THE TRANSACTIONS OF THE IEICE, VOL. E 72, NO. 4 APRIL 1989

PAPER

Systematic Synthesis of Power-Wave Digital Filters

Hisakuzu KIKUCHI†, Hiromichi WATANABE†, Akinori NISHIHARA†† *and* Takeshi YANAGISAWA†††, *Members*

SUMMARY A systematic synthesis is presented to realize any digital filter into a power-wave digital filter. After three canonical matrix representations are introduced, a set of key concepts which comprises cascade interconnection of digital two-ports, pole localization, and computability is presented for the canonical cascade synthesis of lossless digital two-ports. The synthesis procedure consists of global decomposition and local decomposition. The procedure is so general as to give a unified solution to arbitrary frequency responses realization, and is so useful as to find new circuit structures. The synthesized circuits are of robustness and modularity. An illustrative example is included.

1. Introduction

Since wave digital filters (WDFs) were presented by Fettweis[(1)-(3)], many researches have been done to build low-sensitivity digital filters based on the matching concept in power transfer scheme. This is because a WDF bears many excellent properties such as low sensitivity to coefficient quantization, low round-off noise, wide dynamic range, good stability against inevitable nonlinearities caused by finite-precision arithmetic, and the absence of limit cycle oscillations[(2),(3)].

The design of conventional WDFs is indirect ; it is based on digital simulation of analog reference filters by means of wave quantities. Reference filters have to be synthesized by conventional classical network theory. On the other hand, it is difficult to find a WDF realization for some class of transfer functions which are arbitrarily specified in the digital domain. For instance, such difficulty appears when linear phase or group delay response is considered.

In contrast, direct synthesis of WDFs has a considerable significance. This is because a direct procedure calls for neither analog filter theory nor classical network theory. The second reason can be attributed to the difference of manufacturing considerations between analog passive circuit technology and digital circuit technology. Some of the disciplines intended in the former technology are the distinction of frequency band to be handled by lumped or distributed networks, and the minimization of the number of cumbersome circuit components such as ideal transformers, circulators, or nonreciprocal circuit components. On the other hand, digital circuit technology gives higher priority to modularity and throughput rate as well as the minimization of computational complexity. There is no assurance for good circuits in the context of analog passive circuit technology to be so again for digital implementation.

There are only a limited number of direct procedures for WDF systhesis. Orthogonal digital filters presented by Deprettere and Dewilde[(4)-(6)] are the first contribution, and are classified into power-wave digital filters. Their theory is given in highly algebraic and abstract description. The actual computation in the realization procedure requires Givens' algorithm[(7)] for matrix triangularization.

Lossless bounded-real (LBR) theory has been presented by Vaidyanathan and Mitra[(8)-(10)]. The synthesis procedure is characterized by 'one-removal' : the extraction of 'one' on the unit circle from an LBR input function. In their LBR approach, lossy and nonreciprocal two-pair sections are used to realize a general transfer function. Hence exactly speaking, the theory addresses the synthesis of single-input single-output digital filters rather than lossless digital two-ports, and is not suitable for branching (directional) filter applications[(3)].

The authors have described a direct synthesis of power-wave digital filters on the basis of localization of loss poles[(11)-(13)]. A universal circuit structure as a basic section has been derived definitely. The synthesis leads to pipelinable realizations and branching filters realization. However, the synthesis procedure is based on the global decomposition merely characterized by transmission zeros. The procedure requires the preamble operation, and this is the obstacle to get a canonical realization. Moreover, the synthesized filters guarantee external passivity, but lack internal passivity, if a loss pole exists off the unit circle.

To overcome the disadvantage and to improve modularity, this paper addresses canonical cascade synthesis of lossless digital two-ports[(14),(15)]. The synthesis is described by three sorts of scattering representations,

Manuscript received September 30, 1988.
Manuscript revised December 28, 1988.
† The authors are with the Faculty of Engineering, Niigata University, Niigata-shi, 950-21 Japan.
†† The author is with International Cooperation Center for Science and Technology, Tokyo Institute of Technology, Tokyo, 152 Japan.
††† The author is with the Faculty of Engineering, Tokyo Institute of Technology, Tokyo, 152 Japan.

and is systematically performed by a set of tools. The set consists of three sorts of cascade interconnections, pole-localization principle, and the computability condition at interconnection junctions. The synthesis procedure can be divided into two major steps : global synthesis and local synthesis. Global synthesis is characterized by the factorization of three kinds of scattering matrices. On the other hand, local synthesis is characterized by both creation of special poles at zero and infinity and their localization.

This paper is a generalization of Ref.(13), and yields a systematic way to power-wave digital filters which are suitable for CORDIC implementations rather than conventional multiplication-add arithmetics. Moreover, a new realization of a general second-degree section is given in addition to a design example.

2. Scattering Description for Digital Two-Ports

A linear shift-invariant digital two-port is described by a scattering matrix

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \boldsymbol{S}(z) \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad (1)$$

where X_i and Y_i $(i=1,\ 2)$ indicate the input and output at the i th port, respectively. All entries in $\boldsymbol{S}(z)$ are rational functions of z with real coefficients.

The two-port is schematically drawn in Fig. 1. The figure is suitable to represent the transmission and reflection of signals in ladder configuration and the global structure of digital two-ports for digital filtering. Hence only this schematic is used throughout this paper.

As preliminaries, some definitions are given.

[Definition 1] (Causality and Stability) A causal and stable discrete-time transfer function has no poles on the unit circle and at any exterior point.

[Definition 2] (Passivity) A digital two-port is passive, if and only if

(1) Every entry of $\boldsymbol{S}(z)$ is analytic in $|z|>1$, and

(2) $\boldsymbol{S}^*(z)\boldsymbol{S}(z) \leqq \boldsymbol{I}$, for $|z| \geqq 1$ (2)

where the upper asterisk denotes transposed conjugate, and $\boldsymbol{I}=$ diag $\{1, 1\}$.

[Definition 3] (Losslessness) If a digital two-port is lossless,

$$\boldsymbol{S}_*(z)\boldsymbol{S}(z)=\boldsymbol{I}, \quad \text{for all } z \quad (3)$$

where the lower asterisk stands for para-conjugation : $\boldsymbol{S}_*(z)=\boldsymbol{S}^*(1/z^*)$.

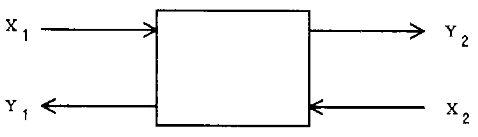


Fig. 1 Block diagram for a digital two-port.

A scattering matrix always exists for an actual digital two-port, because Eq.(1) physically represents the propagation of signals from inputs to outputs. A digital two-port described by Eq.(1) maps an input vector $[X_1,\ X_2]^t$ to an output vector $[Y_1,\ Y_2]^t$, where the superscript indicates tranposition. To the same two-port, if a different pair of vectors is used, another equivalent mapping can be obtained. If S_{21} is not identically zero, another representation

$$\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = \boldsymbol{T}(z) \begin{bmatrix} Y_2 \\ X_2 \end{bmatrix} \quad (4)$$

is algebraically obtained with respect to the same digital two-port. This matrix is referred to as a transfer scattering matrix. The losslessness defined by Eq.(3) as paraunitariness is described in terms of $\boldsymbol{T}(z)$ as

$$\boldsymbol{J}\boldsymbol{T}_*(z)\boldsymbol{J}\boldsymbol{T}(z)=\boldsymbol{I}, \quad \text{for all } z \quad (5)$$

where $\boldsymbol{J}=$ diag $\{1, -1\}$. Such a notation was introduced by Belevitch[16] in classical network theory, and is called J-losslessness after Deprettere and Dewilde[4].

Similarly, if S_{11} is not identically zero, the same two-port may be represented by

$$\begin{bmatrix} X_1 \\ Y_2 \end{bmatrix} = \boldsymbol{H}(z) \begin{bmatrix} Y_1 \\ X_2 \end{bmatrix} \quad (6)$$

where

$$\boldsymbol{J}\boldsymbol{H}_*(z)\boldsymbol{J}\boldsymbol{H}(z)=\boldsymbol{I}, \quad \text{for all } z \quad (7)$$

which we call a hybrid scattering matrix. Algebraic formulas for the conversion between those matrices are summarized in Table 1. Regarding $\boldsymbol{T}$ and $\boldsymbol{H}$, the term 'scattering' will be suppressed for brevity.

With these definitions above, one can prove the following theorem which characterizes a lossless digital two-port.

[Theorem 1] (Canonical Form) The scattering matrix $\boldsymbol{S}(z)$ of a causal, stable, lossless, digital two-port has a

Table 1 Algebraic conversion of scattering description.

	S	T	H
S	$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$	$\frac{1}{T_{11}}\begin{bmatrix} T_{21} & \lvert T\rvert \\ 1 & -T_{12} \end{bmatrix}$	$\frac{1}{H_{11}}\begin{bmatrix} 1 & -H_{12} \\ H_{21} & \lvert H\rvert \end{bmatrix}$
T	$\frac{1}{S_{21}}\begin{bmatrix} 1 & -S_{22} \\ S_{11} & -\lvert S\rvert \end{bmatrix}$	$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$	$\frac{1}{H_{21}}\begin{bmatrix} H_{11} & -\lvert H\rvert \\ 1 & -H_{22} \end{bmatrix}$
H	$\frac{1}{S_{11}}\begin{bmatrix} 1 & -S_{12} \\ S_{21} & \lvert S\rvert \end{bmatrix}$	$\frac{1}{T_{21}}\begin{bmatrix} T_{11} & -\lvert T\rvert \\ 1 & -T_{22} \end{bmatrix}$	$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$

unique representation

$$S(z)=\frac{1}{G(z)}\begin{bmatrix} K(z) & uF_*(z)z^N \\ F(z) & -uK_*(z)z^N \end{bmatrix} \quad (8)$$

which has the following properties.
(1) $G(z)$, $F(z)$, and $K(z)$ are polynomials of z with real coefficients.
(2) $G(z)$ is Hurwitz in the z-plane, and is monic, i. e. its leading coefficient is unity.
(3) N is a certain integer which is expressed by

$$N=\deg |S(z)| \quad (9)$$

where the right-hand side denotes degree of the determinant of $S(z)$.
(4) u is a unimodular constant, i. e. 1 or -1.
(5) $G(z)$, $F(z)$, and $K(z)$ are related by

$$G_*(z)G(z)=F_*(z)F(z)+K_*(z)K(z). \quad (10)$$

The form of Eq. (8) is referred to as the canonical form of a scattering matrix, and a triplet of $G(z)$, $F(z)$, and $K(z)$ is called canonical polynomials[(16),(17)]. Property (3) may seem to be self-contradictory, but it is not the case ; one can evaluate $|S(z)|$ from a given scattering matrix which may be noncanonical. Although thorough treatment on this respect is omitted here, a simple example concerning this issue is given by

$$S(z)=\frac{1}{z}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (11)$$

and is shown in Fig. 2. The proofs for three theorems described in this paper are also omitted, but they are found in Ref. (18).

Regarding the same two-port treated in Theorem 1, if $F(z)$ or $K(z)$ is not identically zero, the canonical form of a transfer matrix or hybrid matrix is obtained by algebraic conversion, as follows,

$$T(z)=\frac{1}{F(z)}\begin{bmatrix} G(z) & uK_*(z)z^N \\ K(z) & uG_*(z)z^N \end{bmatrix} \quad (12)$$

or

$$H(z)=\frac{1}{K(z)}\begin{bmatrix} G(z) & -uF_*(z)z^N \\ F(z) & -uG_*(z)z^N \end{bmatrix}, \quad (13)$$

respectively.

These canonical forms obey the next theorem.

[Theorem 2] Regarding each matrix representation among $S(z)$, $T(z)$, and $H(z)$, a product of two canonical forms is also canonical.

Finally, a trivial comment is given. Any sign of unimodular constant u is permitted in this theory. Nevertheless, if $F_*(z)z^N$ is the mirror image or antimirror image polynomial of $F(z)$, we also apply the convention in classical network theory[(16),(17)] to this sign choice, i. e.

$$u=F_*(z)z^N/F(z). \quad (14)$$

This choice leads to the relationship, $|T(z)|=1$, and it is said that such a two-port is reciprocal.

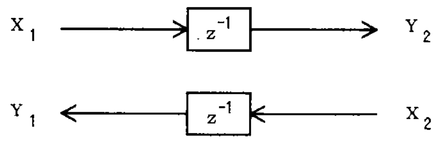


Fig. 2 Lossless digital two-port defined by Eq. (11).

3. Conservation and Creation of Feedback Structures

By Theorem 2, a product of two canonical forms $S_a(z)$ and $S_b(z)$ gives a canonical form of a new scattering matrix

$$S(z)=S_a(z)S_b(z). \quad (15)$$

$S(z)$ is realized by the cascade interconnection as shown in Fig. 3. If $S_a(z)$ and $S_b(z)$ have no common factor in denominator, the poles of $S(z)$ are distributed to and are fixed at $S_a(z)$ and $S_b(z)$. Each member of those poles is locally realized by each internal feedback structure of $S_a(z)$ and $S_b(z)$. The interconnection specified by Fig. 3 never create a new feedback structure. This interconnection conserves two feedback structures in respective circuits, and will be referred to as S-cascade interconnection[(14),(15)].

By contrast, the numerators of the entries in $S(z)$ can be realized only by the result of the interconnection of $S_a(z)$ and $S_b(z)$, because they are the product-sums of those in $S_a(z)$ and $S_b(z)$.

A product of the canonical forms of two transfer matrices is written by

$$T(z)=T_a(z)T_b(z) \quad (16)$$

and is realized by the cascade interconnection shown in Fig. 4 which we call T-cascade interconnection. The T-cascade structure locally realizes the factors of $F(z)$ within $T_a(z)$ and $T_b(z)$. Note that factors of $F(z)$ are the poles of the transfer matrix $T(z)$. On the contrary to

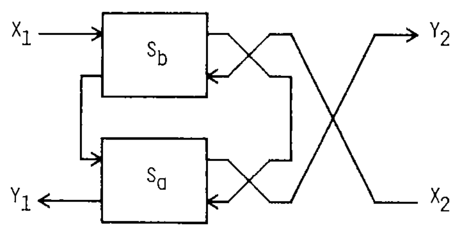


Fig. 3 S-cascade interconnection.

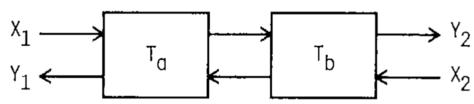


Fig. 4 T-cascade interconnection.

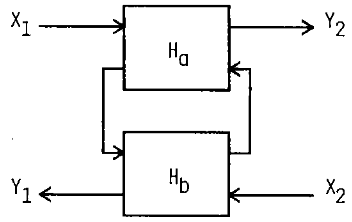


Fig. 5 H-cascade interconnection.

the case for the S-cascade interconnection, whenever two lossless digital two-ports are interconnected by the T-cascade interconnection, a new feedback structure is necessarily created. Alternately speaking, even though both sections to be cascaded have no feedback loops, T-cascade interconnection can create a feedback loop.

Owing to the newly created feedback structure, one must, however, be careful about whether this feedback is computable or not. Computability corresponds to the local, physical realizability of interconnected networks, and is different from the global, physical realizability that means causality involved in a given transfer function. If a product of two transfer matrices is computable, the corresponding digital two-port is physically realizable. Note that the computability problem does not arise in S-cascade interconnection.

In parallel to the case of transfer matrices, a product of the canonical forms of two hybrid matrices, described by

$$\boldsymbol{H}(z)=\boldsymbol{H}_a(z)\boldsymbol{H}_b(z) \tag{17}$$

finds the H-cascade interconnection as shown in Fig. 5. The H-cascade structure independently realizes the factors of $K_a(z)$ and $K_b(z)$ in $\boldsymbol{H}_a(z)$ and $\boldsymbol{H}_b(z)$, respectively. Furthermore, H-cascade interconnection necessarily creates a new feedback structure, even though two sections to be cascaded are devoid of feedback structures. Like the case of transfer matrices, computability must be guaranteed at the junction of interconnection.

The above observation suggests that a cascade interconnection of lossless digital two-ports is characterized by local or independent realization of a certain quantity, which may be $G(z)$, $F(z)$, and $K(z)$ in the case of $\boldsymbol{S}(z)$, $\boldsymbol{T}(z)$, and $\boldsymbol{H}(z)$, respectively. Such an independent realization is provided by the factorization of poles in appropriate scattering descriptions. Conversely, cascade systhesis of lossless digital two-ports may be described by the decomposition of appropriate poles into individual two-ports under leaving the decomposed fractions being still the members of the original poles. The strategy stated above is referred to as the pole-localization[(14),(15)].

Cascade interconnection of digital two-ports, pole localization, and computability are a set of key concepts for the cascade synthesis of lossless digital two-ports presented in this paper. Three sorts of interconnections presented here are essential to represent cascade interconnection of lossless digital two-ports, because each product of $\boldsymbol{S}(z)$, $\boldsymbol{T}(z)$, or $\boldsymbol{H}(z)$ represents the unique factorization of canonical polynomials which charcterize lossless digital two-ports. One can define additional matrix descriptions each of which denominator is G_*z^N, F_*z^N, or K_*z^N. Yet, there is no need to employ those descriptions for the application of lossless digital two-port synthesis. This is because other scattering descriptions derived by the exchange of numerator entries can be mathematically expressed by the combination of the basic three matrices and a permutation matrix defined by

$$\boldsymbol{P}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}. \tag{18}$$

By the way, the pole localization demands the decomposition of $\boldsymbol{S}(z)$, $\boldsymbol{T}(z)$, and $\boldsymbol{H}(z)$ on the basis of respective poles. The next factorization theorem proved using the minimum real-part theorem[(16)] guarantees the passivity of two matrices obtained from the factorization of a given canonical form.

[Theorem 3] If a given canonical form is factored into two lossless matrices without higher-degrees, they are stable.

This also implies that passivity is ensured, even if a matrix factor is constant.

4. Global Synthesis

4.1 Preliminaries

A prescribed rational tranfer function, $H(z)$, with real cofficients can be embedded in any entry of a scattering matrix. For example, if we identify $H(z)$ with $S_{21}(z)$, the canonical form is found by Theorem 1. Through such embedding, the realization problem of a transfer function is restated as the synthesis problem of a lossless digital two-port. Indeed, embedding is possible, if

$$\max\{|H(z)|^2\}=1, \text{ for } |z|=1 \tag{19}$$

is satisfied. The unimodular-bounded property is the necessary and sufficient condition for the existence of a set of canonical polynomials with real coefficients.

Cascade synthesis of lossless two-ports can still do more than for low passband-sensitivity. In classical network theory, it is a fact that a doubly-terminated reactance ladder network can realize a frequency-selective filter with high precision. The first reason is due to matching in power transfer, and this contributes to the low passband-sensitivity. The other is the independent realization of transmission zeros by each arm in the ladder. This in turn contributes to the stopband immunity against parameter variations. According to this lesson, if a prescribed transfer function is embedded into a transmission coefficent, S_{21}, it is reasonable to solve the synthesis problem by means of global factorization of transfer matrices that results in T-cascade interconnection.

Equivalently, one can use another factorization to the same problem. If a transfer function is embedded into a reflection coefficient such as S_{11}, the hybrid matrix factorization is preferable.

If an all-pole filter is considered in the context of the above discussion, the scattering matrix factorization may be better. This is why because such a filter is primarily characterized by a denominator polynomial, whereas a numerator consists of a multiple zero at the origin in the z-plane. On the other hand, this special zero with multiplicity brings about a peculiar situation, because such a zero is realized by a simpler circuit structure. Owing to the special property, an all-pole filter embedded in S_{21} can be realized by T-cascade interconnection.

The same discussion applies to the case for FIR filters. An FIR filter is realized by S-cascade interconnection rather than T-cascade interconnection because of the multiple pole at $z=0$, which permits the simplest parameterization. This causes no contradiction to that a linear-phase response cannot be realized with indispensable feedback loops.

Of course, any kind of global structures among S, T, and H-cascades is permitted to synthesize a given transfer function. In particular, S-cascade global structure offers the simplest pipelinability[13].

So, cascade synthesis of lossless digital two-ports can afford to give many circuit realizations. One of them has to be selected by application-specific considerations. Any way, global synthesis is performed by the localization of poles associated with three scattering matrices. As for most practical applications to frequency-selective filtering, the global structure for circuit realization should be represented by T-cascade or H-cascade interconnection which localizes every loss pole.

4.2 Global Factorization of Transfer Matrices

After the embedding of a transfer function, one obtains the canonical form of the corresponding transfer matrix as

$$\boldsymbol{T}(z)=\frac{1}{F(z)}\begin{bmatrix} G(z) & uK_*(z)z^N \\ K(z) & uG_*(z)z^N \end{bmatrix}. \tag{20}$$

Assume that $\boldsymbol{T}$ is factored into a product of $\boldsymbol{T}_a$ and $\boldsymbol{T}_b$ as in Eq.(16), then both $\boldsymbol{T}_a$ and $\boldsymbol{T}_b$ are passive by Theorem 3, unless their degrees exceed that of $\boldsymbol{T}$.

Among F, specifying a single factor F_a with real coefficients, one can define $\boldsymbol{T}_a$ formally as the canonical form of a transfer matrix, i. e.

$$\boldsymbol{T}_a=\frac{1}{F_a}\begin{bmatrix} G_a & u_aK_{a*}z^L \\ K_a & u_aG_{a*}z^L \end{bmatrix} \tag{21}$$

where F_a, G_a, and K_a are related by the paraunitary condition as same as Eq.(10). Denoting $\boldsymbol{T}_b$ with a formal degree, $N+L$, by $\boldsymbol{T}_c$, and using the formula

$$\boldsymbol{T}_c=\boldsymbol{T}_a^{-1}\boldsymbol{T}=\boldsymbol{J}\boldsymbol{T}_{a*}\boldsymbol{J}\boldsymbol{T} \tag{22}$$

one obtains the formal matrix as

$$\boldsymbol{T}_c=\frac{1}{F_c}\begin{bmatrix} G_c & u_cK_{c*}z^{N+L} \\ K_c & u_cG_{c*}{}^{N+L} \end{bmatrix} \tag{23}$$

where

$$F_c=F_{a*}z^LF \tag{24 a}$$

$$G_c=G_{a*}z^LG-K_{a*}z^LK \tag{24 b}$$

$$K_c=-u_aK_aG+u_aG_aK. \tag{24 c}$$

The problem is to find a condition such that the degree of the formal $\boldsymbol{T}_c$ is actually reduced to $N-L$. Specifically, we are concerned with the first and second degree reduction.

At first, observe that every factor of any numerator entry in lossless scattering matrix descriptions has its mirror image in the corresponding entry at the opposite column. If F_a cancels out between F_c and G_c in Eq.(23), F_a is a factor of G_c, and hence $F_{a*}z^L$ is a factor of $G_{c*}z^{N+L}$. Again in Eq.(23), if F_a cancels out between F_c and $G_{c*}z^{N+L}$, F_a is a factor of $G_{c*}z^{N+L}$, and hence $F_{a*}z^L$ is a factor of G_c. Namely, $F_aF_{a*}z^L$ is a common factor between F_c and both G_c and $G_{c*}z^{N+L}$. Denoting the zero of F_a by a, one obtains a single condition

$$G_c(a)=0 \tag{25}$$

which is necessary and sufficient for the desired reduction in degree by $2L$. The other expected requirement

$$K_c(a)=0 \tag{26}$$

is automatically fulfilled by paraunitariness.

If $|a|=1$, then $F_aF_{a*}z^L$ has a double zero, because the mirror image of such a zero is identical to itself. In this case,

$$G_c'(a)=0 \tag{27}$$

turns out to be the additional condition, where the prime stands for the derivative with respect to z.

After that, $\boldsymbol{T}_a$ is determined by paraunitariness and the degree reduction condition, thereby completing the basic step in transfer matrix factorization. Such a decomposition is repeated until a constant matrix $\boldsymbol{T}_{bn}$ appears. The termination of the procedure results in

$$\boldsymbol{T}=\boldsymbol{T}_{a1}\boldsymbol{T}_{a2}\cdots\boldsymbol{T}_{an}\boldsymbol{T}_{bn}. \tag{28}$$

Paying our attention to the feedback loops in Fig. 6, it is found that if every path depicted by an asterisk has a delay, the T-cascade realization is computable. It is

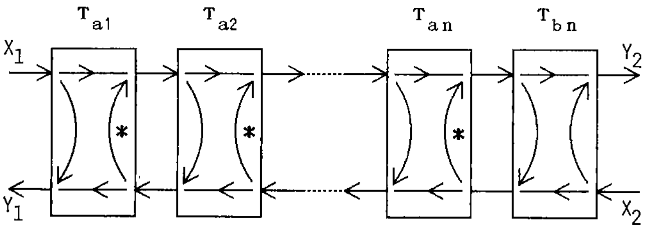


Fig. 6 T-cascade global structure.

thus adequate to incorporate a delay in S_{22} entry. In parallel, if every reflection coefficient at the left-hand port is devoid of delay-free propagation, the T-cascade realization is computable. These conditions are expressed in terms of canonical polynomials by

$$K(0)=0, \tag{29 a}$$

$$K_*(z)z^N|_{z=0}=0 \tag{29 b}$$

at the right-hand and left-hand ports, respectively. On the other hand, if any lossless digital two-port is realized by the global structure shown in Fig. 6 where the rightmost section depicted by $\boldsymbol{T}_{bn}$ has a nonzero reflection without delay at its left-hand port, the adjacent two-port cannot have delay-free reflection at its right-hand port. This statement is true for other interconnection junctions. Therefore, if such a constant section is removed from the rightmost port at first, then the resulting two-port must satisfy Eq. (29 a). At the same time, one can see that the synthesis similar to the realization expressed by Eq. (28) can be performed from the rightmost or the leftmost port.

4. 3 Localization of Real Loss Poles

The canonical form of a transfer matrix with a real pole at $z=a$, which is one of the prescribed loss poles, can be written as

$$\boldsymbol{T}_a=\frac{1}{f(z-a)}\begin{bmatrix} z-p & u_a k \\ kz & u_a(1-pz) \end{bmatrix} \tag{30}$$

where p, f, and k are unknown real parameters. Owing to the paraunitary condition, p, a, and f^2 are related by

$$p=f^2a. \tag{31}$$

The absence of the constant term of the numerator in (2, 1) entry is due to Eq. (29 a). The unimodular constant u_a is arbitrary. For the case of $|a|\neq 1$, from Eqs. (25), (26), one obtains

$$\begin{bmatrix} K(a) & aG(a) \\ aG(a) & K(a) \end{bmatrix}\begin{bmatrix} k \\ p \end{bmatrix}=\begin{bmatrix} G(a) \\ aK(a) \end{bmatrix} \tag{32}$$

and readily finds the solution as follows[13].

$$k=P(a)(1-a^2)/(P^2(a)-a^2) \tag{33 a}$$

$$p=a(P^2(a)-1)/(P^2(a)-a^2) \tag{33 b}$$

$$f^2=(P^2(a)-1)/(P^2(a)-a^2) \tag{33 c}$$

where

$$P(z)=K(z)/G(z). \tag{34}$$

It should be noted that $P(z)$ is complementary to the transfer function $H(z)$ and has a useful relationship shown by

$$P_*(a)P(a)=1. \tag{35}$$

In the case of $|a|=1$, from Eqs. (25), (27),

$$\begin{bmatrix} K(a) & aG(a) \\ K'(a) & aG'(a)+G(a) \end{bmatrix}\begin{bmatrix} k \\ p \end{bmatrix}=\begin{bmatrix} G(a) \\ G'(a) \end{bmatrix} \tag{36}$$

is obtained to find the solution

$$k=-a/(P'(a)-aP(a)) \tag{37 a}$$

$$p=aP'(a)/(P'(a)-aP(a)) \tag{37 b}$$

$$f^2=P'(a)/(P'(a)-aP(a)). \tag{37 c}$$

4. 4 Localization of Complex Loss Poles

A second-degree section $\boldsymbol{T}_a$ for the localization of a complex conjugate pair of loss poles at $z=a$ and a^* is written by

$$\boldsymbol{T}_a=\frac{1}{f\{z^2-(a+a^*)z+aa^*\}} \cdot\begin{bmatrix} z^2-(p+p^*)z+pp^* & u_a(\#) \\ k(z^2-qz) & u_a(\#) \end{bmatrix} \tag{38}$$

where the second column is abbreviated except for the unimodular constant which is assumed to be unity. Unknown parameters are f, k, and q in addition to a complex parameter p which amounts to two real unknowns such as $p+p^*$ and pp^*. Paraunitariness imposes

$$pp^*=f^2aa^*. \tag{39}$$

One can thus obtain the linear system of four equations with respect to k, kq, $p+p^*$, and pp^* by using the complementary function notation. The four equations consists of two equations for degree reduction and their complex conjugates. For $|a|\neq 1$, solving the linear system

$$\begin{bmatrix} -P & aP & -a & a^2 \\ -P^* & a^*P^* & -a^* & a^{*2} \\ a^2 & -a & aP & -P \\ a^{*2} & -a^* & a^*P^* & -P^* \end{bmatrix}\begin{bmatrix} k \\ kq \\ p+p^* \\ pp^* \end{bmatrix}=\begin{bmatrix} -1 \\ -1 \\ a^2P \\ a^{*2}P^* \end{bmatrix} \tag{40}$$

where the argument of $P(a)$ being suppressed, one gets the solution, as follows.

$$k=(a-a^*)[(aP-a^*P^*)(|a|^4+PP^*)$$

$$-(aP^*-a^*P)(1+|a|^4PP^*)$$
$$-(P-P^*)(a+a^*)aa^*(1+PP^*)]/d \quad (41\text{ a})$$

$$kq=(a-a^*)[(a+a^*)(aP^*-a^*P)(aa^*+PP^*)$$
$$-(a^2P^*-a^{*2}P)(1+aa^*PP^*)$$
$$+(P-P^*)(PP^*+|a|^6)]/d \quad (41\text{ b})$$

$$p+p^*=[(a+a^*)(a^2-a^{*2})(aa^*+|P|^4)$$
$$-(aP-a^*P^*)(a^2P^*-a^{*2}P)(1+aa^*)$$
$$+(P-P^*)(aP^*-a^*P)(1+|a|^6)]/d \quad (41\text{ c})$$

$$pp^*=aa^*[(a-a^*)^2(1+|P|^4)$$
$$-2(aP^*-a^*P)(aP-a^*P^*)$$
$$-(P-P^*)^2(1+|a|^4)]/d \quad (41\text{ d})$$

where

$$d=(a-a^*)^2(|a|^4+|P|^4)$$
$$-(aP^*-a^*P)^2(1+|a|^4)$$
$$-2aa^*(a^2P^*-a^{*2}P)(P-P^*) \quad (41\text{ e})$$

For $|a|=1$, the solution to the following system

$$\begin{bmatrix} P & -aP & a & -a^2 \\ P^* & -a^*P^* & a^* & -a^{*2} \\ P' & -P-aP' & 1 & -2a \\ P'^* & -P^*-a^*P'^* & 1 & -2a^* \end{bmatrix}\begin{bmatrix} k \\ kq \\ p+p^* \\ pp^* \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad (42)$$

is of the form

$$k=-(a-a^*)[(a-a^*)(aP'+a^*P'^*)$$
$$-(a+a^*)(P-P^*)]/d \quad (43\text{ a})$$

$$kq=-(a-a^*)[(a-a^*)(P'+P'^*)$$
$$-2(P-P^*)]/d \quad (43\text{ b})$$

$$p+p^*=[(a-a^*)^2\{(a+a^*)|P'|^2-(PP'^*+P^*P')\}$$
$$+2(aP^*-a^*P)(P-P^*)]/d \quad (43\text{ c})$$

$$pp^*=[(a-a^*)^2|P'|^2-(P-P^*)^2]/d \quad (43\text{ d})$$

where

$$d=(a-a^*)aa^*\{|P'|^2-2(PP'^*-P^*P')\}$$
$$+(a^2P^*-a^{*2}P)(P-P^*). \quad (43\text{ e})$$

After the successive application of the described pole-localization to all loss poles, a given transfer matrix is factored into Eq. (28). The global synthesis with respect to loss poles of a lossless digital two-port is thus completed, and results in a canonical factorization. Hence,

$$|p|<1 \quad (44)$$

is guaranteed by Theorem. 3. Thus, every entry of the scattering matrix associated with the factorization is analytic in $|z|\geqq 1$. Then, from the maximum modulus theorem[16] and the paraunitary property,

$$|f|\leqq 1 \quad (45\text{ a})$$
$$|k|\leqq 1 \quad (45\text{ b})$$

are deduced by evaluating $S_{21}(z)$ and $S_{11}(z)$ at infinity, respectively.

5. Local Synthesis with Localization of Poles at Zero and Infinity

The primary concern here is addressed to the local realization of the first/second-degree sections. The procedure to be described below can be still applied to any factorization of scattering matrix descriptions.

In general, a circuit without feedback loops has a simple structure. The simplest canonical sections are a constant paraunitary section and a first-degree allpass section that consists of a single pure delay and an interconnection wire. By Theorem 2, any circuit obtained from arbitrary combinations with three cascade interconnections is also a canonical digital two-port. One of the key concepts says that a new feedback structure can be created from the cascade interconnections. Thus we are led to a possibility stated as any lossless digital two-port can be realized with constant paraunitary sections and pure delays.

5.1 Primitive Sections

A constant paraunitary digital two-port is written by

$$S=\begin{bmatrix} k & uf \\ f & -uk \end{bmatrix} \quad (46)$$

where the constant u may be assigned as one. f and k are related by

$$f^2+k^2=1. \quad (47)$$

Its flow-graph representation is trivial, and is thus omitted.

There are trivial four kinds of the first-degree pure delay sections as follows.

$$S=\frac{1}{z}\begin{bmatrix} z & 0 \\ 0 & -1 \end{bmatrix} \quad (48\text{ a})$$

$$S=\frac{1}{z}\begin{bmatrix} 1 & 0 \\ 0 & -z \end{bmatrix} \quad (48\text{ b})$$

$$S=\frac{1}{z}\begin{bmatrix} 0 & 1 \\ z & 0 \end{bmatrix} \quad (48\text{ c})$$

$$S=\frac{1}{z}\begin{bmatrix} 0 & z \\ 1 & 0 \end{bmatrix} \quad (48\text{ d})$$

where a unimodular constant is stripped. The constant can be added by special forms of the constant paraunitary section, if desired. Their circuit realizations are also trivial. A single pole at $z=0$ is just realized by a delay element.

The pure delay sections and the constant paraunitary section are primitive, because they cannot be factored any more by using the key concepts.

If Eq. (46) and anyone in Eqs. (48) are multiplied, a first-degree paraunitary section with the pole still left at $z=0$ is obtained. By contrast, a new pole at $z \neq 0$ or a feedback structure is created, if either the hybrid matrices for Eq. (46) and one of Eqs. (48 a), (48 b) or the transfer matrices for Eq. (46) and one of Eqs. (48 c), (48 d) are multiplied.

5.2 First-Degree Paraunitary Section

To make an inspection of actual signal propagation, the first-degree paraunitary transfer matrix for localizing a real loss pole is rewritten in the corresponding scattering matrix as follows.

$$\boldsymbol{S}_a = \frac{1}{z-p}\begin{bmatrix} kz & u_a f(1-az) \\ f(z-a) & -u_a k \end{bmatrix} \tag{49}$$

Owing to the computability requirement in the global T-cascade structure, S_{11} entry has a special zero located at $z=0$. To take advantage of this fact, Eq. (49) is converted to the following hybrid matrix

$$\boldsymbol{H}_a = \frac{1}{kz}\begin{bmatrix} z-p & -u_a f(1-az) \\ f(z-a) & -u_a(1-pz) \end{bmatrix} \tag{50}$$

thus a special pole comes out at the origin. Of course, such a pole is realized by a single delay element.

At first, as H-cascade interconnection creates a new feedback structure, it is unnecessary for two sections which are cascaded, to incorporate a feedback loop in advance. Secondly, referring Fig. 7, in order for the H-cascade interconnection to be computable, one observes that a new feedback loop, indicated by ABCD, to be built by the interconnection must be interrupted by a delay element. This statement is generally true for H-cascade interconnections, hence its explicit representation is given in terms of canonical polynomials as follows:

$$F(0)=0, \quad \text{for } \boldsymbol{H}_1 \tag{51 a}$$

or

$$F_*(z)z^N|_{z=0}=0, \quad \text{for } \boldsymbol{H}_2. \tag{51 b}$$

Thirdly, the H-cascaded structure has a delay element in the reflection, indicated by FAE, at the right-hand port. Finally, since the two-port of interest is first-degree, if its realization is canonical, the number of delay elements must be one. Consequently, the delay must be exposed on the path depicted by A. It follows that the first-degree hybrid matrix to be factored is of the form

$$\boldsymbol{H}_1 = \frac{1}{K_1 z}\begin{bmatrix} z & -F_1 \\ F_1 z & -1 \end{bmatrix} \tag{52}$$

where two parameters are related by the paraunitariness, and the degree reduction condition in the hybrid matrix representation imposes

$$F_1 = f. \tag{53}$$

$\boldsymbol{H}_a$ is thus factored as a product of the canonical forms of two hybrid matrices, i. e.

$$\boldsymbol{H}_a = \boldsymbol{H}_1 \boldsymbol{H}_2 \tag{54}$$

where

$$\boldsymbol{H}_2 = \frac{1}{K_2}\begin{bmatrix} 1 & u_a F_2 \\ F_2 & u_a \end{bmatrix} \tag{55}$$

and is parameterized by

$$K_2 = k/K_1. \tag{56}$$

The above local synthesis for the first-degree section is summarized as

$$\boldsymbol{T}_a \sim \boldsymbol{H}_a, \qquad \boldsymbol{H}_a = \boldsymbol{H}_1 \boldsymbol{H}_2 \tag{57}$$

where the tilde $\sim$ denotes the equivalent transformation between two representations of the same two-port. By inspection of the second column of Eq. (55), it is found that there are no negative signs which are expected in the notation of hybrid matrices. Hence, to save an inverter to be involved in the implementation, it is effective to set $u_a=-1$ for the first degree section. The circuit

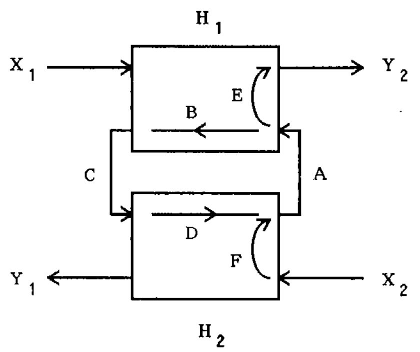


Fig. 7 Computability in H-cascade interconnection.

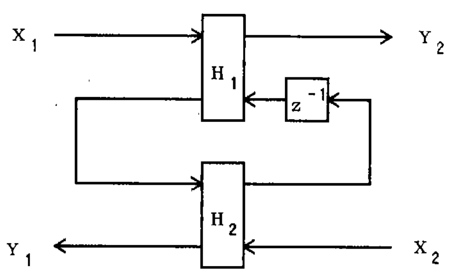


Fig. 8 Realization of the first-degree section.

realization is shown in Fig.8.

5.3 Second-Degree Paraunitary Section

The second-degree transfer matrix for the global T-cascade structure is irreducible in real coefficient polynomials. One of the canonical polynomial, $F(z)$, which is in the T_{21} entry, is still reducible. In particular it contains a factor z due to computability. If the matrix is converted to the equivalent hybrid matrix

$$\boldsymbol{H}_a=\frac{1}{kz(z-q)} \cdot\begin{bmatrix} z^2-(p+p^*)z+pp^* & -u_a(\#) \\ f\{z^2-(a+a^*)z+aa^*\} & -u_a(\#) \end{bmatrix}, \tag{58}$$

a special pole occurs at the origin. In the same manner to the first-degree section, $\boldsymbol{H}_a$ is decomposed into the first-degree section $\boldsymbol{H}_1$ which has the pole at $z=0$ and the other first-degree section $\underline{\boldsymbol{H}}_1$:

$$\boldsymbol{H}_a=\boldsymbol{H}_1\underline{\boldsymbol{H}}_1, \tag{59}$$

where $\boldsymbol{H}_1$ is identical to Eq.(52) accompanied with Eq.(53), and $\underline{\boldsymbol{H}}_1$ is of the form

$$\underline{\boldsymbol{H}}_1=\frac{1}{(z-q)k/K_1} \cdot\begin{bmatrix} z-\{p+p^*-(a+a^*)f^2\}/K_1^2 & u_a(\#) \\ -f\{(p+p^*-a-a^*)z-(pp^*-aa^*)\}/K_1^2 & u_a(\#) \end{bmatrix} \tag{60}$$

In $\underline{\boldsymbol{H}}_1$, there are no factors of naked z which we wish to get, but we can create such a factor by removing a constant paraunitary section $\boldsymbol{H}_2$ from $\underline{\boldsymbol{H}}_1$ to bring about the first-degree remainder $\underline{\boldsymbol{H}}_2$;

$$\underline{\boldsymbol{H}}_1=\boldsymbol{H}_2\underline{\boldsymbol{H}}_2. \tag{61}$$

Referring Fig.9 which illustrates the process up to the present, it is readily found that in order for the H-cascade of $\boldsymbol{H}_2$ and $\underline{\boldsymbol{H}}_2$ to be computable, path D among the loop indicated by ABCD has to be interrupted by a delay element. Other possibilites are excluded as follows. If path A has a delay, the totally composite second-degree section has the reflection with a couple of unit delays at the right-hand port, but this is in contradiction with the form of Eq.(58). As $\boldsymbol{H}_2$ is a constant section, path B is devoid of delay elements. If path C has a delay elment, the composite section gives a unit delay in the left-hand port reflection. This again causes a contradiction to Eq(58). Hence, the constant paraunitary section $\boldsymbol{H}_2$ is determined in such a way that $\underline{\boldsymbol{H}}_2$ meets Eq.(51 b). The result is as follows.

$$\boldsymbol{H}_2=\frac{1}{K_2}\begin{bmatrix} 1 & -F_2 \\ F_2 & -1 \end{bmatrix} \tag{62}$$

where

$$F_2=f(a+a^*-p-p^*)/K_1^2, \tag{63}$$

$$\underline{\boldsymbol{H}}_2=\frac{K_1K_2}{(z-q)k} \cdot\begin{bmatrix} z-qk^2/K_1^2K_2^2 & -u_a(\#) \\ [faa^*K_1^2-\{p+p^*-(a+a^*)f^2\}F_2]/K_1^2K_2^2 & -u_a(\#) \end{bmatrix} \tag{64}$$

The numerator of (2, 1) entry in $\underline{\boldsymbol{H}}_2$ is a constant, and a zero at infinity has been certainly created in that entry. To fix the factor as a local pole, $\underline{\boldsymbol{H}}_2$ is converted to the transfer matrix counterpart $\underline{\boldsymbol{T}}_2$, and then the infinite pole is localized by a first-degree section $\boldsymbol{T}_3$, thereby producing the final constant section $\boldsymbol{T}_4$. The result is given as

$$\boldsymbol{T}_3=\frac{1}{F_3}\begin{bmatrix} z & K_3 \\ K_3z & 1 \end{bmatrix} \tag{65}$$

$$\boldsymbol{T}_4=\frac{1}{F_4}\begin{bmatrix} 1 & u_aK_4 \\ K_4 & u_a \end{bmatrix} \tag{66}$$

where

$$K_3=k/K_1K_2 \tag{67}$$

$$K_4=-qk/K_1K_2. \tag{68}$$

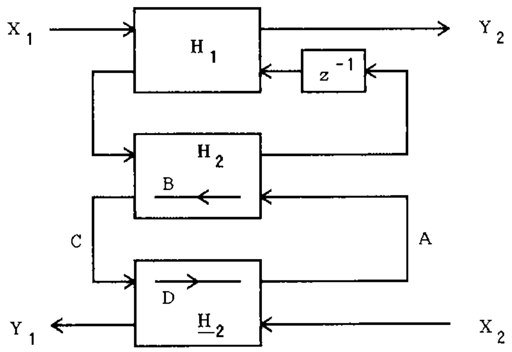


Fig. 9 Illustration of the pole-localization process for the second-degree section.

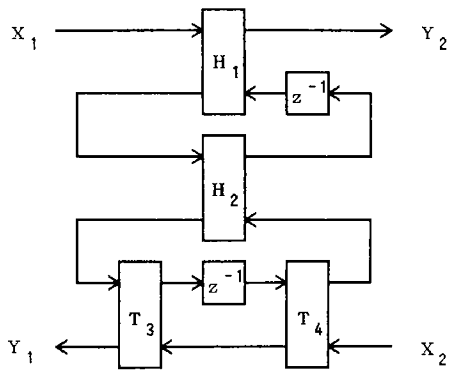


Fig. 10 Realization of the second-degree section.

The local synthesis for the second-degree section is summarized as

$$T_a \sim H_a,\ H_a = H_1 \underline{H}_1,\ \underline{H}_1 = H_2 \underline{H}_2,\ \underline{H}_2 \sim \underline{T}_2,\ \underline{T}_2 = T_3 T_4. \tag{69}$$

Figure 10 shows the realization circuit under the choice of $u_a = 1$ for saving an inverter.

In this way, a general canonical synthesis of lossless digital two-ports is completed. This yields an alternative and systematic design of power-wave digital filters equipped with high modularity.

A constant paraunitary matrix, Eq. (46), can be interpreted as unitary mapping from an input vector $[X_1, X_2]^t$ onto an output vector $[\hat{Y}_2, Y_1]^t$. If we define a plane rotation matrix by

$$\boldsymbol{R} = \boldsymbol{P}\begin{bmatrix} k & f \\ f & -k \end{bmatrix}, \tag{70}$$

then $\boldsymbol{R}$ is expressed by

$$\boldsymbol{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{71}$$

where

$$\theta = \tan^{-1}(k/f). \tag{72}$$

Plane rotation is well performed by CORDICs (Coordinate Rotation Digital Computers)[(5),(19)-(21)]. The synthesized circuits are thus suitable for CORDIC-based implementations.

Here, it is appropriate to make a comment on variations permitted in the synthesis. We have described basic three sorts of global structures in Sect.4, and the topics in this paper is mainly tailored to give a clear-cut presentation. Yet, the local synthesis procedure presented can offer many other variations. Any synthesis can be performed by the local synthesis without the global synthesis. As demonstrated in Sect.5.3, a constant paraunitary section can be removed from any scattering representation to create a pole at zero or infinity. Such a removal keeps the passivity because of Theorem 3. Therefore, the proposed synthesis procedure allows us to find many equivalent realizations which may be the mixture of S, T, and H-cascade structures.

6. An Example

A 6th-degree lowpass filter is designed by the method in Ref. (22) to give a realization example for the proposed synthesis. The specifications for this filter are given for a branching filter application dealt in Ref. (3) where a 7th-degree realization is given. The passband and stopband are split into two parts, respectively. Every edge frequency is an integer-multiple of 1/12. The transition band is from 2/12 to 4/12. Two ripples in the lower and upper passbands are 17 μB and 0.17 μB, respectively (1 $\mu\text{B} = 10^{-5}$ dB). The minimum attenuation

Table 2 Coefficients of the transfer function.

DEGREE	NUMERATOR	DENOMINATOR
6	6.15449E-02	1.00000E+00
5	2.47573E-01	-6.84019E-03
4	5.10444E-01	1.03642E+00
3	6.35419E-01	-5.22801E-03
2	5.10444E-01	2.43270E-01
1	2.47573E-01	-7.20307E-04
0	6.15449E-02	7.67985E-03

Table 3 Rotation angles for CORDIC-implementation.

GLOBAL SECTION INDEX	LOCAL SECTION INDEX			
	1	2	3	4
1	0.942653	2.101194	-0.246552	-1.052533
2	1.283795	1.968184	0.779937	-0.824199
3	1.191050	2.192306	1.064982	-0.205005
4	0.061885			

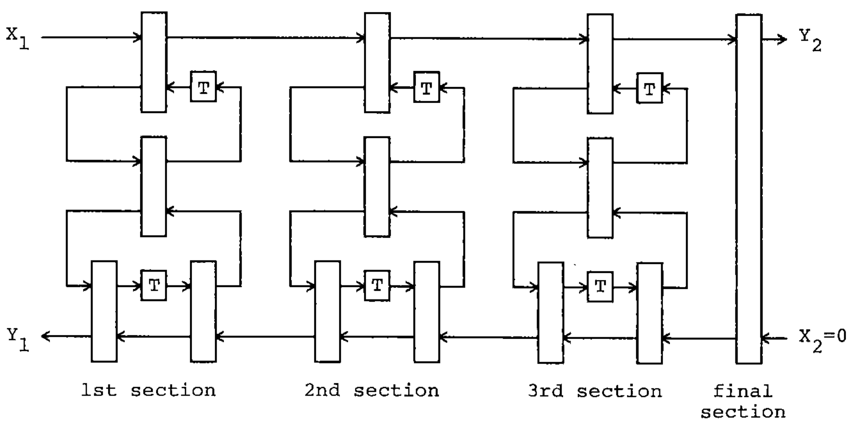


Fig. 11 Sixth-degree lowpass filter that serves for a branching filter with the complementary output.

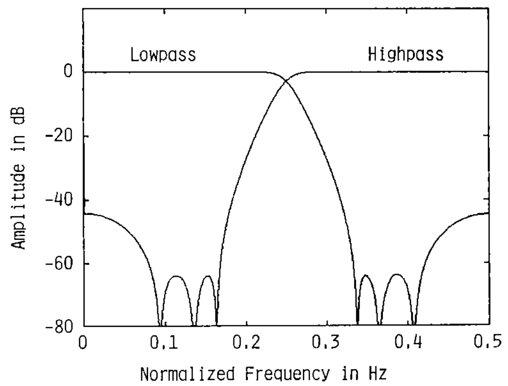


Fig. 12 Amplitude responses of the transfer function and the complementary function realized with 20-bit fixed-point binary.

in the lower stopband is 64 dB, and that in the other is 44 dB.

The transfer function is summarized in Table 2. The realized circuit is shown in Fig. 11, where a small box indicated by T represents a unit delay element. The realization parameters are given in Table 3 in terms of rotation angles, in radian, to be used for a CORDIC-based implementation. In the table, the global section index specifies a section in the global factorization of the transfer matrix, and the local section index denotes a section which appears in the same order as described in Sect.5.3.

A CORDIC performs plane rotations with successive bit-by-bit operations, and is strictly a nonlinear system. No implementations with CORDICs have a transfer function in the strict sense of linear systems. Of course, finite wordlength effects may be simulated by rounding rotation angles to get a virtual transfer function representation. This is, however, different from the reality.

Figure 12 has been obtained by the 1024-point discrete Fourier transform of the actual impulse response samples generated by bit-by-bit calculation with 20-bit fixed-point binary. It is found that a branching filter response is completed. This is a direct consequence of the lossless two-port synthesis.

7. Conclusion

In this paper, we have described a systematic way to power-wave digital filters through the synthesis of lossless digital two-ports. The synthesis is based on a set of physical concepts : pole-localization, cascade interconnection of digital two-ports, and computability. The canonical realization obtained ensures both external and internal passivities and produces high modularity, because it consists of constant paraunitary digital two-ports and pure delay elements. In process of the synthesis, it has been demonstrated that the described approach is useful and general. While the application technique to find new circuits is outlined, its actual demonstration and applications will be given in other opportunities.

Although the aspect on pipeline processing has been dropped in this paper, note that it is possible both to introduce redundancy required for pipelining and to find pipelinable realizations[13].

Acknowledgement

The authors wish to thank Prof. A. Fettweis of Ruhr-Universität Bochum and Prof. M. Tsuboi of Niigata University for their encouragement.

References

(1) A. Fettweis: "Digital filter structure related to classical filter networks", Arch. Elek. Übertrag., **25**, 2, pp. 79-89 (Feb. 1971).
(2) A. Fettweis and K. Meerkötter: "Suppression of parasitic oscillations in wave digital filters", IEEE Trans. Circuits & Syst., **CAS-22**, 3, pp. 239-246 (March 1975).
(3) A. Fettweis: "Wave digital filters: theory and practice", Proc. IEEE, **74**, 2, pp. 270-327 (Feb. 1986).
(4) E. F. Deprettere and P. Dewilde: "Orthogonal cascade realization of real multiport digital filters", Int. J. Circuit Theory & Appl., **8**, pp. 245-272 (1980).
(5) E. F. Deprettere, P. Dewilde and C. V. K. P. Rao: "Orthogonal filter design and VLSI implementation", Proc. Int. Conf. on Comput., Syst. Signal Process., pp. 779-790, Bangalore (Dec. 1984).
(6) E. F. Deprettere, P. Dewilde and C. V. K. P. Rao: "Cascade orthogonal filter design: a tutorial", Delft University of Technology, The Netherlands (1985).
(7) W. Givens: "Computation of plane unitary rotations transforming a general matrix to triangular form", J. Soc. Indust. Appl. Math., **6**, pp. 26-50 (March 1958).
(8) P. P. Vaidyanathan and S. K. Mitra: "Low passband sensitivity digital filters: a generalized viewpoint and synthesis procedures", Proc. IEEE, **72**, 4, pp. 404-423 (April 1984).
(9) P. P. Vaidyanathan: "A unified approach to orthogonal digital filters and wave digital filters, based on LBR two-pair extraction", IEEE Trans. Circuits & Syst., **CAS-32**, 7, pp. 673-686 (July 1985).
(10) P. P. Vaidyanathan: "The discrete-time bounded real lemma in digital filtering", IEEE Trans. Circuits & Syst., **CAS-32**, 9, pp. 918-924 (Sept. 1985).
(11) H. Kikuchi, Y. Tomidokoro and H. Watanabe: "A synthesis method for low coefficient sensitivity digital filters", Paper of Technical Group, **TGCAS84**-93, IECE Japan (Oct. 1984).
(12) H. Kikuchi, Y. Tomidokoro and H. Watanabe: "A z-domain synthesis of wave digital filters using a new universal adaptor", Proc. Int. Symp. Circuits & Syst., pp. 507-510, Kyoto (June 1985).
(13) H. Kikuchi, H. Watanabe, A. Nishihara and T. Yanagisawa: "Pipelinable low-sensitivity digital filters based on the localization of transmission zeros", Trans. IEICE, **E71**, 5, pp. 493-504 (May 1988).
(14) H. Kikuchi, H. Watanabe and T. Yanagisawa: "New second-degree section for orthogonal digital filters", in Digest 6th Kobe Int. Symp. Electron. Inf. Sci., pp. P2/1-9, Kobe (Nov. 1987).
(15) H. Kikuchi, H. Watanabe, A. Nishihara and T. Yanagis-

awa: "A systematic synthesis of power-wave digital filters", Proc. Joint Tech. Conf. Circuits/Syst., Comput. & Commun., pp. 414-419, Seoul (Nov. 1988).
(16) V. Belevitch: "Classical Network Theory", Holden-Day, San Francisco (1968).
(17) A. Fettweis: "Factorization of transfer matrices of lossless two-ports", IEEE Trans. Circuit Theory, **CT-17**, pp. 86-94 (Feb. 1970).
(18) H. Kikuchi: "Digital filter design with transfer function approximation based on circuit structure constraints", Doctoral dissertation, Tokyo Institute of Technology, Tokyo (May 1988).
(19) J. E. Volder: "The CORDIC trigonometric computing technique", IRE Trans. Electronic Comput., **EC-8**, pp. 330-334 (Sept. 1959).
(20) G. L. Haviland and A. A. Tuszynski: "A CORDIC arithmetic processor chip", IEEE Trans. Comput., **C-29**, 2, pp. 68-79 (Feb. 1980).
(21) M. Nakashizuka, H. Kikuchi and H. Watanabe: "A CORDIC-based digital oscillator for a pair of sinusoids", IEICE Technical Report, **CAS88**-32 (July 1988).
(22) H. Kikuchi, H. Watanabe and T. Yanagisawa: "Iterative design of constrained IIR digital filters requiring no initial values", Trans. IECE Japan, **E69**, 5, pp. 601-609 (May 1986).

Appendix

The derivation of Eq. (25) is outlined below. In Sect. 4.2, the cancellation of common polynomial factors between numerator and denominator is discussed with the aid of canonical forms of lossless digital two-ports. Through the discussion, it is apparent that the degree reduction is ensured by Eqs. (25), (26). At least, either of them is necessary.

If Eq. (25) is satisfied, Eq. (26) is also fulfilled by Eqs. (25), (10) with respect to G_c, F_c and K_c, as follows; since a is the zero of F_a, and since F_c has the factor F_a,

$$F_c(a)=0. \qquad (\mathrm{A}\cdot 1)$$

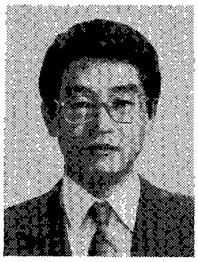
On the other hand, Eq. (10) gives a relationship

$$G_{c*}(a)G_c(a)=F_{c*}(a)F_c(a)+K_{c*}(a)K_c(a). \qquad (\mathrm{A}\cdot 2)$$

The left-hand side vanishes because of Eq. (25). Hence we obtain

$$K_{c*}(a)K_c(a)=0. \qquad (\mathrm{A}\cdot 3)$$

According to the same discussion on the cancellation between numerator and denominator, $F_aF_{a*}z^L$ is a common factor between F_c and both K_c and $K_{c*}z^{N+L}$. This leads to Eq. (26).



Hisakazu Kikuchi was born in Niigata Prefecture, Japan, on March 16, 1952. He received B. E. and M. E. degrees from Niigata University in 1974 and 1976, and Dr. Eng. degree in electrical and electronic engineering from Tokyo Institute of Technology in 1988, respectively. From 1976 to 1979 he worked at the System Laboratory, FUJITSU LIMITED, Tokyo. Since 1979 he has been with Niigata University, where he is now Associate Professor. His research interests are in digital signal processing, CAD for digital filters, and circuit theory. Dr. Kikuchi is a member of IEEE.

Hiromichi Watanabe was born in Hokkaido, Japan, on August 22, 1943. He received the B. E., M. E. and Dr. Eng. degrees in electronic engineering from Tokyo Institute of Technology, Tokyo in 1967, 1969 and 1974, respectively. In 1974, he joined Niigata University, Niigata, where he is now an Associate Professor of Electronic Engineering. He has engaged in research of circuit theory and electronic circuits. Dr. Watanabe is a member of the Institute of Electrical Engineers of Japan.

Akinori Nishihara was born in Fukuoka on February 26, 1951. He received B. E., M. E. and Dr. Eng. degrees in electronics from Tokyo Institute of Technology in 1973, 1975 and 1978, respectively. Since 1978 he has been with Tokyo Institute of Technology, where he is now Associate Professor of International Cooperation Center for Science and Technology. He is responsible not only for research cooperation with ASEAN countries but also for education and research in the field of circuits and systems. His main research interests are in filter design and applications of single-chip signal processors. He received Young Engineer Award in 1982. Dr. Nishihara is a member of IEEE.

Takeshi Yanagisawa was born in Nagasaki Prefecture, Japan, on March 14, 1931. He received the B. E., M. E. and Dr. Eng. degrees in electronic engineering from Tokyo Institute of Technology, Tokyo in 1953, 1955 and 1958, respectively. He joined the Tokyo Institute of Technology in 1958, and currently, he is a Professor at the Department of Electrical and Electronic Engineering. From 1980 to 1986, he also served as Director of the International Cooperation Center for Science and Technology at the same Institute. His research interests are in the fields of circuit theory, electronic circuits, and CAD for electronic circuits. Dr. Yanagisawa is the recipient of the 1986 Achievement Award of the Institute of Electronics and Communication Engineers of Japan. He is the author of many books including "Fundamentals of Electronic Circuits" and "Fundamentals of Circuit Theory". He is a member of the Institute of Electrical Engineers of Japan.