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PAPER

Linear Phase FIR Digital Filters with Cyclotomic Resonators

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SUMMARY This paper describes a design method of linear phase recursive FIR digital filters. The basic structure consists of a transversal part cascaded with a cyclotomic resonator, which is characterized by cyclotomic polynomials and has no multipliers. The digital filters implemented by this method require the short wordlength both for multiplier coefficients and for signals in their transversal part. By introducing integer arithmetic, the filtering operation proceeds fast and exactly. As a bonus, it is possible to employ a multiplier-less implementation in most practical applications. While the stability of this type of structure requires an integer-valued impulse response, a satisfactory procedure assures the requirement. A parameter to control the approximation error is found somewhat predictively rather than tentatively.

1. Introduction

In practice a linear phase FIR digital filter requires many multipliers, especially when narrow-band characteristics are realized. Thus it is left an open problem to achieve improvement with respect to cost, size and speed. One way to solve this problem is to rely on a programmable signal processor[1],[2]. Another way is to implement the digital filter efficiently as a special-purpose hardware. In the latter case it is necessary to decrease the number of multipliers and the wordlengths of both multiplier coefficients and signals in the digital filter. This paper is prepared as an approach along this line.

Since it is easy to implement an FIR digital filter in some well-known structures[3],[4], there has been devoted to a transfer function approximation in the field of designing FIR digital filters. On the other hand, Quach et al.[5] have pointed out : the approximation and the realization will be related to each other in the implementation with a finite wordlength. Thus an optimal implementation may be searched to yield a better approximation depending on the structure used. Saramäki[6] has also stated the similar idea regarding to IIR digital filters, but his concentration is still directed toward approximation.

Yet attempts from a viewpoint of structure to solve the above mentioned problem are rare. The difference routing digital filter[7] originally presented by van Gerwen et al. occupies a pioneering position. Although its structure offers several advantages, the design facility is limited. The main reasons are twofold. The first is due to the shortage of a detailed examination of the structure[7]-[9]. The other is that available resonators are small in number[7],[9] or are constructed with the sacrifice of efficiency[8] to some extent.

This paper first discusses a recursive structure of FIR digital filters consisting of a transversal part cascaded with a resonator. The discussion leads to a guideline to arrive at an efficient design. Secondly a family of cyclotomic resonators is introduced from the viewpoint of efficient structure. This is a systematic extension of conventional resonators, and the family of cyclotomic resonators contributes the structural improvement. The choice of cyclotomic resonators used depends on the combination of the center frequency and the passband width of a desired FIR digital filter, according to the guideline derived.

This method leads to the substantial reduction of multiplier coefficient wordlength as well as signal wordlength in the transversal part. Moreover in the case of narrow-band FIR digital filters, reduction of the net number of multipliers results as well. Finally a few examples of designed filters are given to illustrate the effectiveness of the proposed method.

2. Effect of Resonators

The frequency response of a linear phase FIR digital filter with symmetrical impulse response is expressed as[4]

$$H(e^{j\omega})=H^*(e^{j\omega})e^{-j\omega(N-1)/2} \tag{1}$$

where $H^*(e^{j\omega})$ is a continuous real function and N is the impulse response duration. Let the passband of the filter be (ω_1, ω_2). Then the magnitude of the impulse response $h(n)$ is evaluated as

$$|h(n)|\leqq\frac{1}{\pi}\int_{\omega_1}^{\omega_2}|H^*(e^{j\omega})|d\omega, \quad n=0,1,\cdots,N-1 \tag{2}$$

This equation means that the magnitude of the impulse response does not exceed the spectrum area of the filter. Note that $H^*(e^{j\omega})$ has the constant sign in the passband. Otherwise zero-crossing happens at a sign changing

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point, and this zero will give rise to a contradiction to the definition of a passband. Hence

$$\frac{1}{\pi}\int_{\omega_1}^{\omega_2}|H^*(e^{j\omega})|d\omega=\left|\frac{1}{\pi}\int_{\omega_1}^{\omega_2}H^*(e^{j\omega})d\omega\right| \qquad (3)$$

holds. Furthermore, when N is odd, the right-hand side of Eq.(3) really finds a particular sample among the impulse response as

$$\left|h\left(\frac{N-1}{2}\right)\right|=\left|\frac{1}{\pi}\int_{\omega_1}^{\omega_2}H^*(e^{j\omega})d\omega\right|. \qquad (4)$$

Thus the peak value of the impulse response $h_{\max}$ is of the form

$$h_{\max}=\frac{1}{\pi}\int_{\omega_1}^{\omega_2}|H^*(e^{j\omega})|d\omega. \qquad (5)$$

Consider that a digital transfer function $H(z)$ comprises a transversal part $D(z)$ and a resonator $1/R(z)$ as shown in Fig. 1. $H(z)$ is then described by

$$H(z)=D(z)/R(z). \qquad (6)$$

Obviously, the peak value $d_{\max}$ of the impulse response of the transversal part is evaluated as

$$d_{\max}\leqq\frac{1}{\pi}\int_{\omega_1}^{\omega_2}|H(e^{j\omega})|\,|R(e^{j\omega})|d\omega. \qquad (7)$$

Regarding to the frequency response of $R(z)$, if we assume

$$|R(e^{j\omega})|<1, \quad \text{for } \omega_1<\omega<\omega_2 \qquad (8)$$

then from the comparison between Eqs. (5) and (7), it follows that

$$d_{\max}<h_{\max}. \qquad (9)$$

As a consequence, if we use such a resonator consistent with Eq.(8) in the configuration of Fig. 1, the maximum magnitude of multiplier coefficients of the transversal part is smaller than that of the overall filter in a well-known direct or canonic form.

When N is even or when $h(n)$ is antisymmetrical, it is difficult to show the precise equation corresponding to Eq.(4). Nevertheless in such a wide-sense that a differentiator and a Hilbert-transformer are of interest as well as a band-selective digital filter, it seems fact: the impulse response of a linear phase FIR digital filter has its peaks around the center of its duration and that peak value is not so smaller than the value given by the left-hand side of Eq.(3). This observation has been demonstrated by a number of computer simulations. Hence the implication stated with regard to the function

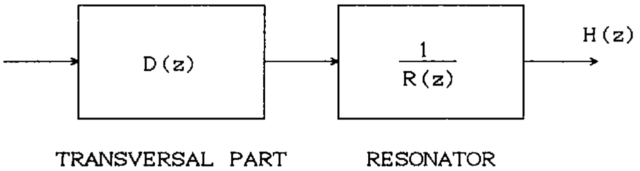

Fig. 1 A recursive FIR structure.

of resonators will be true in general.

In other words, one of the salient effects of a useful resonator in the form shown in Fig. 1 is to suppress the spectrum area of the overall FIR digital filter. This will be a guideline(10),(11) to reduce the coefficient wordlength of an FIR digital filter by means of resonators.

Similarly the magnitude-sum of the transversal coefficients

$$\sum_n|d(n)|=\sum_n\left|\frac{1}{\pi}\int_{\omega_1}^{\omega_2}H(e^{j\omega})R(e^{j\omega})e^{jn\omega}d\omega\right| \qquad (10)$$

estimates the signal wordlength required for the transversal part. Thus the use of appropriate resonators will also yield a reduction in signal wordlength of the transversal part.

3. Realization with Cyclotomic Resonators

Before cyclotomic resonators are introduced from the viewpoint of efficient structure, a brief preparation is given.

Cyclotomic polynomials $C_k(z)$ arise from a factorization for the polynomial z^K-1 as a product of irreducible polynomials with integer coefficients(12)-(14). There is one $C_k(z)$ for each divisor k of K, including $k=1$ and $k=K$. The roots of $C_k(z)$ are the primitive kth roots of unity. The number of such roots is given by $\varphi(k)$, Euler's function. $\varphi(k)$ is equal to the number of positive integers smaller than k and prime to k. Therefore, the degree of $C_k(z)$ is $\varphi(k)$. $C_k(z)$ is defined by

$$C_k(z)=\prod_{(i,k)=1}(z-e^{-j2\pi i/k}) \qquad (11)$$

where $(i,k)=1$ denotes that i and k are mutually prime.

The important property for our aim is as follows: If k has no more than two distinct odd prime factors, the coefficients of $C_k(z)$ will be 1, -1 or 0. The smallest integer k with three prime factors is $k=105=3\cdot5\cdot7$. Hence, to get multiplierless resonators, we define a cyclotomic resonator by

$$1/R(z)=1/C_k(z)z^{-\varphi(k)}. \qquad (12)$$

Each cyclotomic resonator is composed of shift registers and adders/subtracters. The structure is inherently free from multiplications. In addition, it is feasible with the modern silicon technology to efficiently implement those resonators in LSI/VLSI.

Now Fig. 2 indicates two basic structures of an FIR digital filter. The first structure has serial delay elements. The other structure is derived from the first one by interchanging the FIR and IIR parts in the transpose. In the latter structure, every delay element is interrupted by an adder. Yet every group of multiplications is addressed by a single signal such as the input or the output. Each structure consists of a transversal part cascaded with a cyclotomic resonator. The overall impulse response $h(n)$ is of the form

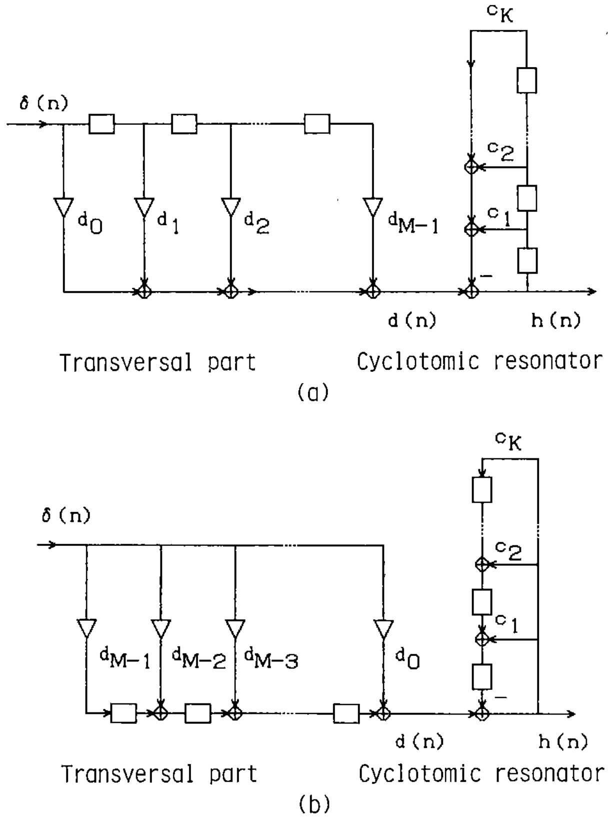

Fig. 2 The basic structures of an FIR digital filter with a cyclotomic resonator. (a) serial delay form, (b) interrupted delay form.

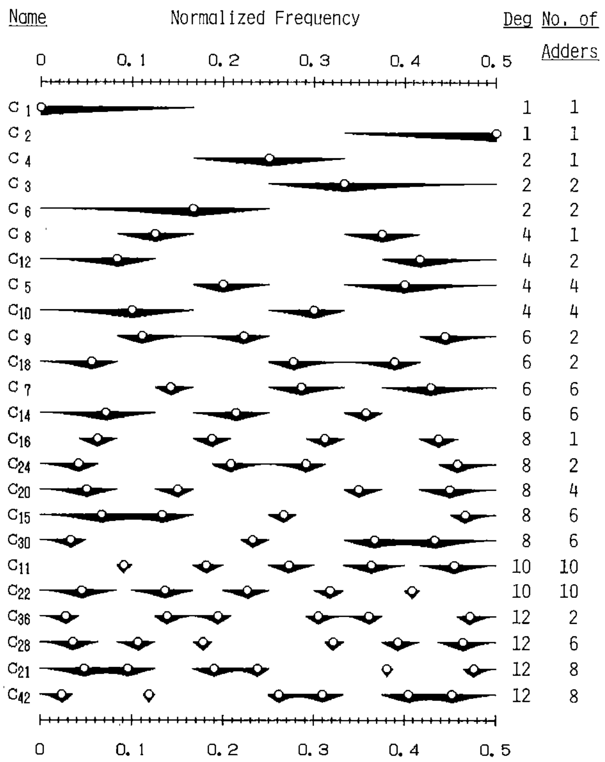

Fig. 3 Suppression bands and root locations of 24 cyclotomic polynomials.

$$h(n)=d(n)+\sum_{k=1}^{K} c_k h(n-k) \tag{13}$$

$$d(n)=\sum_{m=0}^{M-1} d_m \delta(n-m) \tag{14}$$

where each c_k is a coefficient of the cyclotomic resonator, pertaining to 1, -1, and 0. If the sequence $h(n)$ is given as a set of integers, the sequence $d(n)$ turns out to be integers from Eq.(13). Thereby, if overflow is prevented by scaling of the input signal, the filtering proceeds under integer arithmetic without round-off errors nor limit cycle oscillations. How to approximate $h(n)$ as integers is deferred until the Sect. 5.

On the other hand, according to the guideline given in the previous section, cyclotomic resonators can afford to reduce the wordlength for the transversal coefficients. Since there are infinitely a great number of cyclotomic polynomials, it is impossible to list all of the responses. Instead, for a practical use, we select 24 of cyclotomic polynomials, and they are included in the Appendix. Among this set, every implementation of the useful resonators has no more complexity than ten adders and twelve delay elements. Heavy segments in Fig. 3 show the frequency ranges on which the magnitude of each cyclotomic polynomial is less than unity, and for convenience we will refer to those ranges as suppression bands throughout this paper. Light dots indicate the root locations of each polynomial. The figure will enable us to select effective cyclotomic polynomials to meet given specifications.

Figure 4 illustrates how the spectrum area of $H(z)$ be reduced to that of $D(z)$ by multiplying $R(z)$. The frequency responses of $H(z)$ and $R(z)$ are drawn in solid lines. The resulting spectrum area of $D(z)$ is shaded.

The suppression bands of a few kinds of cyclotomic polynomials may possibly cover the passband of a particular filter, as is the case with wide passband applications. One polynomial may have a zero close to the center frequency of the filter, as shown in Fig. 5(a) by a broken line. By contrast, another may have a zero far from it, but its amplitude can be smaller over the whole passband. This case is drawn by a solid line in the same figure. In this situation, by paying much attention to a passband width rather than a center frequency, it is

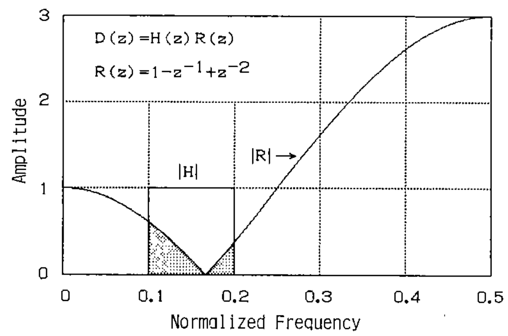

Fig. 4 Illustration of spectrum area suppression.

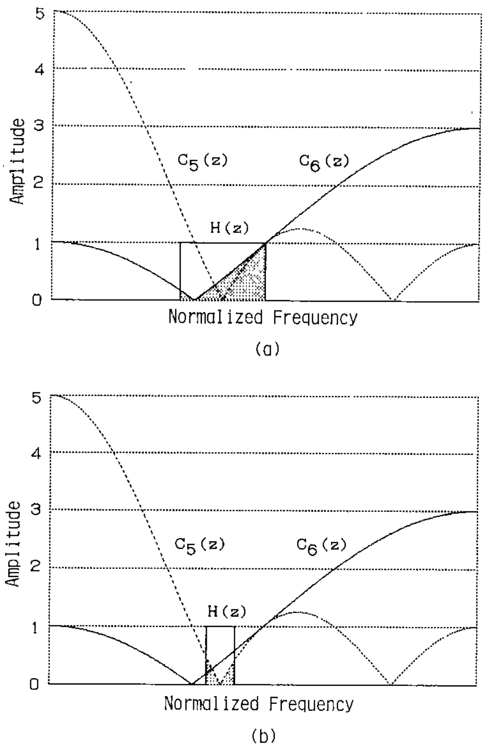

Fig. 5 Difference of spectrum area suppression depending on the specifications. (a) for wide band filtering, (b) for narrow-band filtering.

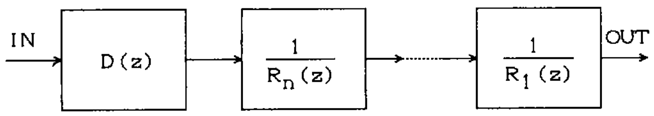

Fig. 6 Resonator cascade form.

better to employ the cyclotomic polynomial with wider suppression band. The resulting spectrum area from the application of such a polynomial is shaded in the figure.

On the contrary, for narrow-band applications the more effective choice will be to use a cyclotomic polynomial with zeros near the center frequency of the filter. Figure 5(b) illustrates such a case.

The guideline allows us to use cyclotomic resonators depending on the combination of the pass-band width and the center frequency of a desired filter. There is nothing but to suppress the spectrum area of the filter of interest. Thus it is natural to use cyclotomic resonators in a cascade configuration as shown in Fig. 6 to achieve a better performance.

4. Discussion

4.1 Zero-Pole Cancellation

In general the spectrum of an impulse response represents the distribution of the frequency components in a steady-state response. Let us consider a particular structure that consists of a transversal part cascaded with a resonator. Suppose that the filter is excited by a frequency component coincident with an in-band zero of the transversal part. Its steady-state output is then nothing. One may thus wonder that the non-vanishing response appears at the total output of the filter.

On the contrary, we can take another filter structure that comprises a resonator followed by a transversal part. Applying a sinusoidal excitation which coincides with the frequency of the resonator pole on the unit circle, the resonator shows the undamped sinusoidal oscillation. One may again suppose that it is strange for the total filter response to vanish within a finite duration.

The way to the solution is to consider the transient behavior or the complex frequency variable in terms of the Z transform. This is evident from the fact that the complete description of the system dynamics requires an initial condition. In the Z domain, the reason for the finite duration response is due to zero-pole cancellation under an appropriate initial condition.

Alternately the answer is interpreted in the time domain. Let us consider a simple example of the form

$$H(z)=D_1(z)/R_1(z),$$

where

$$D_1(z)=1-z^{-3}, \quad R_1(z)=1+z^{-1}+z^{-2}.$$

The impulse response of $D_1(z)$ is a unit pulse at $n=0$, followed by a negative unit pulse at $n=3$. On the other hand, the impulse response of the resonator is represented by

$$r_1(n)=\sum_{m=0}^{\infty}[\delta(n-3m)-\delta(n-3m-1)]$$

$$=U(n)\frac{2}{\sqrt{3}}\sin\frac{2\pi}{3}(n+1),$$

where $U(n)$ denotes the unit step function. The envelope of $r_1(n)$ is completely sinusoidal. For simplicity we assume that all of the internal states are zero at $n=0$. As illustrated in Fig. 7, the leading pulse of the transversal part drives the cascaded resonator, and the resonator starts its proper oscillation. The final pulse of the transversal part again excites the resonator in anti-phase to the earlier excitation. Because of superposition of the two opposite phase responses, the output of the total filter shows the time limited sinusoidal response.

4.2 Some Aspects on the Implementations

In practice the transversal part involves a number of symmetric or anti-symmetric pulse pairs. Thereby in order for the resonator to yield the desired finite-duration response, all of the initial states must be zero.

According to system theory[15], the basic structures shown in Fig. 2 are uncontrollable, owing to zero-pole cancellation. This fact is, however, not fatal to our purpose of shift-invariant filtering. In fact there is no

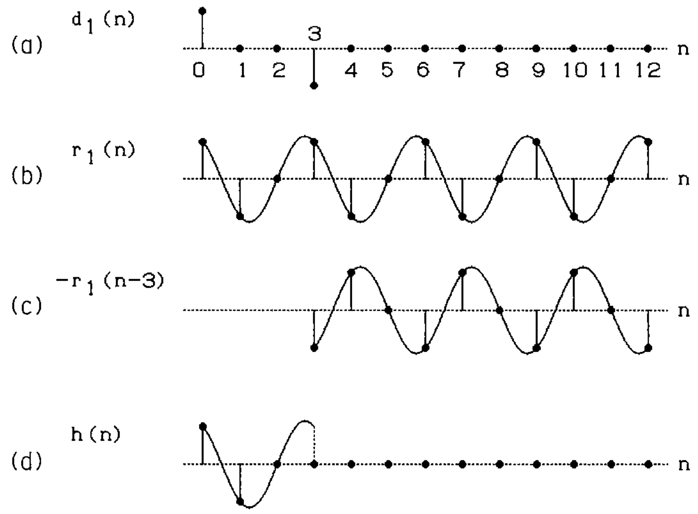

Fig. 7 Behavior of the filter.
(a) the response of the transversal part,
(b) the first response of the resonator,
(c) the second response of the resonator,
(d) the total output.

need to control or monitor the internal states by remote control through the input or by remote sensing through the output. It is enough to set the zero initial condition by any means. The need arises on the actual activation of a digital filter, that is at the instance of power-on.

The internal states of a digital filter are directly and independently accessible by digital integration technology. Therefore the internal states can be vanished at once, as desired.

As for a special implementation of the set of delay elements, the situations may be slightly complicated. This is a fact, if the set is devoid of some direct paths from the outside to an arbitrary inner position. Thereby, it is impossible to do instantaneous elimination of the internal states. Moreover, since the system is uncontrollable, no means are available to reset the internal states. Nevertheless, if the structure is torn by inserting a multiplexer into the common feedback loop, the resulting system recovers controllability. As a result, the internal states of the system can be cleared after a finite set-up time.

To ensure the perfect zero-pole cancellation, the implementation invokes the following two conditions. The first is an input scaling to avoid overflow, and the other is integer arithmetic to prevent from accumulating quantization errors. These conditions guarantees the external stability of the total system. Owing to those facts, it is possible to continue neither transient nor steady-state oscillations.

Hence the three facts are crucial to our implementation of the digital filter: That is the initial reset of the internal states, an input scaling and integer arithmetic. These rules assure the desired behavior, in spite of the absence of controllability or observability as well as the lack of internal stability.

Finally additional comment is given about the connecting order among the transversal part and the resonators. As estimated by Eq. (10), the signal wordlength of the transversal part can be reduced than that of the original impulse response. To take this advantage, the resonator must follow the transversal part, as depicted in Fig. 2. In practice several resonators are used as shown in Fig. 6, according to the spectrum suppression guideline. The best order to cascade them is determined by the amount of their spectrum suppressing effects. That is to place more effective resonator closer to the total filter output.

5. Design Procedure

An ideal impulse response for a given task is obtainable by resort to several approximation methods, for instance the discrete Fourier transform, windowing techniques and the equiripple design[(3),(4),(16)]. One can suppose that such a response has been already available at the outset of the filter design. From the ideal impulse response $h_w(n)$, the integer-valued impulse response $h(n)$ is calculated by

$$h(n)=\mathrm{INT}[h_w(n)P/h_{w\max}] \qquad (15)$$

where INT[ ] stands for rounding into an integer, $h_{w\max}$ is the peak value of $h_w(n)$, and P is a certain integer.

The integer P in Eq. (15) is determined somewhat predictively rather than tentatively from the minimum stopband attenuation required. The reason is explained below.

The approximated impulse response $h(n)$ given by Eq. (15) is a sum of $h_w(n)P/h_{w\max}$ and the error due to quantization. The sequence of the quantization error will be statistically independent and has a uniform probability distribution with the probability density P in the range from $-1/2P$ to $1/2P$, because the peak value of $h(n)$ becomes P. Thus the root-mean-square of the error will be $\sqrt{1/12P^2}$. As a matter of fact the error spectrum is not completely flat. Then, if the spectrum shape is assumed to be sinusoidal, its peaks will be greater by 3 dB than the average value. Those peaks will be almost uniformly distributed and so they will determine the minimum stopband attenuation $A_{\min}$. Therefore a simple calculation yields

$$P=10^{(A_{\min}-7.8)/20}. \qquad (16)$$

Figure 8 shows considerable agreement between Eq. (16) and computer simulations for the raised-cosine roll-off characteristics. Figure 9 illustrates the characteristic, where f_c and f_b are the center frequency and the bandwidth between the center and a half amplitude frequencies, respectively. α denotes the roll-off factor. It should be noted that a single parameter P controls over both the passband and stopband ripples through Eq. (15).

By the way, the most difficult issue in our design is to assess the efficiency or the effectiveness. For the sake of only convenience, we here introduce a figure of assessment defined by

(the total number of transversal taps)

times (the coefficient wordlength required).

We consider that the smaller the figure, the more efficient will be the realization. The total number of transversal taps may be different from that of multipliers depending on particular implementations. For example, it is the case, if there are the same multiplier coefficients, and if the multiplication follows the summations. In such a case, the figure of assessment will overestimate the amount of a hardware complexity.

Exploiting this figure as a measure of complexity, we can provide a rather systematic procedure to design FIR digital filters, as follows.

(Step 1) To get the integer-valued impulse response $h(n)$, quantize an ideal impulse response $h_w(n)$ through Eq.(15). The quantization effects are checked by computing the frequency response. If the quantization error stays within a given tolerance limit, proceed to the next step. Otherwise after increasing the integer P, repeat Step 1.

(Step 2) According to the guideline of spectrum area suppression, select an appropriate cyclotomic resonator. It is effective to use Fig. 3 as a selecting chart. Let i refer to the stage number of selection. When the i th cyclotomic resonator has been chosen, compute the associated transversal part to get its multiplier coefficients. At the same time the computation also yields the figure of assessment associated with that choice.

(Step 3) If the figure of assessment at the current stage of the design is less than the last one, try to cascade another cyclotomic resonator: go to step 2. Otherwise stop the design procedure. Note that the design itself has been already obtained as the result of the last design stage.

The design procedure can be well performed with the aid of computer graphics in a man-machine interactive mode.

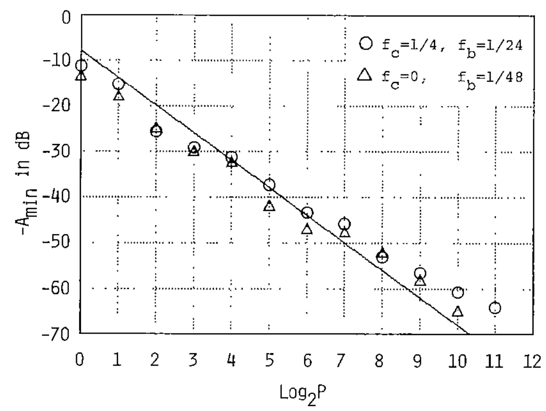

Fig. 8 Dependence of A_{min} on P. f_c center frequency, f_b bandwidth between f_c and a half-amplitude frequency.

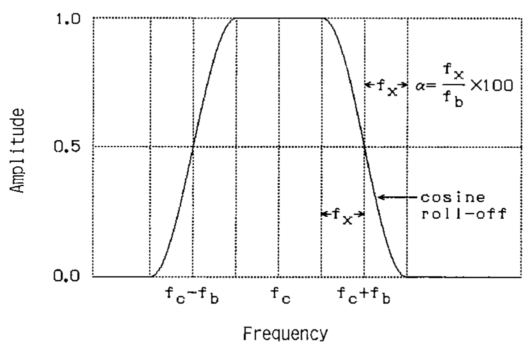

Fig. 9 Raised cosine α% roll-off characteristic.

6. Design Examples

Three examples of linear phase FIR digital filters are included to demonstrate the efficiency of the method described in the preceding sections. All of them have the raised cosine α% roll-off characteristics and are designed by the discrete Fourier transform.

(Example 1) Let us consider a 100 % roll-off bandpass filter with the specifications

$$f_c=11/256,\ f_b=1/48,\ A_{min}=45(\text{dB}).$$

The integer P is chosen as 64 for this example from Eq. (16). Figure 10 (a) shows the overall impulse response. The impulse response of the transversal part depicted in Fig. 10 (b) results from the use of the type A_1 resonator[9]. This choice has been exclusively suggested by the nearness of the center frequency to a pole of a resonator. By contrast, Fig. 10 (c) comes of the use of triple cyclotomic resonators defined by $C_1(z)$ so as to suppress the spectrum area.

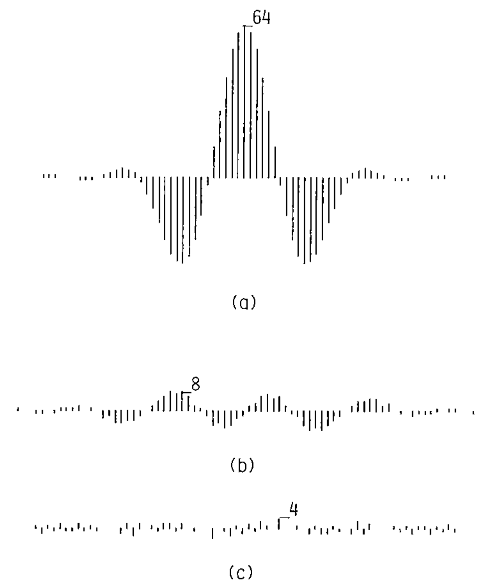

Fig. 10 Example 1. Impulse responses of (a) the overall filter, (b) the transversal part for A_1 used, (c) the transversal part for a triple of C_1 used.

The degree of $C_1(z)$ is one but that of type A_1 is 13 according to Ref.(9). The coefficient wordlength in Fig. 10(c) is a half of that in Fig. 10(b). Hence it is efficient to rely on the design method presented in this paper, even though only conventional resonators are available. This demonstrates the design improvement.

(Example 2) The specifications of a 50% roll-off bandpass filter are now as follows.

$$f_c=0.3,\ f_b=0.05,\quad A_{min}=50(\text{dB})$$

As can be found in Fig. 8, with increase of A_{min}, P given

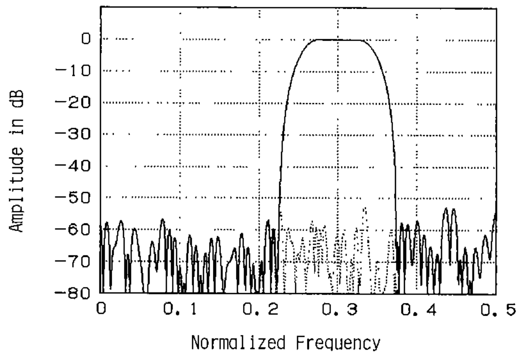

Fig. 11 Example 2. Responses of (solid) the filter and (dotted) the quantization error.

by Eq.(16) will be less satisfactory. So while the equation suggests taking P as 129, we set it as 1024. The resulting response of the digital filter is shown in Fig. 11, and it clears the required attenuation.

Using cyclotomic resonators defined by C_3, C_4, C_3, C_4 and C_{18}, the degree in the implementation increases by 10 % from 150 to 164. On the contrary the net number of multipliers decreases from 26 to 12. The wordlength of multiplier coefficients decreases from 10 bits to 4 bits, excluding the sign bit. Furthermore the signal wordlength in the transversal part reduces to 1/13. This proves the structural improvement.

(Example 3) To give a demonstration of the behavior in the successive spectrum area suppression, a 25 % roll-off bandpass filter is considered. The specifications are

$$f_c=0.1,\ f_b=0.03125,\quad A_{min}=55(\text{dB}).$$

Selecting P as 1024, the amplitude response of Fig. 12(a) is obtained. The remainder of the figure illustrates how the spectrum area subsequently decreases. For the overall response (a), the successive application of C_{10}, C_{15}, C_{12} and C_8 produces each response of the relevant transversal parts, (b)-(d), respectively.

A comparison with respect to hardware complexity is made, in Table 1, between conventional canonic forms

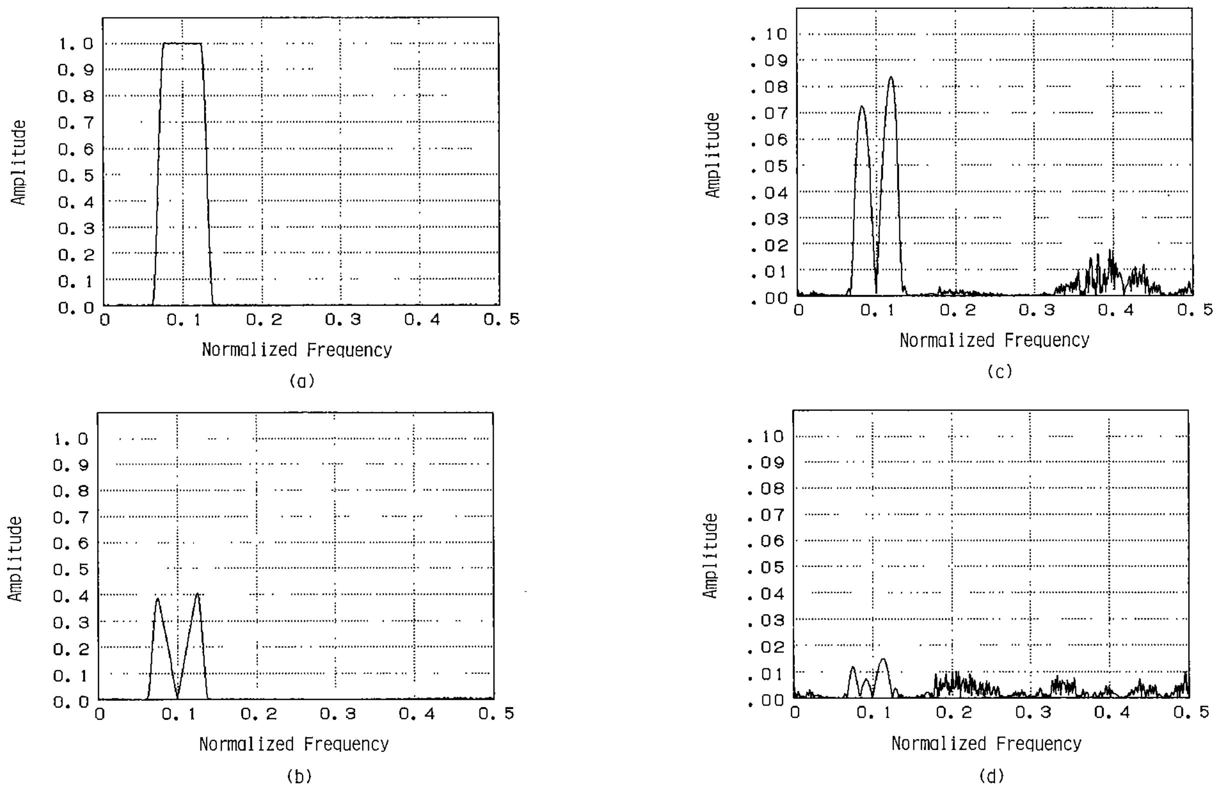

Fig. 12 Example 3. Amplitude responses of
(a) the overall filter,
(b) the transversal part produced by the application of C_{10},
(c) the transversal part produced by the application of C_{10} and C_{15},
(d) the transversal part produced by the application of C_{10}, C_{15}, C_{12}, and C_8.

Table 1 Comparison on hardware complexity.

	Example 1		Example 2		Example 3	
	CANONIC	PROPOSED	CANONIC	PROPOSED	CANONIC	PROPOSED
DEGREE	66	69	150	164	400	420
NO. OF MULTIPLIERS	21(17)	7(3)	31(26)	20(12)	59(51)	8(4)
NO. OF TAPS	59	54	97	151	263	297
MAX. OF COEFFICIENTS	64	4	1024	13	1024	5
MAGNITUDE-SUM*	998	88	8766	656	17044	496

( ) DENOTES THE NUMBER WHEN THE DISTINCT SIGNS ARE IGNORED.
* DENOTES REGARDING TO THE TRANSVERSAL COEFFICIENTS.

and the proposed forms. As can be seen from Table 1, the net number of multipliers is decreased in addition to the reduction of the maximum value of coefficients and the magnitude-sum of the transversal tap gains.

7. Concluding Remarks

In order to reduce the coefficient wordlength of a linear phase FIR digital filter, it is efficient to suppress the spectrum area by means of appropriate resonators. On the other hand, cyclotomic resonators defined in this paper have simple structures and efficient implementations are feasible within the modern silicon technology.

The above two facts are combined to yield an efficient implementation of FIR digital filters. As a consequence, they achieve not only the structural but also design improvements. In spite of its recursive structure, the filtering operation proceeds fast and exactly. When the coefficient wordlength is very small, the implementation requires no multipliers, and this happens sometimes in practice.

A portion of these features is supported by a suitable approximation: the impulse response having integer values is obtained rather predictively from the prescribed specifications.

To take the full advantage of this structure, a special type of hardware implementations may be found. One of the promising applications will be the video-band filtering.

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Appendix

The explicit formulae of the 24 cyclotomic polynomials are listed for a practical use.

$$C_1(z)=z-1$$

$$C_2(z)=z+1$$

$C_4(z)=z^2+1$

$C_3(z)=z^2+z+1$

$C_6(z)=z^2-z+1$

$C_8(z)=z^4+1$

$C_{12}(z)=z^4-z^2+1$

$C_5(z)=z^4+z^3+z^2+z+1$

$C_{10}(z)=z^4-z^3+z^2-z+1$

$C_9(z)=z^6+z^3+1$

$C_{18}(z)=z^6-z^3+1$

$C_7(z)=z^6+z^5+z^4+z^3+z^2+z+1$

$C_{14}(z)=z^6-z^5+z^4-z^3+z^2-z+1$

$C_{16}(z)=z^8+1$

$C_{24}(z)=z^8-z^4+1$

$C_{20}(z)=z^8-z^6+z^4-z^2+1$

$C_{15}(z)=z^8-z^7+z^5-z^4+z^3-z+1$

$C_{30}(z)=z^8+z^7-z^5-z^4-z^3+z+1$

$C_{11}(z)=z^{10}+z^9+z^8+z^7+z^6+z^5+z^4+z^3+z^2+z+1$

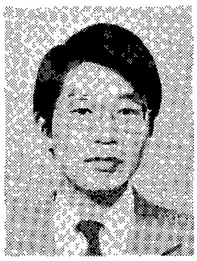
$C_{22}(z)=z^{10}-z^9+z^8-z^7+z^6-z^5+z^4-z^3+z^2-z+1$

$C_{36}(z)=z^{12}-z^6+1$

$C_{28}(z)=z^{12}-z^{10}+z^8-z^6+z^4-z^2+1$

$C_{21}(z)=z^{12}-z^{11}+z^9-z^8+z^6-z^4+z^3-z+1$

$C_{42}(z)=z^{12}+z^{11}-z^9-z^8+z^6-z^4-z^3+z+1$



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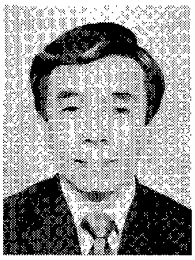

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