the repeated elements they behave as weights for corresponding elements, therefore they does not have effect on enhancement for DOFs and DOA resolution.

The beamformer for this extended data vector can be defined by

$$P_{BF}(\theta) = \frac{\bar{a}(\theta)^H R_{\bar{z}\bar{z}}^{1/2} \bar{a}(\theta)}{\bar{a}(\theta)^H \bar{a}(\theta)},\tag{9}$$

where $R_{\bar{z}\bar{z}}$ is the correlation matrix of \bar{z} . The square root processing is required because amplitude of each incident wave is squared in the processing by (6). Note that the additive noise term $vec[\sigma^2 I]$ in (6) is no longer uniform in the extended (virtual) elements. Therefore, peaks of spectrum will be biased due to the noise term especially for low SNR echoes.

Obviously ULA is assumed in this study although the KR processing can enhance array aperture more effectively for non-uniform array such as nested arrays [5]. This is because ULAs have already deployed many along the seashores in Japan.

4. Experimental Results

Experimental data used in this analysis were obtained by the Frequency Modulated Interrupted Continuous wave (FMICW)-OSCR of 8-element ULA at Ariake Sea, Kyushu, Japan, in Oct. 22, 2006. Detailed specification of the data and the radar is listed in Table 1. The FMICW is a kind of FMCW radar which introduce interrupted sequence to deal with high power transmission in a monostatic radar configuration [9].

Figures 2(a) and (b) show an example of 2-dimensional spectrum for angular-Doppler frequency at the 15 km range bin without and with the KR product processing, respectively. This can be obtained by the conventional beamforming and FFT with 8-element data. Dominant echo can be seen at around Doppler frequency of -0.5 Hz. The angular

 Table 1
 Specification of the experimental data and FMICW radar.

Date	Oct. 22, 2006
Observation Site	Ariake Bay, Kyushu, Japan
Array Shape	Uniform Linear Array
Element	2-el. Yagi-Uda array
Number of elements (L)	8
Center frequency	24.515 [MHz] (λ ≈12.26 [m])
Element spacing	7 [m] (0.57λ [m])
Frequency bandwidth	100 [kHz]
Swept time	500 [ms]
Number of sampling points in a pulse	512
Number of chirp pulses	1024

spectrum at the frequency is shown in Fig. 3 where spectrum of the proposed technique by the Khatri-Rao product array is also plotted. Obviously, the main beam at -4° becomes narrower by the KR product array processing. Fig-



Fig. 2 Estimated angular-Doppler spectrum at 15 km range bin. L = 8. (a) Conventional beamformer. (b) Proposed beamformer with the KR preprocessing.



Fig. 3 Estimated angular spectrum for the -0.4961 Hz Doppler component at 15 km range bin. L = 8.



Fig.4 Estimated Doppler frequency spectrum for the waves coming from -4° at 15 km range bin. L = 8.

ure 4 show the Doppler spectrum of the original and KR processed array data at -4° . The dominant Doppler component can be resolved clearly and adjacent components are suppressed. These suppressed components will be the interference components close to the dominant component at -4° received due to the wide mainlobe of the original beamformer. As can be seen in these figures, improvement of the angular resolution can also yield improvement of the quality of Doppler spectrum.

Derivation of the extended data vector in (8a), or (7), we assume that the incoming waves by ocean surface currents are uncorrelated. This is the key assumption of this proposed preprocessing scheme. The number of incident waves K in the OSCR is usually greater than the number of elements L and/or the waves may often distributed in angle as a clutter, so direct verification of wave correlation is difficult. In this paper, validity of the proposed method is checked as follows. If the discussion in the previous section is valid in actual data set, the estimated spectrum by the conventional method for a physical 2L-1-element array will be almost coincide with that by the proposed method for a virtual 2L - 1 array with KR product array processing method of physical *L*-element array. Fortunately, we have 8 element data in this experimental data set, hence we can verify the results estimated by the virtual 7-element array and the physical 7-elemet array (L = 4). Virtual 7-elemet array can be derived by a 4-element subarray. As shown in Fig. 5, there are 4 ways of subarray selection in this data set. In addition, there are two physical 7-elemet subarrays. To decrease experimental error due to imperfection of calibration, averaged spectrum estimated by each subarray is used in the following analysis.

Figures 6(a), (b) and (c) show the estimated angular and Doppler spectrum for 4-element array without the KR processing, with the KR processing, and 7-element array without the KR-processing, respectively. We can say that the spectrum in Fig. 6(a) is the original spectrum by the physical 4-element array, and the spectrum in Fig. 6(b) is the preprocessed spectrum estimated by the virtual 7-



Fig.5 Selection of subarrays. (a) 4-element subarrays. (b) 7-element subarrays.

element array of the proposed scheme. Clearly, DOA resolution is also improved as discussed in Fig. 2. The 2D angular-Doppler spectrum estimated by the 7-elemet subarrays in Fig. 5(b) is also shown in Fig. 6(c) whose array length corresponds to 4-element array with the KR preprocessing scheme. As can be seen in Figs. 6(b) and (c), there is a good correspondence between them. When we apply the KR-processing to the 7-element array, angular resolution improvement can be improved. This effect is already discussed in Fig. 2 with 8-element case. We focus on the comparison of the angular and Doppler spectrums obtained by the 7-element array (without the KR processing) and the 4-element array with the KR-processing.

To see the details of the spectrums, let us focus on the Angular spectrums for the Doppler component of -0.4961 Hz as shown in Fig. 7. The solid blue line shows the spectrum estimated by using the proposed preprocessing scheme with the physical 4-element array. Clearly the angular spectrum width at around -4° becomes almost half in comparison with the original spectrum (without the KR processing) plotted by the dashed green line. In this figure, the spectrum estimated by the physical 7-element array is also plotted by the broken red line. The angular spectrums of the 4-element array with the KR processing and 7element array without the KR processing are not well agree in this case. Calibration error, limitation of the number of snapshots and SNR may cause the error. However similar narrow spectrum property can be seen in high level peak. To sharpen the main-beam helps us to reduce interference of other components out of the peak. This effect can also be seen in the estimated spectrums of this peak angle as shown in Fig. 8. The estimated peak components at around -0.5 Hz shown in this figure have almost the same shape, while the Doppler components in some other frequencies have different levels. This will be caused by the beamwidth difference. The estimated spectrum by the proposed KR preprocessing scheme agree again with that by the physical 7-element array.

In this paper, we have discussed angular resolution im-

YAMADA et al.: ANGULAR RESOLUTION IMPROVEMENT OF OCEAN SURFACE CURRENT RADAR BASED ON THE KHATRI-RAO PRODUCT ARRAY PROCESSING 2473



Fig. 6 Estimated angular-Doppler spectrum at 15 km range bin. (a) Conventional beamformer of physical 4-element subarray. (b) Proposed beamformer with the KR preprocessing of virtual 7-element subarray. (c) Conventional beamformer of physical 7-element subarray.

provement for the data at specific range bin (15 km), but almost the same effect can be seen for the data at the other range bins. As a result, we can say that the proposed KR product preprocessing scheme is available to enhance angular resolution and Doppler spectrum separation for the OSCR without any modifications of the radar system. Obvi-



Fig. 7 Estimated averaged angular spectrum for the -0.4961 Hz Doppler component at 15 km range bin.



Fig.8 Estimated averaged Doppler frequency spectrum for the waves coming from -4° at 15 km range bin.

ously, the KR product array processing uses data covariance matrix, then performance of the processing also depends on the estimation accuracy of matrix. Insufficient number of snapshots will deteriorate the performance. To realize quantitative evaluation of the proposed scheme, we need a statistical model of radar echo for ocean surface current. However, precise model including higher order statistical property has not been established yet. To derive effective and efficient signal model is necessary to evaluate the radar performance. This will be done in near future.

5. Conclusion

In this paper, we propose to apply the concept of the Khatri-Rao product array processing to the FMCW-OSCR. Performance of the proposed method has been demonstrated by the experimental results. The proposed method will be able to improve angular resolution performance of the existing FMCW-OSCR. Main objective of the OSCR is to detect velocity and direction of ocean surface current. To show the availability for the objective, OSCR datasets at two distinct test sites are required. Numerical and experimental study for such a scenario will be provided in near future.

References

- D.E. Barrick, "First-order theory and analysis of MF/HF/VHF scatter from the sea," IEEE Trans. Antennas and Propagat., vol.AP-20, no.1, pp.2–10, Jan. 1972.
- [2] A. Nadai, H. Kuroiwa, M. Mizutori, and S. Sakai, "Measurement of ocean surface currents by CRL HF ocean surface radar of the FMCW type. Part 1. Current velocity," J. Oceanography, vol.53, pp.325–342, 1997.
- [3] B.J. Lipa and D.E. Barrick, "Least-square methods for the extraction of surface currents from CODAR crossed -loop data: Application at ARSLOE," IEEE J. Oceanic Engineering, vol.OE-8, no.4, pp.226– 253, Oct. 1983.
- [4] W.K. Ma, T.H. Hsieh, and C.Y. Cui, "DOA estimation of quasistationary signals with less sensors than sources and unknown spatial covariance: A Khatri-Rao subspace approach," IEEE Trans., Signal Processing, vol.58, no.4, pp.2168–2180, April 2010.
- [5] P. Pal and P.P. Vaidyanathan, "Nested arrays: A novel approach to array processing with enhanced degree of freedom," IEEE Trans. Signal Processing, vol.58, no.8, pp.4167–4181, Aug. 2010.
- [6] B. Porat and B. Friedlander, "Direction finding algorithms based on high-order statistics," IEEE Trans. Signal Processing, vol.39, no.9, pp.2016–2023, Sept. 1991.
- [7] P. Chevalier, A. Ferreol and L. Albera, "High-resolution direction finding from higher order statistics: The 2q-MUSIC algorithm," IEEE Trans. Signal Processing, vol.54, no.8, pp.2986–2997, Aug. 2006.
- [8] N. Ozawa, H. Yamada, Y. Yamaguchi, K. Hirano, and H. Ito, "Angular resolution improvement of ocean surface current radar," Proc. International Symposium on Antennas and Propagation 2012 (ISAP2012), Nagoya, Japan, Oct. 2012.
- [9] R.H. Khan and D.K. Mitchell, "Waveform analysis for high-frequency FMICW radar," IEE Proceedings, Radar and Signal Processing, Pt. F, vol.138, no.5, pp.411–419, Oct. 1991.



Naoki Ozawa received a B.E. degree in information engineering from Niigata University, Niigata, Japan, in 2012. He is now a graduate student pursuing an M.E. degree in Electrical and Information Engineering at Niigata University, where he is engaging in Ocean surface current radar.



Yoshio Yamaguchi received a B.E. degree in electronics engineering from Niigata University in 1976 and M.E. and Dr.Eng. degrees from the Tokyo Institute of Technology in 1978 and 1983, respectively. In 1978, he joined the Faculty of Engineering, Niigata University, where he is a professor. During 1988 and 1989, he was a research associate at the University of Illinois, Chicago. His interests include the propagation characteristics of electromagnetic waves in a lossy medium, radar polarimetry, and mi-

crowave remote sensing and imaging. He has served as Chair of IEEE GRSS Japan Chapter (2002–2003), vice chair (2000–2001), Chair of URSI-F Japan since 2006. He received the Best Tutorial Paper Award from Comm. Soc. of IEICE in 2007. He is a fellow of the IEEE.



Keizo Hirano received the B.E, M.E. and Dr.Eng. degrees in electronic engineering from Shinshu University, Nagano, Japan, in 1986, 1988, and 1994, respectively. In 1988, he joined a Nagano Japan Radio Co., Ltd, where he is engaging in Ocean surface current radar.



Hiroyoshi Yamada received the B.E, M.E. and Dr.Eng. degrees in electronic engineering from Hokkaido University, Sapporo, Japan, in 1988, 1990 and 1993, respectively. In 1993, he joined the Faculty of Engineering, Niigata University, where he is a professor. From 2000 to 2001, he was a Visiting Scientist at the Jet Propulsion Laboratory, California Institute of Technology, Pasadena. His current interests include superresolution techniques, array signal processing, and microwave remote sensing and

imaging. He has served as Vice Chair of IEEE AP-S Japan Chapter (2011–2012), secretary (2009–2010) and a Chair since 2013. He also has served as an Editor of IEICE Transaction on Communications, and a Editor of IEICE Communications Express (ComEX) since 2011. Dr. Yamada received the Young Engineer Award of IEICE Japan in 1999, the Kiyasu-Zen'ichi Award and the Best Paper Award of IEICE both in 2010, and the Best Tutorial Paper Award from Comm. Soc. of IEICE in 2010. Dr. Yamada is a member of the IEEE.



Hiroyuki Ito received a B.E. degree in electronic engineering from Musashi Inst. of Technology, Tokyo, Japan, in 1984. In 1984, he joined a Nagano Japan Radio Co., Ltd, where he is engaging in Ocean surface current radar.