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PAPER

Interpolated FIR Filters Based on the Cyclotomic Polynomials

Hisakazu KIKUCHI†, Yoshito ABE†,
Hiromichi WATANABE† *and* Takeshi YANAGISAWA††, *Members*

SUMMARY Based on the cyclotomic polynomials, this paper describes a family of efficient and practical interpolators for interpolated FIR filters. The family can be applied to bandpass filters as well as lowpass/highpass filters without any multiplications. It also mitigates the inconvenience to select a practical interpolation factor, and gains a further saving in computational complexity required. Several examples are given to demonstrate the effectiveness for reducing the computational complexity required.

1. Introduction

In practice finite impulse response (FIR) digital filters require many multiplications and additions. The filtering operation to do a given task regularly proceeds by convolving a subsequent input signal with the impulse response. Such regularity is favorable to software simulated and signal processor based implementations. Yet this is not the all desired. As for a special purpose hardware implementations, the property contributes inadequate efficiency. FIR digital filters still remain expensive and bulky.

A full implementation of a digital filter of course requires a special body of knowledge, as found in an example(1). It depends on particular applications in addition to basic structures. Discarding the detailed aspects in individual applications, one can focus a basic structure for efficient implementations with respect to computational complexity.

There are several literatures to decrease the computational complexity. Van Gerwen et al. have proposed difference routing digital filters(2) to replace the complicated multiplications with simple shift-add operations. The filter consists of an FIR part cascaded with a resonator. Owing to its recursive structure, one must take into account of special criteria both in the design and in the implementation(3). Another example is the prefilter-equalizer design presented by Adams and Willson(4)-(6). It aims at the reduction of multiplications and additions. The computational complexity of the corresponding implementation usually amounts to two thirds, compared to conventional implementations.

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† The authors are with the Faculty of Engineering, Niigata University, Niigata-shi, 950-21 Japan.
†† The author is with the Faculty of Engineering, Tokyo Institute of Technology, Tokyo, 152 Japan.

A further significant saving in arithmetic operations can be obtained by interpolated FIR (IFIR) digital filters presented by Neuvo, Dong, and Mitra(7). An IFIR filter consists of two FIR sections: that is a model filter with the impulse response inserted by several null samples each and an interpolator. The efficiency offered by IFIR filters is a consequence of exploiting the redundancy in filter coefficients. In principle the number of arithmetic operations required for the implementation is inversely proportional to an interpolation factor. Moreover, since the original version is inherently free from recursion, it guarantees the absolute stability and the absence of limit cycle oscillations.

This paper deals with a practical family of efficient interpolators for IFIR digital filters(8). In the original IFIR filter implementations(7), practical and efficient interpolators are strictly available only to a special interpolation factor of powers of two. In addition, the interpolation with no multiplications is limited to lowpass and highpass types. The inconvenience will be mitigated by a novel interpolator family. Since the family is characterized by the cyclotomic polynomials, it inherits two properties: no multiplications and no recursion.

The designed filters gain a salient saving in computational complexity over the conventional filters, and they will compete with the modified versions of IFIR filters by Saramäki et al.(9).

2. Interpolated FIR Filters

The transfer function of interpolated FIR filters(7) is described by

$$H(z)=H_M(z^L)G(z) \qquad (1)$$

where $H_M(z)$ and $G(z)$ are referred to as a model filter and an interpolator, respectively. An integer L denotes the interpolation factor.

The structure of IFIR filters is a cascade of two FIR sections as shown in Fig. 1. The first section generates a

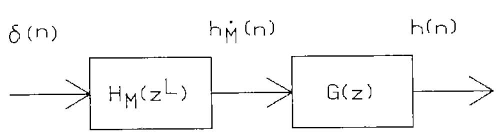

Fig. 1 Block diagram of an interpolated FIR filter.

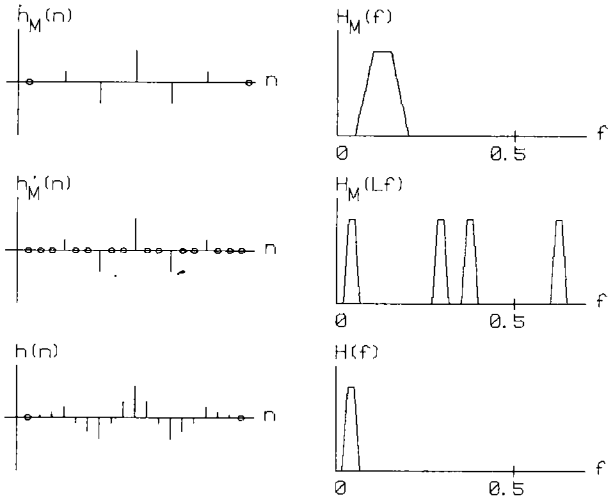

Fig. 2 Illustration of the IFIR filter principle.

sparse sequence $h'_M(n)$ of impulse response samples with every Lth sample being non-zero. The sequence is produced by inserting $L-1$ null samples between the original sequence $h_M(n)$. The other section interpolates the sparse sequence, and produces the desired impulse response $h(n)$. The left column of Fig. 2 illustrates this process.

As shown in the right column of the same figure, the frequency domain interpretation facilitates understanding of band-selective filtering. Regarding to a frequency response, the exponential argument is simply abbreviated to the normalized frequency throughout this paper. A model filter $H_M(f)$ can span the long interval of a passband and a transition band. The interval is reduced to $1/L$ by replacing each delay of the model filter with L delays. Since the process inevitably produces L replicas of the desired passband, an interpolator follows the model filter to attenuate the unwanted replicas. The total system thus displays the desired frequency behavior $H(f)$, if the interpolator less affects the frequency response of the desired passband.

It is a well-known fact that the impulse response duration of an FIR digital filter is inversely proportional to the transition band width[10]. On the other hand, the frequency response of an FIR digital filter can be shortened by inserting null samples between the original impulse response. Hence, if the interpolator to suppress the unwanted replicas is implemented by using a simple structure, the computational complexity of IFIR filters only amounts to the reciprocal of the interpolation factor.

The availability of IFIR filters depends on that of efficient interpolators. The most fundamental interpolators are represented by linear and quadratic polynomials[7]. The first-order interpolators are of the forms

$$G_0(z)=(1+z^{-1})/2, \quad (2)$$

$$G_1(z)=(1-z^{-1})/2. \quad (3)$$

The former has a zero at $z=-1$, and can attenuate a possible replica at the corresponding frequency. The latter applies to the opposite situation. In parallel the second-order interpolator

$$G_2(z)=(1+2\cos\omega_0 z^{-1}+z^{-2})/K \quad (4)$$

has a pair of zeros on the unit circle at $\pi\pm\omega_0$.

As mentioned earlier, the replacement of each delay with L delays produces L periodic replicas of the model filter response. The replicas occur at integral multiples of $1/L$ in the normalized frequency. The above three types of interpolators might be used to suppress all of the unwanted replicas for an arbitrary interpolation factor L. If those interpolators are exclusively applied, the total interpolator composed from the cascaded sections of them may require an extra amount of multiplications, compared to an estimate offered by the IFIR principle. This may spoil the potential efficiency of IFIR filters to some extent.

Instead, to take advantage of interpolation effects, some realistic restrictions have been introduced in the usage of those interpolators. The issue of inconvenience from the restrictions will be dealt in part of the following section, by contrast to a novel approach.

3. Cyclotomic Polynomials as Interpolators

Cyclotomic polynomials $C_k(z)$ arise from a factorization for the polynomial z^K-1 as a product of irreducible polynomials with rational coefficients[11].

$$z^K-1=\prod_{k|K} C_k(z) \quad (5)$$

where $k|K$ denotes that k is a divisor of K. There is one $C_k(z)$ for each divisor k of K, including $k=1$ and $k=K$. The roots of $C_k(z)$ are the primitive kth roots of unity. The number of those roots is given by Euler's function $\varphi(k)$. It is equal to the number of positive integers prime to k and smaller than k. Therefore, $\varphi(k)$ specifies the degree of $C_k(z)$. $C_k(z)$ is defined by

$$C_k(z)=\prod_{(i,k)=1}(z-e^{-j2\pi i/k}) \quad (6)$$

where $(i,k)=1$ denotes that i and k are co-prime.

The cyclotomic polynomials $C_k(z)$ have an interesting property: $C_k(z)$ has coefficients from the set $\{0, 1, -1\}$, if k has no more than two distinct odd prime factors. The smallest integer k with three prime factors is $k=105=3\cdot5\cdot7$. Note that even if k is not less than 105, there exist infinitely a great number of cyclotomic polynomials of which coefficients pertain to 0, 1, -1. For instance, see the case for $k=128$. As long as k is a composite number without more than two odd prime factors, the coefficients of the cyclotomic polynomials are 0, 1, and -1.

These cyclotomic polynomials of course keep their amplitude responses at lower levels smaller than unity around their roots. Hence a cyclotomic polynomial is a

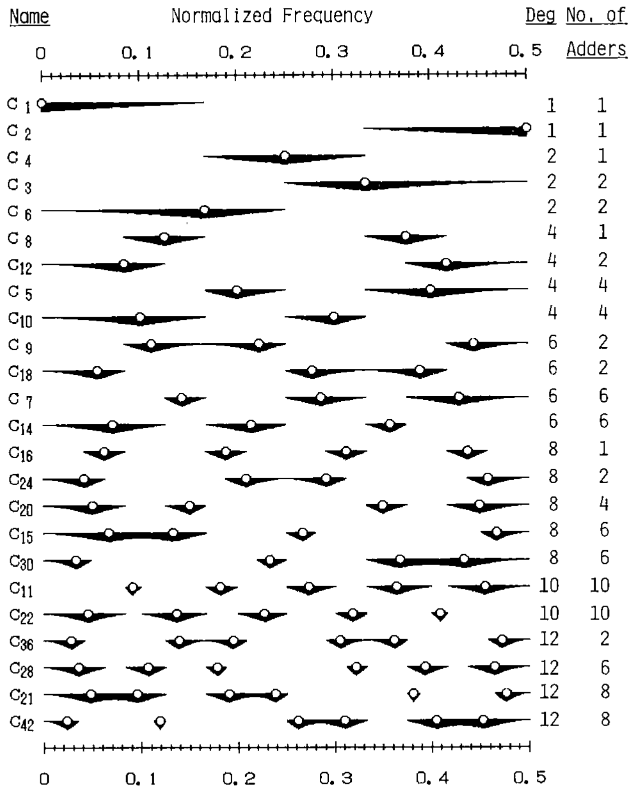

Fig. 3 Suppression bands and root locations of 24 cyclotomic polynomials.

candidate for a multiplication-free interpolator. This observation leads to the introduction of a novel family of efficient and practical interpolators. Each member of the interpolator family is characterized by $C_k(z)$, and the transfer function is of the form

$$C_k(z)z^{-\varphi(k)}. \quad (7)$$

Since this interpolator is based on a polynomial, it is inherently free from recursion. This fact is also preferable to practical applications, because it guarantees the absolute stability and the absence of limit cycle oscillations.

The interpolators based on the cyclotomic polynomials show various amplitude responses. Although the cyclotomic polynomials are infinite in number, a set of 24 members is selected for their simplicity. The explicit forms of them are included in the Appendix. Every interpolator from this set requires no more complexity than ten additions and twelve delay elements.

Heavy segments in Fig. 3 designate the frequency ranges on which the magnitude of each cyclotomic polynomial is smaller than unity. For convenience those ranges are referred to as suppression bands[12],[13]. Notice that the heavier the segment grows, the smaller the magnitude becomes. Light dots indicate the root locations of each polynomial. The figure will enable us to select effective cyclotomic polynomials to attenuate unwanted replicas. One can thus see that a variety of combinations among the family allows us to apply the interpolators to bandpass filters in addition to lowpass/highpass filters without any multiplications.

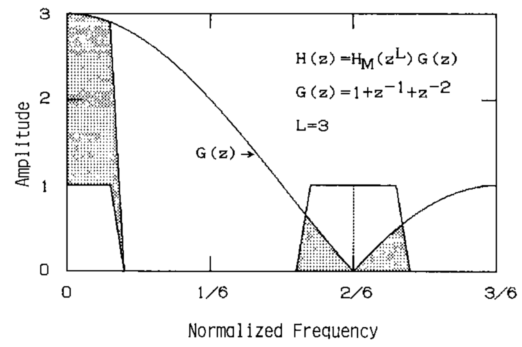

Fig. 4 Illustration of spectrum area suppression.

Figure 4 illustrates how undesired replicas of a model filter, $H_M(z^L)$, folded by L times is suppressed so that suitable interpolators may result in the desired response. The frequency responses of $H_M(z^L)$ and the cyclotomic interpolators are drawn in solid lines. The resulting spectrum shape of $H(z)$ is shaded. Therefore, in order to achieve sufficient attenuation of undesired replicas, it is natural to cascade several kinds of the cyclotomic polynomials as a whole interpolator.

In the original IFIR structures[7], individual interpolators are implemented to do linear and quadratic interpolations between missing samples. Furthermore, a power of two is in fact assigned to an interpolation factor. This usage of the fundamental interpolators fills a non-zero sample at the midpoint of two sampling instances in every successive interpolation stage. By contrast, in the cyclotomic polynomial-based interpolation, one can use different polynomials in addition to multiple use of the same polynomials in a cascade configuration. This means that successive stages perform not only the lower-order interpolations but also higher-order interpolations to produce the desired impulse response. Thereby the facility of IFIR filters can be extended.

4. Design Procedure

The first step to design an IFIR filter starts with selecting a suitable interpolation factor L. While L is preferred as large as possible, one of the replicas of a model filter folded by L times must span the interval over the desired passband and transition band. Thus, from the given stopband frequency, f_s, of a desired lowpass filter, the maximum value for possible interpolation factors is found as

$$L_{\max} = \lfloor 0.5/f_s \rfloor \quad (8)$$

where the brackets denote truncation. By replacing f_s with $0.5-f_s$, Eq. (8) applies to highpass filters.

When a phantom model filter is a lowpass type, its replicas are centered at the normalized frequencies of integral multiples of $1/L$. If one of those frequencies fits the center frequency of the desired IFIR bandpass filter, the lowpass model filter can be used. In this case, L_{max} is specified also by Eq. (8) by replacing f_s with $(f_{s2}-f_{s1})/2$, where f_{s1} and f_{s2} are the lower and the upper edges of the bandpass filter.

Selecting a bandpass type as a model filter alleviates the restriction about the desired center frequency. Equation (8) also applies to this case, by replacing f_s with $f_{s2}-f_{s1}$.

After the calculation of L_{max}, an actual interpolation factor L is assigned to the number smaller than L_{max} among highly composite numbers. Since a composite number is a product of several factors, such a number specifies many cyclotomic polynomials of lower-degree rather than less those of higher-degree. The richness in the members of those polynomials contributes the high facility of selection.

The second step is to compose an efficient set of interpolators characterized by several cyclotomic polynomials. At this step, one can apply Fig. 3 to suppressing the unwanted replicas. Thereby a suitable cyclotomic polynomial is selected in such a way that its suppression band covers the frequencies corresponding to unwanted replicas.

A combination of several cyclotomic polynomials must offer a sufficient attenuation in total. Thus such a selection is repeated, until the successive process achieves the objective. Each suppressing effect is at once evaluated by calculating the amplitude response. At the same time, every effect is successively accumulated into the last one to assure that the total response accumulated up to the current stage is sufficient. Consequently, the cyclotomic polynomials selected in this way constitute the interpolating section $G(z)$. This procedure can be well performed with the aid of computer graphics in a man-machine interactive mode.

The final step is to find the model filter $H_M(z)$ that determines the passband response of the desired IFIR filter. It is necessary for the model filter to compensate the amplitude distortion produced by the interpolator $G(z)$ in the passband. Provided that the computer program for designing FIR filters[14] is used with slight modifications, the designing equation is of the form

$$-\delta \leq W(f)[D(f)-H_M(f)] \leq \delta \tag{9}$$

where δ denotes an unknown deviation that arises from the optimization. $W(f)$ and $D(f)$ represent the weighting function and the desired function, respectively. Both functions are described by the filter specifications, $G(z)$, and L, as follows.

$$W(f)=(\delta_p/\delta_s)G(f/L) \tag{10 a}$$

$$D(f)=\begin{cases}1/G(f/L), & \text{for } f\in B_p \\ 0, & \text{for } f\in B_s\end{cases} \tag{10 b}$$

where δ_p and δ_s denote the ripples in passband and in stopband, respectively. Two sets B_p and B_s are the passband and the stopband of a model filter, respectively. When the model filter is, for example, a lowpass filter specified as a passband edge f_p and a stopband edge f_s, it follows that

$$B_p=[f\ ;\ 0\leq f\leq f_p], \tag{11 a}$$

$$B_s=[f\ ;\ f_s\leq f\leq 0.5]. \tag{11 b}$$

Even though the desired response is a bandpass filter, the corresponding model filter may be designed as a lowpass filter. This choice leads to a significant saving in the arithmetic operations needed. The reasons are twofold; first, the degree of a lowpass filter is roughly half that of the bandpass counterpart. The other is due to the capability of employing the higher interpolation factor that is twice the factor for a bandpass model filter.

In general, it is yet difficult to set one of the replicas of a lowpass model filter at the desired position for an IFIR filter. Selecting a bandpass model filter is still a better strategy to relax the difficulty.

The above three steps gives the two FIR constituents, $G(z)$ and $H_M(z)$, for the desired IFIR filter.

It should be noted that there is the possibility to simplify the primitive set of cyclotomic polynomials selected as the interpolator $G(z)$. The simplification contributes to the reduction of additions. The essence for the simplification can be found from Eqs. (5) and (6), by paying attention to which numbers specify the divisors of K and which is prime to the index k. Some useful properties have been summarized by McClellan and Rader in Ref. (11) and are cited in Appendix B. By means of the simplification, one can find that it is enough to use a few simple cyclotomic polynomials in many cases. Those polynomials are specified by any prime numbers.

5. Design Examples

Several examples are given to demonstrate the effectiveness of the method described in the preceding sections. Table 1 shows all of the specifications of them and the results due to the 8-bit coefficient quantization. Table 2 summarizes each computational complexity. For brevity, the trivial argument (z) for cyclotomic polynomials is suppressed unless otherwise needed. The multiple use of the same polynomial is specified by a super-script.

(Example 1) The first example is a lowpass filter with the same specifications as in Ref. (7), and $L_{max}=8$. Although the IFIR filter obtained by Neuvo et al. has been implemented with the interpolation factor $L=2$, the factor can be increased up to the upper limit of interpolation factors to decrease the multiplications required. Figure 5(a) shows the amplitude response

Table 1 Specifications of the design examples and coefficient quantization error.

Edge Frequencies	Approximation Errors in dB			
Example 1	Conventional	IFIR with L=2	IFIR with L=5	IFIR with L=8
pass(0.0000, 0.0404)	0.41(0.46)	0.41(0.52)	0.44(0.60)	0.42(0.87)
stop(0.0556, 0.5000)	-32.6 (-30.4)	-32.3 (-32.2)	-31.8 (-30.8)	-32.0 (-32.0)
Example 2	Conventional	IFIR with L=6	IFIR with L=8	IFIR with L=10
pass(0.0000, 0.0250)	0.20(0.23)	0.17(0.49)	0.20(0.48)	0.32(0.85)
stop(0.0500, 0.5000)	-60.7 (-44.9)	-61.2 (-50.6)	-61.3 (-61.1)	-60.2 (-61.4)
Example 3	Conventional	IFIR with L=4	IFIR with L=5	IFIR with L=8
stop(0.0000, 0.0200)	-27.8 (-24.9)	-27.8 (NA)	-29.4 (-28.7)	-30.0 (-32.1)
pass(0.0300, 0.0500)	1.54(1.54)	1.65(NA)	1.00(1.04)	1.13(1.15)
stop(0.0600, 0.5000)	-27.8 (-24.1)	-27.8 (NA)	-28.6 (-28.6)	-27.7 (-27.7)
Example 4	Conventional	IFIR with L=20		
pass(0.0000, 0.0100)	0.17(0.25)	0.17(0.18)		
stop(0.0195, 0.5000)	-40.1 (-35.9)	-40.1 (-40.6)		
Example 5	Conventional	IFIR with L=15	IFIR with L=30	
stop(0.0000, 0.3183)	-49.2 (-29.4)	-49.1 (-44.6)	-53.4 (-53.4)	
pass(0.3283, 0.3383)	0.08(0.08)	0.10(0.23)	0.92(0.76)	
stop(0.3483, 0.5000)	-49.2 (-44.5)	-48.9 (-46.6)	-49.0 (-49.0)	

In a pair of figures drawn as ###(###), the leading and the following in parentheses denote the approximation errors using the coefficients in infinite wordlength and 8-bit, respectively.

Table 2 Comparison on the computational complexity.

	Conventional			IFIR with L=2			IFIR with L=5			IFIR with L=8		
Example 1	M	A	D	M	A	D	M	A	D	M	A	D
Model Filter				25	48	96	10	19	95	8	14	112
Interpolator				0	2(2)	2	0	12(12)	12	0	12(12)	36
Total Filter	50	98	98	25	50(50)	98	10	31(31)	107	8	26(26)	148

	Conventional			IFIR with L=6			IFIR with L=8			IFIR with L=10		
Example 2	M	A	D	M	A	D	M	A	D	M	A	D
Model Filter				9	17	102	7	12	96	5	8	80
Interpolator				0	12(24)	24	0	13(21)	36	0	20(36)	57
Total Filter	55	108	108	9	29(41)	126	7	25(33)	132	5	28(44)	137

	Conventional			IFIR with L=4			IFIR with L=5			IFIR with L=8		
Example 3	M	A	D	M	A	D	M	A	D	M	A	D
Model Filter				NA	NA	NA	12	22	110	8	14	112
Interpolator				NA	NA	NA	0	10(16)	16	0	12(26)	36
Total Filter	56	110	110	18	30	110	12	32(38)	126	8	26(40)	148

	Conventional			IFIR with L=20		
Example 4	M	A	D	M	A	D
Model Filter				6	10	200
Interpolator				0	21(53)	67
Total Filter	111	220	220	6	31(63)	267

	Conventional			IFIR with L=15			IFIR with L=30		
Example 5	M	A	D	M	A	D	M	A	D
Model Filter				9	16	240	5	8	240
Interpolator				0	15(34)	40	0	23(85)	121
Total Filter	131	260	260	9	31(50)	280	5	31(93)	361

M, A, and D stand for the numbers of multiplications, additions, and delays, respectively.
( ) denotes the number of additions before simplifying the sets of cyclotomic polynomials.

with $L=5$. The interpolator to this case is a triple cyclotomic polynomial C_5.

Figure 6 illustrates that the combination of the model filter with $L=8$ and the interpolator produces the desired IFIR filter. The interpolator consists of C_8^7, C_4^3, and C_2^2.

As can be seen from Table 2, the implementation with $L=5$ has gained a considerable saving in arithmetic operations at the expense of a slight increase of delay operations. While the highest value for L results in the

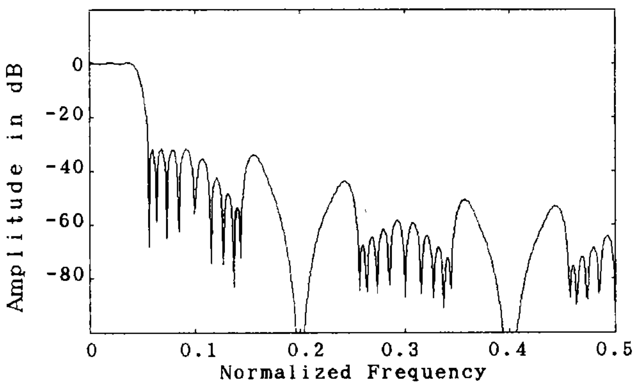

Fig. 5 Example 1. Amplitude response of IFIR filter with $L=5$.

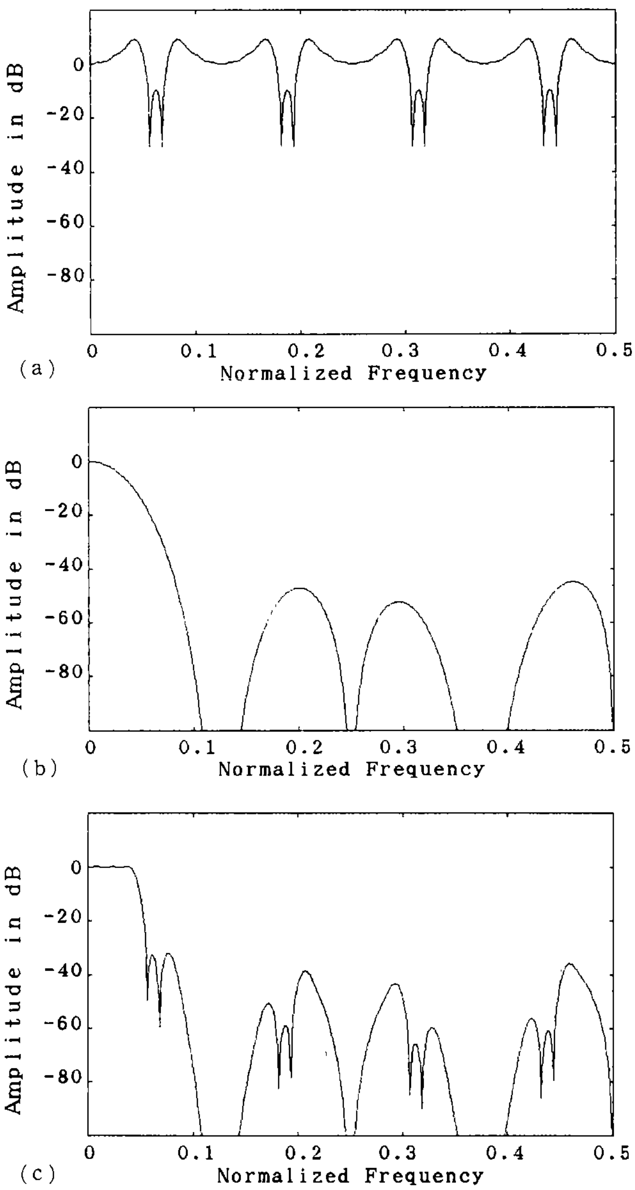

Fig. 6 Amplitude responses of (a) model filter, (b) interpolator, (c) IFIR filter with $L=8(=L_{\max})$ for the first example.

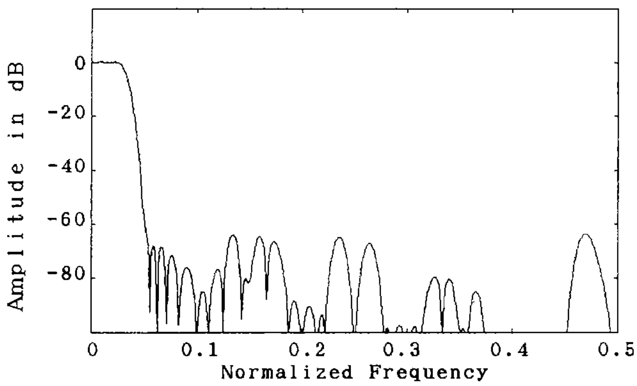

Fig. 7 Example 2. Amplitude response of IFIR lowpass filter with $L=10(=L_{\max})$.

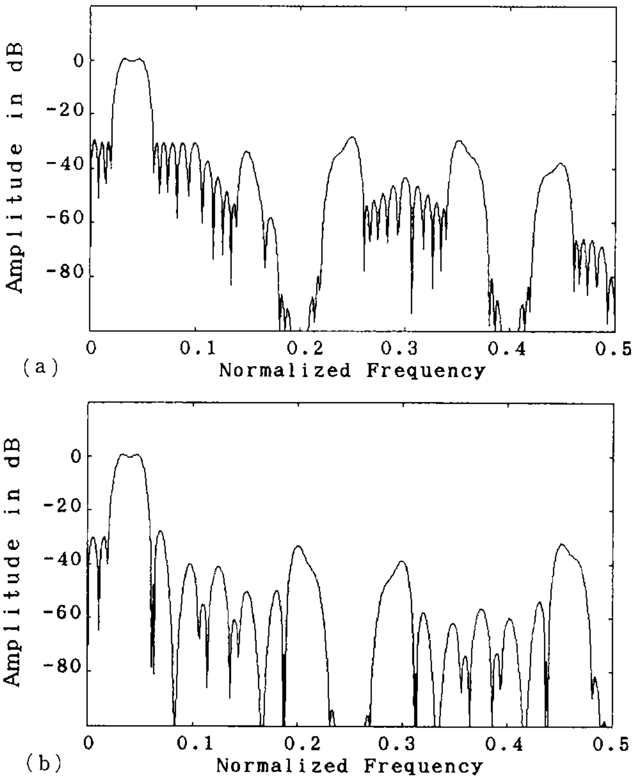

Fig. 8 Example 3. Amplitude responses of IFIR bandpass filters with (a) $L=5$, (b) $L=8$.

fewest multiplications, this leads to much more delay elements.

(Example 2) The lowpass filter as the second example has been also dealt with in Ref. (9). Figure 7 shows the response of the IFIR filter with $L=10$ that is the permissible limit to this example. The cyclotomic interpolator comprises C_{16}, C_8, $C_7(z^2)$, $C_5(z^2)$, $C_3(z^6)$, $C_3(z^2)$, $C_2(z^6)$, and C_2^3.

The computational complexity for this implementation is listed in Table 2 in addition to the data for different implementations. Those additional data can be drawn by applying $C_3(z^2)^4$, $C_2(z^3)^2$, and C_2^2 for $L=6$, and C_8^3, C_4^3, $C_3(z^2)$, $C_2(z^6)$, $C_2(z^5)$, and C_2^3 for $L=8$. By comparing those with the results by Saramäki et al.[9],

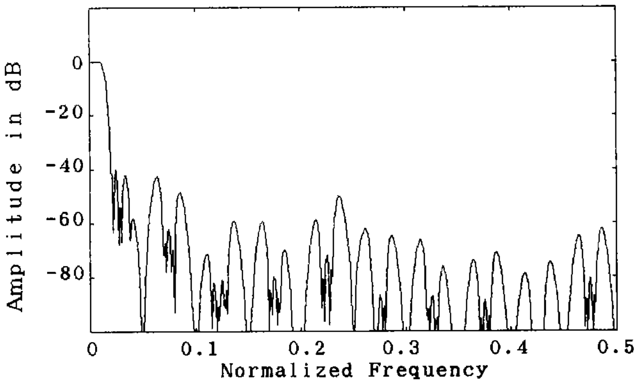

Fig. 9 Example 4. Amplitude response of IFIR lowpass filter with $L=20$.

one can find that all of them are competitive to each other. The efficiency will have to be determined by a full implementation for individual applications.

(Example 3) From a bandpass model filter, a bandpass IFIR filter has been designed with increased interpolation factors than 4. Whereas the factor 4 was used by Neuvo et al., it has been selected as $L=5$ and $L=8$. Although the virtual L_{max} is 12 according to Eq. (8), the real L_{max} reduces to 8 because of the requirement of a passband position.

For $L=5$, the interpolator based on the cyclotomic polynomials has its factors such as C_5^2, $C_2(z^3)$, and $C_1(z^5)$. The designed IFIR filter has the amplitude response shown in Fig. 8 (a). By making a comparison between the data in Table 2 and those in Ref. (7), one can see that the designed filter requires 12 multiplications fewer than 18 in Ref. (7). The price payed for the reduction amounts to the increase of delays by 15 %.

As for the case of $L=8$, the interpolator is factored as $C_3(z^2)^3$, $C_2(z^6)^3$, $C_1(z^4)$, and C_2^2. The resulting amplitude response is shown in Fig. 8 (b).

(Example 4) A very narrow-band lowpass filters has been designed as an IFIR filter. In this case, $L_{max}=25$, and L has been determined as 20. The interpolator for this example comprises C_{20}, $C_5(z^2)^3$, $C_2(z^{13})$, $C_2(z^{10})^2$, and C_2^2. Figure 9 shows the amplitude response of the IFIR filter, and Table 2 proves its prominent efficiency.

(Example 5) The final example is a very narrow-band IFIR bandpass filter with $L=15$. It has been designed from a lowpass model filter having the permissible interpolation factor $L_{max}=33$. The largest factor is of course acceptable for the cyclotomic polynomial based interpolators. However, this introduces much more delays than those in the conventional FIR filter.

The factors of the interpolator used are $C_5(z^3)^3$, $C_1(z^2)$, and C_1^2. Figure 10 shows the amplitude responses of the model filter with the sampling rate increased by 15, the interpolator, and the total IFIR filter. The comparison with the conventional FIR filter is listed in Table 2. It is evident that a desirable trade-off has been achieved in terms of the computational complexity.

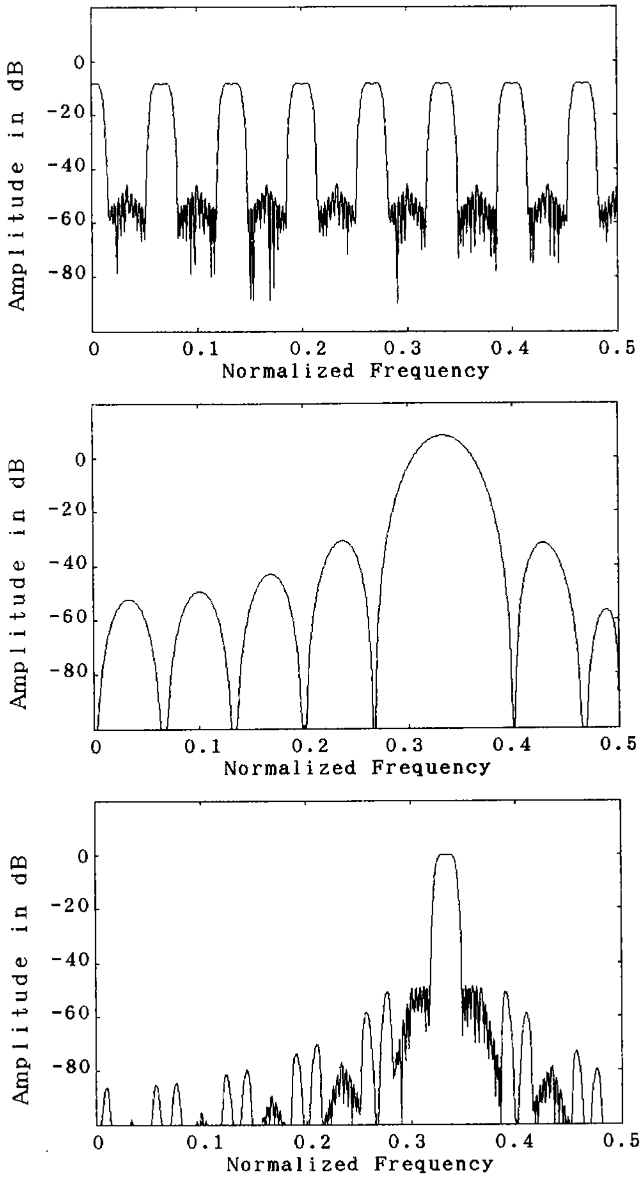

Fig. 10 Example 5. Amplitude responses of (a) model filter with L times sampling rate, (b) interpolator, (c) IFIR filter with $L=15$.

6. Discussion

Finite wordlength properties of IFIR filters have been already analyzed in Ref. (7) in terms of the standard deviation of the error in the frequency response due to coefficient quantization and output roundoff noise variance. The results are right if the length of a model filter is long. All of them also apply to the cyclotomic polynomial based IFIR filters.

However, as the interpolation factor goes high, the model filter can be implemented with a short length of its impulse response. The high interpolation factor may destroy the assumption of the statistical analysis, especially regarding the frequency response degradation caused by coefficient quantization.

Table 1 shows the degradation behavior due to the 8-bit coefficient quantization in fixed-point binary representation. The number of bits includes the sign bit and the coefficients have been scaled by the greatest one to fit its effective figures for 8 bits.

As seen from Table 1, the coefficient quantization property of IFIR filters is superior than that of conventinal FIR filters in a stopband, but the situation is opposite in a passband.

In general, a cyclotomic polynomial based interpolator has a single peak in the desired passband, because a vacancy of a equi-spaced zero on the unit circle controls the spectral shape. As the interpolation factor increases, the distance between the zeros next to the vacancy must be shortened. Deeper attenuation requires multiple use of similar interpolators in a cascade configuration. Both facts make the interpolator peak sharp. Therefore the corresponding model filter must compensate the sharp droop in the passband. One can observe this effect in Fig. 6 (a). The model filter should realize stringent zeros closer to the unit circle. This is the reason why the coefficient sensitivity of the IFIR filters requiring very small amount of multiplications is poor in a passband.

The poor behavior will be overcomed by introducing a spectral shaping section having the structurely-insensitive property as in the improved prefilter-equalizer design[15].

As for the stopband, there are no considerable disadvantages, because an interpolator attenuates the undesired replicas of the model filter in a stopband.

Finally miscellaneous issues related to hardware implementations are added. Since a cyclotomic polynomial based interpolator has a large gain constant, an additional multiplier should be inserted between the model filter section and the interpolator to adjust the passband gain to be unity. Otherwise the internal signal wordlength must be increased. Note that there is the need for the extra multiplier even in the conventional FIR filters, if the internal signal wordlength is fully used. This means that the signal wordlength required in IFIR filter implementations are same with that in conventional FIR filters.

Several examples dealt in this paper reveals that there are many variations in interpolated FIR digital filters. Those variations are competitive to each other in terms of the computational complexity. In the hardware design for implementing a practical IFIR filter, one of the variations has to be selected by the specific design considerations. The decision is an open problem and will depend on future studies.

7. Conclusion

An efficient and practical family of interpolators has been described for interpolated FIR digital filters. That is characterized by the cyclotomic polynomials, and inherits the multiplication-free and recursion-free properties. The family is applicable to bandpass filters as well as lowpass/highpass filters with no multiplications. This extends the potential facility of IFIR filters. In addition, the described approach together with the interpolator family permits to employ a larger interpolation factor. As a result, the IFIR filters can afford to gain a further saving with respect to arithmetic operations.

Finite wordlength properties in the implementations with larger interpolation factors and hence with the model filters having rather short length have been discussed with several examples.

Further improvements will be gained by introducing a spectral shaping section.

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Appendix A

The explicit formulas of the 24 cyclotomic polynomials are listed for a practical use with their alternative expressions if available.

$$C_1(z)=z-1$$
$$C_2(z)=z+1$$
$$C_4(z)=z^2+1=C_2(z^2)$$
$$C_3(z)=z^2+z+1$$
$$C_6(z)=z^2-z+1=C_3(-z)$$
$$C_8(z)=z^4+1=C_2(z^4)$$
$$C_{12}(z)=z^4-z^2+1=C_3(-z^2)$$
$$C_5(z)=z^4+z^3+z^2+z+1$$
$$C_{10}(z)=z^4-z^3+z^2-z+1=C_5(-z)$$
$$C_9(z)=z^6+z^3+1=C_3(z^3)$$
$$C_{18}(z)=z^6-z^3+1=C_3(-z^3)$$
$$C_7(z)=z^6+z^5+z^4+z^3+z^2+z+1$$
$$C_{14}(z)=z^6-z^5+z^4-z^3+z^2-z+1=C_7(-z)$$
$$C_{16}(z)=z^8+1=C_2(z^8)$$
$$C_{24}(z)=z^8-z^4+1=C_3(-z^4)$$
$$C_{20}(z)=z^8-z^6+z^4-z^2+1=C_5(-z^2)$$
$$C_{15}(z)=z^8-z^7+z^5-z^4+z^3-z+1$$
$$C_{30}(z)=z^8+z^7-z^5-z^4-z^3+z+1=C_{15}(-z)$$
$$C_{11}(z)=z^{10}+z^9+z^8+z^7+z^6+z^5+z^4+z^3+z^2+z+1$$
$$C_{22}(z)=z^{10}-z^9+z^8-z^7+z^6-z^5+z^4-z^3+z^2-z+1$$
$$=C_{11}(-z)$$
$$C_{36}(z)=z^{12}-z^6+1=C_3(-z^6)$$
$$C_{28}(z)=z^{12}-z^{10}+z^8-z^6+z^4-z^2+1=C_7(-z^2)$$
$$C_{21}(z)=z^{12}-z^{11}+z^9-z^8+z^6-z^4+z^3-z+1$$
$$C_{42}(z)=z^{12}+z^{11}-z^9-z^8+z^6-z^4-z^3+z+1$$
$$=C_{21}(-z)$$

Appendix B[11]

[Theorem 1] If p is a prime,

$$C_p(z)=z^{p-1}+z^{p-2}+\cdots+z+1.$$

[Theorem 2] For n odd and $n\geq 3$,

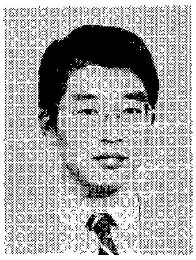
$$C_{2n}(z)=C_n(-z).$$

[Theorem 3] For any choice of the integers m and n,

$$C_{mn^k}(z)=C_{mn}(z^{n^{k-1}}).$$

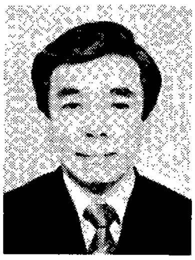
[Theorem 4] If p is prime, and p does not divide m,

$$C_m(z^p)=C_{pm}(z)C_m(z).$$

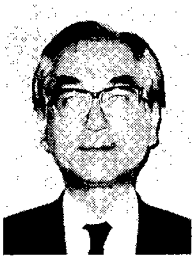


Hisakazu Kikuchi was born in Niigata Prefecture, Japan, on March 16, 1952. He received the B. E. and M. E. degrees in electronic engineering from Niigata University, Niigata in 1974 and 1976, respectively. From 1976 to 1979 he worked at the System Laboratory, FUJITSU LIMITED, Tokyo. Since 1979, he has been a Research Associate at the Department of Electronic Engineering, Niigata University. His research interests are in digital signal processing, CAD for digital filters, and circuit theory. He is a member of IEEE.

Yoshito Abe was born in Niigata, Japan, on May 28, 1962. He received the B. E. degree in electronic engineering from Niigata University, Niigata in 1985. From 1986 he has been a postgraduate student at the Department of Electronic Engineering, Niigata University. His research interests are in digital signal processing and CAD for digital filters.



Hiromichi Watanabe was born in Hokkaido, Japan, on August 22, 1943. He received the M. E. and Dr. Eng. degrees in electronic engineering from Tokyo Institute of Technology, Tokyo in 1969 and 1974, respectively.
In 1974, he joined Niigata University, Niigata, where he is now an Associate Professor of Electronic Engineering. He has engaged in research of circuit theory and electronic circuits.
Dr. Watanabe is a member of the Institute of Electrical Engineers of Japan.



Takeshi Yanagisawa was born in Nagasaki Prefecture, Japan, on March 14, 1931. He received the B. E., M. E. and Dr. Eng. degrees in electronic engineering from Tokyo Institute of Technology, Tokyo in 1953, 1955 and 1958, respectively. He joined the Tokyo Institute of Technology in 1958, and currently, he is a Professor at the Department of Physical Electronics. From 1980 to 1986, he also served as Director of the International Cooperation Center for Science and Technology at the same Institute. His research interests are in the fields of circuit theory, electronic circuits, and CAD for electronic circuits.
The recipient of the 1986 Achievement Award of IECE Japan, Dr. Yanagisawa is a member of the Institute of Electrical Engineers of Japan.