A Dynamic Frequency Assignment Algorithm in Mobile Radio Communication Systems

Masakazu SENGOKU*, Kiyohiko ITOH *and* Tadashi MATSUMOTO, *Regular Members*

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SUMMARY The efficient use of frequency spectrum is one of the most important problems in mobile radio communication systems. In a small zone system (the so-called cellular system), the algorithm used to make the channel assignment for a call has a great effect on system performance (traffic characteristics). This paper presents the relationships between the traffic-carrying capacity and the graph which represents the relation among a zone and its buffer zones in the system. Furthermore, using these relationships, an algorithm for the dynamic frequency (channel) assignment which has all channel available at all base stations is proposed. The performance characteristics of this algorithm obtained from computer simulations are also presented. · The computer simulation treats the two examples (systems), one is the system in which the mobile units does not cross a zone boundary during a communication (the zone size is sufficiently large and the probability of the occurrence can be neglected.), the other is the system in which the mobile unit may cross a zone boundary during a communication. The results from these simulations show that the algorithm carries more traffic per channel at a given blocking rate than the RING method.

1. Introduction

The frequency bands in mobile radio communication systems are limited. Thus the efficient use of frequency spectrum is one of the most important problems in the system design.[6]. In a small zone (cell) system, it is possible to reuse the same frequency channel at the same time in the other zones and the channel assignment method has a great effect on the traffic-carrying capability. The channel assignment methods are classified roughly into two types, that is, a fixed channel assignment method and a dynamic channel assignment method.[5],[6]. In the fixed channel assignment method, a subset of the channels available to the radio system is permanently served for use within each coverage zone. In the most general form of dynamic channel assignment, it is assumed that any channel can be used in any coverage zone.[7],[8],[9]. In the others (they may be contain in the class of the dynamic channel assignment methods.) there are the hybrid method [12] containing both fixed channels and dynamic channels and the channel borrowing method [10],[11] in which if a channel of the nominal channel set of the zone is not available then a channel is borrowed from the nominal channel sets of the suroounding caller's zone.

Although these methods are not directly comparable, it is known that the method containing the dynamic channels produces an increased channel occupancy on the order of 25-50% over fixed channel assignment at blocking rate in the range of 1–5%.[1].

In this paper, only dynamic channel assignment method which has all channels available at all zones (base stations) is considered. In this system, channels are assigned to serve calls (through the central processing unit) based on the state of the system and a channel assignment algorithm. Some algorithms of the dynamic channel assignment in this system have been proposed.[7],[8],[9]. In this paper, the relationship between the traffic-carrying capacity and the graph which represents the structure of zone arrangements in the system are presented and using these relationships, a new algorithm for dynamic channel assignment is proposed. Furthermore; computer simulations for 37 zones and 19 zones areas as representative system layouts are described. This algorithm can be also applied to the system which uses the hybrid method.

2. The Model of the System and Traffic Characteristics

In a small zone system, whole of the service area is divided into a number of zones. (Refer to (2), (3) and (4) for the methods to divide the service area.) The advantage of this system, as far as traffic-carrying capacity is concerned, is that the low power communication on a given RF channel in a certain zone allows the same channel to be reused in a number of different zones simultaneously so long as there is sufficient geographical separation between the zones. If a frequency is used in a zone it may not be used in some zones. These are called the buffer zones of the zone.

Let us define the following nonoriented graph# which represents the relations between a zone and its buffer zones. This graph is convenient for considering the traffic characteristics of the system.

Graph $G^*=(V^*, E^*)$: V^* and E^* are the set of vertices and edges in graph G^* respectively. $v_i (\in V^*)$ corresponds to a zone $z_i (i=1, 2, \cdots, n)$ and an edge $e_k=(v_i, v_j)$ represents that a zone z_i belongs to the buffer zones

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The authors are with the Faculty of Engineering, Hokkaido University, Sapporo, 060 Japan.
*Presently, the Faculty of Engineering, Niigata University, Nagaoka, 940 Japan.
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*1: If the buffer zones of a zone consist of all zones that are on this side of k zones away from it, the buffering system is called the k-belt buffering.

of z_j .

To show an example of a graph G^* , consider a service area of Fig. 1 (a). If the buffer zones of a zone consist of the contiguous zones to it (that is, 1-belt buffering *1), the graph G^* is Fig. 1 (b). Fig. 1 (c) is the complement # of G^* .

From the definition of graph G^*, the minimum number of frequencies required to communicate in all zones simultaneously is equal to the chromatic number # of the graph G^* .

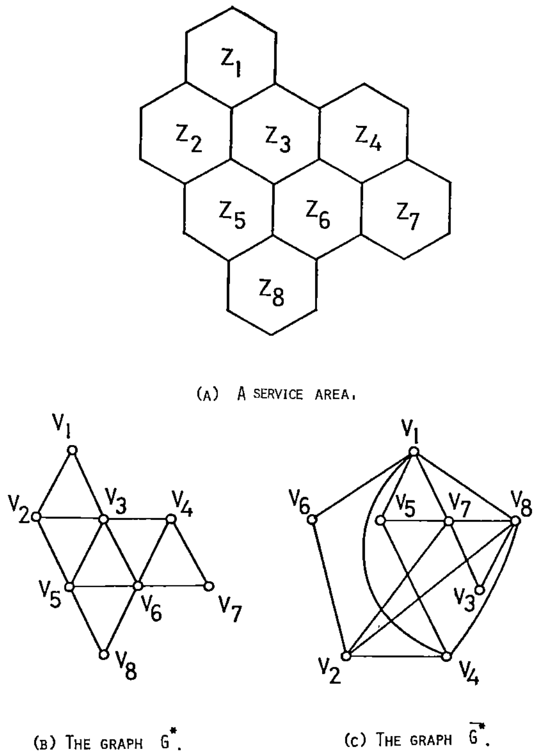

Fig. 1 An example of a service area and its graph.
(a) A service area.
(b) The graph G^*.
(c) The graph $\bar{G}^*$.

Let us assume that the traffic distribution is uniform over the service area, so that the average arrival rate is identical in each zone and that the mobile unit does not cross a zone boundary during a communication. Let the total traffic intensity be a and let the system be a loss system. Let us consider the case of one channel operation. *2 And we make the following two mathematical assumptions for a call in a zone. (that is, M/M/S type.[20])

(i) Arrivals occur according to a Poisson process with intensity λ .

(ii) The duration of a message (the holding time) is an exponential random variable with mean length $1/\mu$.

There are many ways that a channel is used in some zones simultaneously. The each different way may be considered as a state of the system. Let Θ be the set of the states and a element θ_i of the set consists of the vertices (of the graph G^* or $\bar{G}^*$) corresponding to the zones in which a channel is busy. $S^+(\theta_i)=\{\theta^+_{i_1}, \theta^+_{i_2}, \cdots, \theta^+_{i_{\alpha(\theta_i)}}\}$ is the set whose element $\theta^+_{i_j}$ is a state gaining one call in a state θ_i. $S^-(\theta_i)=\{\theta^-_{i_1}, \theta^-_{i_2}, \cdots, \theta^-_{i_{\beta(\theta_i)}}\}$ is the set whoes element $\theta^-_{i_j}$ is a state losing one call in state θ_i . From the assumptions (i) and (ii), the probability of gaining or losing one call in very short time Δt in a state θ_i is $\alpha(\theta_i)\lambda\Delta t$ or $\beta(\theta_i)\mu\Delta t$ respectively. The probability of transition from a state $\theta_j \in S^+(\theta_i)$ or $\theta_j \in S^-(\theta_i)$ to a state θ_i is $\mu\Delta t$ or $\lambda\Delta t$ respectively. Let us show a simple example. In the system represented by the graphs G^* and $\bar{G}^*$ in Fig. 2 (a) and (b),

$$\Theta=\{\phi, v_1, v_2, v_3, v_4, v_5, v_1v_3, v_3v_4, v_3v_5, v_4v_5, v_3v_4v_5\}$$
$$=\{\theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}\}.$$

The state flow diagram of this system is shown in Fig. 2 (c). In this figure, Δt is omitted and an arrow line shows the probability of transition between the states.

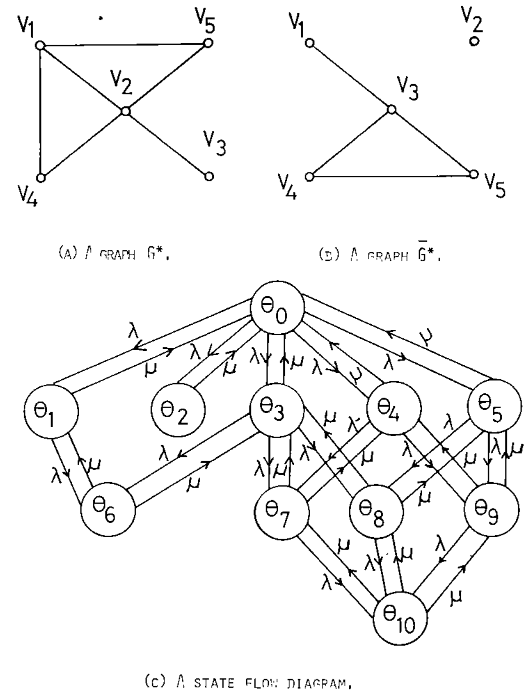

Fig. 2 An example of a state flow diagram.
(a) A graph G^*.
(b) A graph $\bar{G}^*$.
(c) A state flow diagram.

Let the probability of being in a state θ_i be $P(\theta_i)$. We assume the system to be in statistical equilibrium, then (as an example, see the state flow diagram in Fig. 2 (c).), it follows that

$$P(\theta_i)\alpha(\theta_i)\lambda\Delta t+P(\theta_i)\beta(\theta_i)\mu\Delta t = \sum_{\theta_j\in S^+(\theta_i)} P(\theta_j)\mu\Delta t+\sum_{\theta_j\in S^-(\theta_i)} P(\theta_j)\lambda\Delta t \qquad (1)$$

*2: In the system having more than one channel, the traffic carrying capacity depends on the channel assignment algorithm. So, the strict solution to it has not been obtained.

where the terms of order $(\Delta t)^2$ and higher are neglected as $\Delta t \to 0$. Let n be the number of zones in the system. Since $\lambda/\mu = a/n$,

$$P(\theta_i)\{\alpha(\theta_i)a/n+\beta(\theta_i)\} = \sum_{\theta_j \in S^{+}(\theta_i)} P(\theta_j)+(a/n)\sum_{\theta_j \in S^{-}(\theta_i)} P(\theta_j) \tag{2}$$

Then we can see that the equation

$$P(\theta_i)=(a/n)^k P(\theta_0) \tag{3}$$

is the solution of (2), where $\theta_i=\{v_{i_1}, v_{i_2}, \cdots, v_{i_k}\}$ and $\theta_0=\{\phi\}$, that is, θ_0 is the state in which a channel is not used in any zones.

Let us classify the set of states Θ as follows,

$$\Theta=\{\Theta_0, \Theta_1, \Theta_2, \cdots, \Theta_{k_m}\}, \quad \Theta_k=\{\theta_i \mid \theta_i \in \Theta, \ |\theta_i|=k\} \tag{4}$$

where $|\theta_i|$ denotes the cardinal number of θ_i. Let P_x be the probability that a channel is used in x zones simultaneously. Then,

$$P_x=\sum_{\theta_i \in \Theta_x} P(\theta_i)=(a/n)^x \nu_x P(\theta_0),$$

where

$$\nu_x=|\Theta_x| \tag{5}$$

Using $\sum_{x=0}^{\infty} P_x=1$,

$$P(\theta_0)=1\Big/\sum_{i=0}^{k_m}(a/n)^i \nu_i .$$

Therefore

$$P_x=(a/n)^x \nu_x \Big/ \sum_{i=0}^{k_m}(a/n)^i \nu_i \tag{6}$$

where $x=0, 1, \cdots, \kappa_m$.

The total load carried

$$a_c=\sum_{i=1}^{k_m} i P_i = \sum_{i=1}^{k_m} i(a/n)^i \nu_i \Big/ \sum_{i=0}^{k_m}(a/n)^i \nu_i \tag{7}$$

The probability of blocking in the system

$$B=1-a_c/a \tag{8}$$

In equations (7) and (8), we can see that the relation between B and a is obtained from ν_i and κ_m. ν_i is the number of i-cliques # of the graph $\bar{G}^*$ (which is the complement of G^*) and κ_m is the point independent number # of G^*.(13) An i-clique in $\bar{G}^*$ means that a channel can be used in the zones corresponding to its vertices simultaneously. In the service area of Fig. 1, $n=8$, $\nu_0=1$, $\nu_1=8$ (the number of vertices of $\bar{G}^*$), $\nu_2=15$ (the number of edges of $\bar{G}^*$) and $\nu_3=7$ (the number of triangles of $\bar{G}^*$). It is not so easy to find all kinds of cliques is the graph $\bar{G}^*$ of large system. (e.g., refer to (18) for the methods to find all maximal cliques of a graph.) In case of a high way system as shown in Fig. 3, the number of cliques is given in the simple form. That is,

$$\nu_0=1, \ \nu_1=n,$$

$$\nu_i=\sum_{j_{i-1}=1}^{n-(i-1)(k+1)} \sum_{j_{i-2}=1}^{j_{i-1}} \cdots \sum_{j_2=1}^{j_3} \sum_{j_1=1}^{j_2} j_1 , \tag{9}$$

where the buffer system is the κ-belt buffering and $2 \le i \le [1+(n-1)/(\kappa+1)]$.

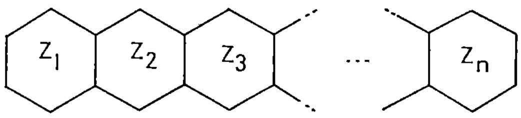

Fig. 3 A high way system.

From (7) and (8), we obtain the following relationships between the traffic-carrying capacity and the graph $\bar{G}^*$.

(a) When the blocking rate is high $(B \simeq 1)$, the number of large cliques *3 of the graph $\bar{G}^*$ produces a more effect on the traffic-carrying capacity than the number of small cliques of $\bar{G}^*$.

(b) Expanding eq. (7) in powers of (a/n),

$$a_c=f(a/n)=f(0)+f'(0)(a/n)+\frac{1}{2}f''(0)(a/n)^2+\cdots$$
$$=\nu_1(a/n)+(2\nu_2-\nu_1^2)(a/n)^2+\frac{1}{3}(3\nu_1^3-\nu_1\nu_2+9\nu_3)\cdot(a/n)^3+\cdots \tag{10}$$

When $(a/n) \ll 1$ and consequently the blocking rate is low, the number of small cliques of the graph $\bar{G}^*$ produces a more effect on the traffic-carrying capacity than the number of large cliques of $\bar{G}^*$ and in two systems with the same number of 1-cliques, the system having the more 2-cliques carries the more traffic.

3. A Channel Assigment Algolithm and Computer Simulations

In the previous section, we obtained that the traffic characteristics of mobile radio communication systems using dynamic channel assignments depend on the number of cliques in the graph $\bar{G}^*$. Using this fact, we propose a channel assignment algorithm (the clique method) as follows.

Let the system have n_c channels. Let a new call attempt occurs in a zone (vertex) v_a in the graph $\bar{G}^*$ and let C_a be the set of channels which are not used in a zone v_a and its buffer zones. Let U_i be the set of vertices in which the channel i is used and $\overline{Adj}(v_j)$ be the set of vertices which are not adjacent # to v_j in the graph $\bar{G}^*$. The graph $\bar{G}_i^{*'}$ for $i \in C_a$ obtained from $\bar{G}^*$ by deleting *4 the vertices $U_i \cup \{\bigcup_{v_j \in U_i} \overline{Adj}(v_j)\}$

*3: A small clique or a large clique of a graph contains a small or a large number of vertices, respectively.

*4: Deletion of a vertex implies the deletion of all edges incident on v_j.

contains fewer cliques than the graph $\bar{G}^*$. The vertices in the graph $\bar{G}_i^{*\prime}$ mean the zones which can use the channel i . Now if a channel $i \in C_a$ is used in the zone (vertex) v_a , then the graph $\bar{G}_i^{*\prime}$ is transformed into the graph $(\bar{G}_i^{*\prime\prime})$ obtained from $\bar{G}_i^{*\prime}$ by deleting the vertices $v_a \cup \overline{Adj}(v_a)$ and cliques of the graph $\bar{G}_i^{*\prime}$ decrease in number. As an example, let us consider the graphs G^* and $\bar{G}^*$ in Fig. 4(a) and (b). When $U_i=\{v_5\}$ and $v_a=v_1$, the graphs $\bar{G}_i^{*\prime}$ and $\bar{G}_i^{*\prime\prime}$ are shown in Fig. 4 (c).

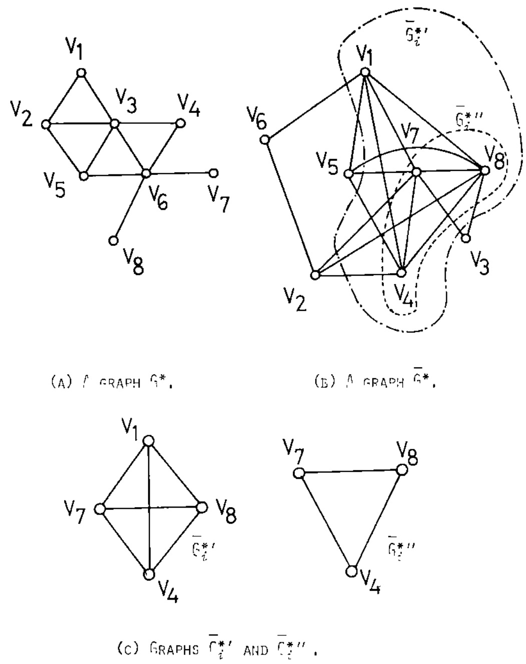

Fig. 4 An example of graphs G^* , $\bar{G}^*$, $\bar{G}_i^{*\prime}$ and $\bar{G}_i^{*\prime\prime}$.
(a) A graph G^*.
(b) A graph $\bar{G}^*$.
(c) Graphs $\bar{G}_i^{*\prime}$ and $\bar{G}_i^{*\prime\prime}$.

The algorithm assigning a channel in C_a which minimizes this decrease of cliques is called *clique method*.

In the above definition of the clique method, the expression "the decrease of cliques" is ambiguous, since a graph contains many kinds of cliques. So, we can consider the various algorithms. As one of them, let us consider a method examining the number of cliques in order of 1-cliques, 2-cliques, 3-cliques,... . A search is made through the set C_a to determine which channel(s) minimizes the difference of the number of 1-cliques between the graphs $\bar{G}_i^{*\prime}$ and $\bar{G}_i^{*\prime\prime}$. If more than one channel satisfies the above criteria, a channel minimizing the difference of the number of 2-cliques between $\bar{G}_i^{*\prime}$ and $\bar{G}_i^{*\prime\prime}$ is selected. If more than one channel satisfies the above criteria, 3-cliques, 4-cliques, ... are examined in turn. In the last search (maximum cliques), if more than one channel is selected, the first available channel in the search is assigned.

This algorithm is based on the fact that the number of small cliques in $\bar{G}^*$ has more effect on the total load carried than the number of large cliques at low blocking rates. A disadvantage of this method is that if the graph $\bar{G}^*$ contains many vertices and edges, it is difficult to find all cliques of $\bar{G}^*$ in short computation time. Thus, as a simple method there is an algorithm which selects a channel by examining only the number of small cliques.

Now, let us show the simple example in which only the number of 1-cliques of the graph $\bar{G}^*$ is examined to select a channel. That is, for a new call-attempt at v_a zone, a channel $i \in C_a$ which maximizes

$$|\{\bigcup_{v_j \in U_i} \overline{Adj}(v_j)\} \cap \overline{Adj}(v_a)\}| \qquad (11)$$

is assigned. This method has following meanings. For a new call-attempt at v_a zone, a channel of C_a may be already used in some zones, so that there are some zones (forbidden zones) in which the channel can not be used for a call. A channel $i \in C_a$ assigned by this method maximized the common zones among the buffer zones of v_a and the forbidden zones of the channel i . That is, the channel i minimizes an increase of forbidden zones for a next new call. As an example, let us consider the graph $\bar{G}^*$ of Fig. 4 again. Let $v_a=v_1$ and $C_a=\{i, j\}$. If the channels i and j are used in zones v_4 and v_5 respectively, that is $U_i=\{v_4\}$ and $U_j=\{v_5\}$, then the graphs $\bar{G}_i^{*\prime}$, $\bar{G}_i^{*\prime\prime}$, $\bar{G}_j^{*\prime}$ and $\bar{G}_j^{*\prime\prime}$ are shown in Fig. 5 (a) and (b). Since $|\overline{Adj}(v_4) \cap \overline{Adj}(v_1)| = |\{v_3\}| = 1$ and $|\overline{Adj}(v_5) \cap \overline{Adj}(v_1)| = |\{v_2, v_3\}| = 2$, the channel j are assigned for a call at v_1 zone.

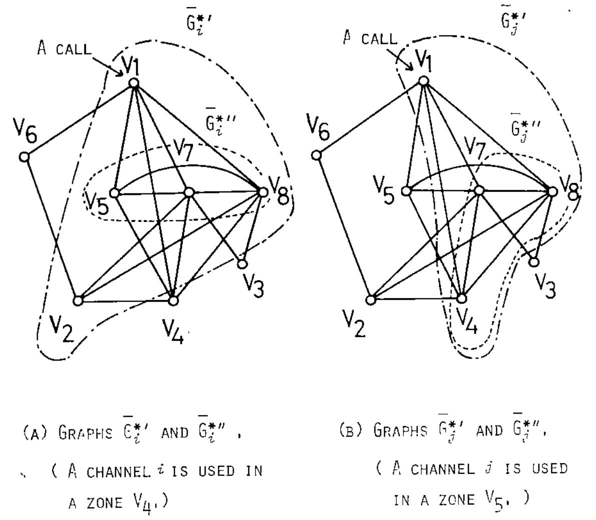

Fig. 5 Graphs $\bar{G}_i^{*\prime}$, $\bar{G}_i^{*\prime\prime}$, $\bar{G}_j^{*\prime}$ and $\bar{G}_j^{*\prime\prime}$.
(a) Graphs $\bar{G}_i^{*\prime}$ and $\bar{G}_i^{*\prime\prime}$.
(A channel i is used in a zone v_4)
(b) Graphs $\bar{G}_j^{*\prime}$ and $\bar{G}_j^{*\prime\prime}$.
(A channel j is used in a zone v_5)

In this method, if more than one channel satisfies the above condition, the selection is made on a first available basis in those channels.

The service area for a computer simulation is shown in Fig. 6. In this system, let the buffering zones

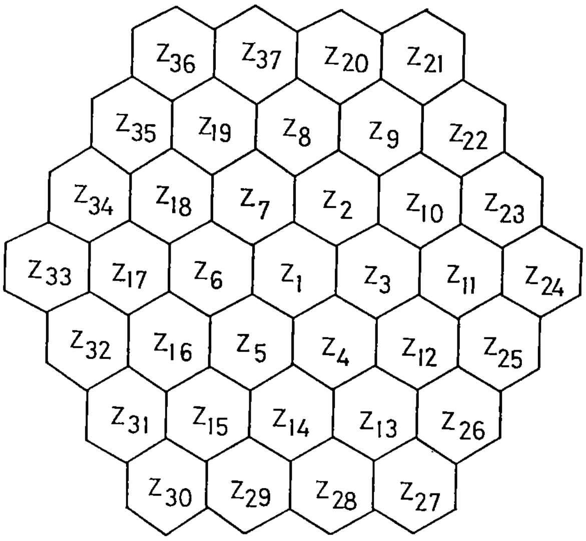

Fig. 6 A service area for computer simulations.

be the 2-belt buffering and the following assumptions are made.

1. Telephone traffic distribution is uniform over the service area.
2. The arrival of initiations (requests for service) forms a Poisson process and the holding time is an exponential random variable or a constant.
3. The mobile unit does not cross a zone boundary during a communication. (The zone size is sufficiently large and the probability of the occurrence can be neglected.)

As channel assignment algorithms, the first available method, *5 the RING method *5 and the above method (1-cliques method) are examined. The result of the computer simulation (the number of channels in the system is 20.) is shown in Fig. 7.

Next, let us show the example in which the mobile unit may cross a zone boundary during a communication. The service area consists of $z_1 \sim z_{19}$ zones in Fig. 6 and the radius of a zone is 1.25 (km) . (see Fig. 8) Let the buffer zones of a zone in this system consist of its contiguous zones (1-belt buffering). Let the mean of holding time be 180 seconds. In general, it is not so easy to make a model of the movement of the mobile subscriber in the service area. Let us define the model by the following assumption[14]. These serve for the assumption 3. of the above example.

3_1 · Each point within a zone is equally likely to be the starting point.

3_2 · The mobile subscriber moves in a straight line until he leaves the zone.

3_3 · His traveling direction is uniformly distributed from 0° to 360°.

*5: It is known that the first available method is far from an optimum assignment algorithm and this, however, minimizes the amount of data processing necessary to set up a call and that the RING method maximizing usage on the reuse ring gives good performance (traffic carried per channel at a given blocking rate)(7)~(9).

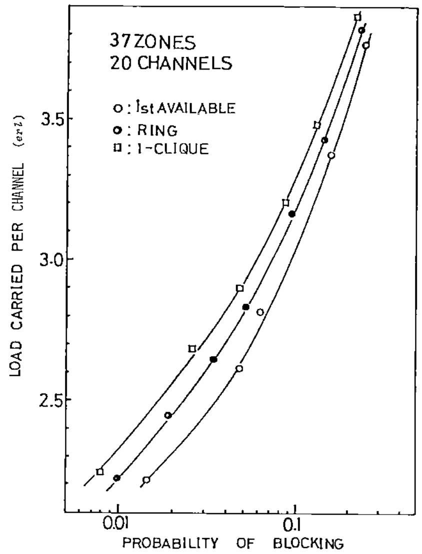

Fig. 7 Traffic carried for some dynamic frequency assignment algorithms in a 37-zone system.
(The mobile unit does not cross a done boundary during a communication.)

The assumptions are applicable to the period from the origination of the call unitl the first hand-off. For subsequent hand-offs, it is known that the mobile subscriber starts from the zone-edge. Hence, the first and third assumption are modified as follows.

$3_1'$ · Each point on the zone boundary is equally likely as a starting point.

$3_3'$ · His traveling direction is uniformly distributed over the angle which leads to the interior of the zone.

The speed distribution of the mobiles is shown in Fig. 9. This shows that the mobile subscribers of 15 (km/h), 30 (kn/h), 45 (km/h) and 60 (km/h) are 20%, 20%, 20% and 10%, respectively and 30 percent of the mobile subscribers park vehicles. Then, in this system, nearly 40 percent of the mobile subscribers cross a zone boundary during a communication.

In this example, three channel assignment methods in the previous example are also examined. The data from the simulation, which show the relationships between traffic carried per channel and the blocking of a

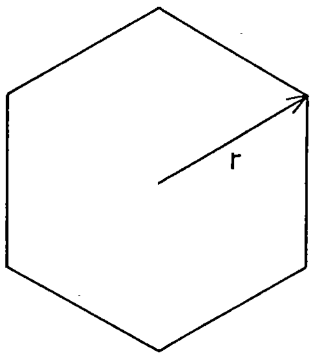

Fig. 8 A zone.

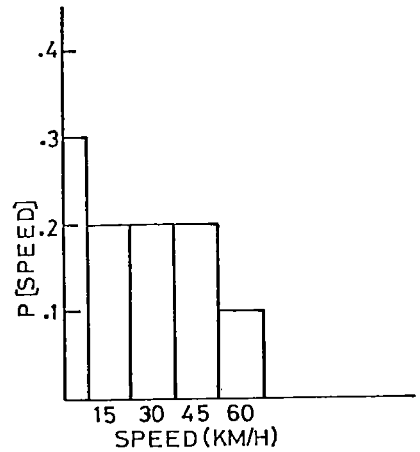

Fig. 9 Speed distribution.

new callattempt, are plotted in Fig. 10. These data are the mean in the central part ($z_1 \sim z_7$) of the service area.

From Fig. 7 and Fig. 10, the method of RING carries approximately 5 percent more traffic at a given blocking rate than the first available method and the 1-clique method carries $3 \sim 5$ percent more traffic at the blocking rate than the RING method.

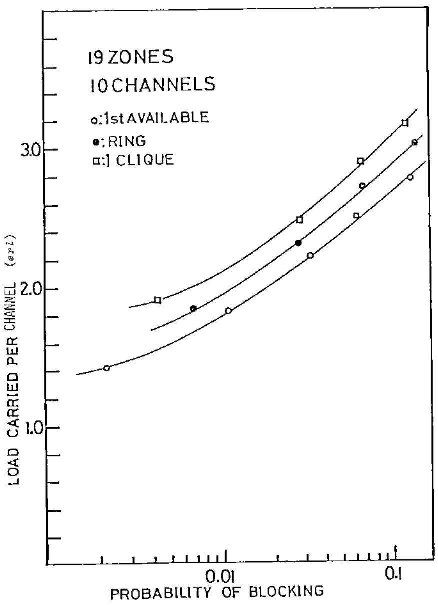

Fig. 10 Traffic carried for some dynamic frequency assignment algorithms in a 19-zone system.

4. Conclusions

The graph representing the relation among a zone and its buffer zones in a mobile radio system is defined and relationships between the graph and traffic carried in the system are presented. Using these relationships, the clique method as a channel assignment algorithm are proposed. The results from computer simulations show that this method is somewhat better (carrying 3 - 5 percent more traffic at a given blocking rate) than the RING method. In this computer simulation, only the number of 1-cliques of the graph is examined. If large cliques are also examined, the performance may be improved. However, it is reasonable to examine the number of 1-cliques and 2-cliques (and 3-cliques at most) in large system because of the computing efforts.

Appendix : Basic Concepts and Teams in Graph Theory[16],[17],[19]

A nonoriented *linear graph* (or simply a graph) $G=(V, E)$ consists of a set of objects $V=\{v_1, v_2, \cdots\}$ called *vertices* (*points* or *nodes*), and another set $E=\{e_1, e_2, \cdots\}$, whose elements are called edges (*lines* or *branches*), such that each edge e_κ is identified with an unordered pair (v_i, v_j) of vertices. The vertices v_i, v_j associated with edge e_κ are called the *end vertices* of e_κ, and v_i (or v_j) and e_κ are said to be *incident with* (*on* or *to*) each other. A graph g is said to be *subgraph* of a graph G if all the vertices and all the edges of g are in G, and each edge of g has the same end vertices in g as in G.

Two vertices v_i and $v_j (v_i \neq v_j)$ are said to be *adjacent* if there exists an edge $e_\kappa=(v_i, v_j)$. Painting all the vertices of a graph with colors such that no two adjacent vertices have the same color is called the *proper coloring* (or sometimes simply *coloring*) of a graph. A graph G that requires κ different colors for its proper coloring, and no less, is called a *κ-chromatic* graph, and the number κ is called the *chromatic number* of G.

An edge e_κ having the same vertex as both its end vertices is called a *self-loop*. The edges $e_\kappa=(v_i, v_j)$, $e_l=(v_i, v_j), \cdots (\kappa \neq l \neq \cdots)$ are referred to as *parallel edges* and a graph that has neither self-loops nor parallel edges is called a *simple graph*. A simple graph in which there exists an edge between every pair of vertices is said to be *complete*. A complete subgraph of a graph G is called a *clique* and it is also called an *n-clique* if it contains n vertices.

A simple graph $\bar{G}$ obtained by deleting the edges of a given simple graph G from a complete graph having the same vertices is called the *complement* of G.

A set of vertices in a graph G is said to be *independent* if no two of them are adjacent, and the largest number of vertices in such a set is called the *vertex (point) independent number* of G.

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