

# Antenna arrangements realizing a unitary matrix for $4 \times 4$ LOS-MIMO system

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**Abstract:** This paper proposes antenna arrangements which realize a unitary matrix for  $4 \times 4$  multiple-input multiple-output (MIMO) system in line-of-sight (LOS) environment. It is shown that the channel matrix is unitary matrix by properly adjusting the element spacing and the high channel capacity is obtained by the proposed antenna arrangements with directional antennas even if the multipath signals in indoor scenario are considered.

**Keywords:** MIMO, unitary matrix, antenna arrangement, directional antenna

**Classification:** Antennas and Propagation

## References

- [1] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*, Cambridge, U.K.: Cambridge Univ. Press, 2003.
- [2] F. Bohagen, P. Orten, and G. E. Oien, "Design of optimal high-rank line-of-sight MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 4, pp. 1420–1425, April 2007.
- [3] K. Nishimori, N. Honma, T. Seki, and K. Hiraga, "On the Transmission Method for Short Range MIMO Communication," *IEEE Trans. Vehicular Tech.*, vol. 60, no. 3, pp. 1247–1251, March 2011.
- [4] R. Kataoka, K. Nishimori, and H. Makino, "Simple analog beamforming method for short range MIMO transmission," *Proc. Asia-Pacific Microwave Conference (APMC) 2012*, pp. 812–814, Dec. 2012.
- [5] H. Y. E. Chua, K. Sakaguchi, K. Araki, H. Iwai, T. Sakata, and K. Ogawa, "Theoretical and experimental verification of the effects of mutual coupling on a  $2 \times 2$  MIMO system," *IEICE Technical Report*, WBS2004-118, AP2004-299, RCS2004-386, MoMuC2004-169, MW2004-296, March 2005.
- [6] Y. Hori, K. Iguchi, D. Uchida, and H. Arai, "Design of optimal MIMO channel under line-of-sight environment by using directional antennas," *IEICE Commun. Express*, vol. 2, no. 2, pp. 62–66, March 2013.

## 1 Introduction

Due to the recent popularity of smart phones and wireless local area network (WLAN), multiple-input multiple-output (MIMO) systems are incorporated into commercial broadband wireless systems to achieve higher transmission rate without expanding the frequency band [1]. *Multipath-rich* environments are generally considered for MIMO systems, and independent identically distributed (i.i.d.) channels are usually assumed to explain such environments in a simple manner [1].

Recently, the existence of *optimal* antenna spacing is confirmed that the channel capacity is maximized for given signal to noise power ratio (SNR) even when LOS environment is considered [2, 3], because the channel matrix at the optimal antenna spacing becomes unitary matrix [4]. Moreover, the channel capacity by the difference of element spacing is evaluated when  $2 \times 2$  MIMO system with mutual coupling effect is considered [5].

Hori et al. proposed the antenna arrangement for  $2 \times 2$  MIMO system which is effective even in the multipath environment by utilizing a distributed antenna arrangement and directional antennas [6]. Although the unitary condition in this antenna arrangement is not guaranteed due to multipath signals when using omni-directional antennas, the channel matrix is unitary matrix and the high channel capacity is obtained even in multipath environment when the directional antennas are adopted.

In this letter, we propose two types of antenna arrangements for  $4 \times 4$  MIMO system whose channel matrixes are unitary matrix. It is shown that the optimal condition regarding the element spacing exists when considering only direct wave. Although the channel capacity by these antenna arrangements is decreased when considering the multipath environment, we clarify that the use of directional antennas enables the improvement on the channel capacity even in the multipath environment. It is verified that the arrangement with the optimal antenna spacing is effective by using a ray-tracing simulation when considering an indoor scenario.

## 2 Antenna arrangement for maximizing channel capacity of $4 \times 4$ LOS-MIMO system

Fig. 1 (a) shows the proposed antenna arrangements. The channel matrixes which are obtained by the Arrangement A and B in Fig. 1 (a) ( $\mathbf{H}_{Ad1}$ ,  $\mathbf{H}_{Ad2}$  and  $\mathbf{H}_{Bd1}$ ,  $\mathbf{H}_{Bd2}$ ) are denoted in Fig. 1 (b), respectively. The transmit distance between transmitter (Tx) and receiver (Rx) is assumed to be much longer than the element spacing,  $d$  in Fig. 1 (a). As shown in Fig. 1 (a), Rx1 in Arrangement A is located with the direction of 45 degrees from Y-axis around the origin. When the proper element spacing ( $d = d_1(n) = \sqrt{2}(1/4 + (n - 1))\lambda_0$ ,  $n$ : natural number,  $\lambda_0$ : wavelength) with a square antenna arrangement is selected, the phase differences is  $\theta_{Rx1} - \theta_{Rx2} = 90^\circ$ ,  $\theta_{Rx2} - \theta_{Rx3} = 0^\circ$ ,  $\theta_{Rx1} - \theta_{Rx4} = 180^\circ$ , respectively. Hence, the elements of channel matrix regarding Tx1 ( $h_{1j}$ ) are  $h_{11} = 1$ ,  $h_{12} = -j$ ,  $h_{13} = -j$ , and  $h_{14} = -1$  as shown in  $\mathbf{H}_{Ad1}$  of Fig. 1 (b). The elements of channel matrix

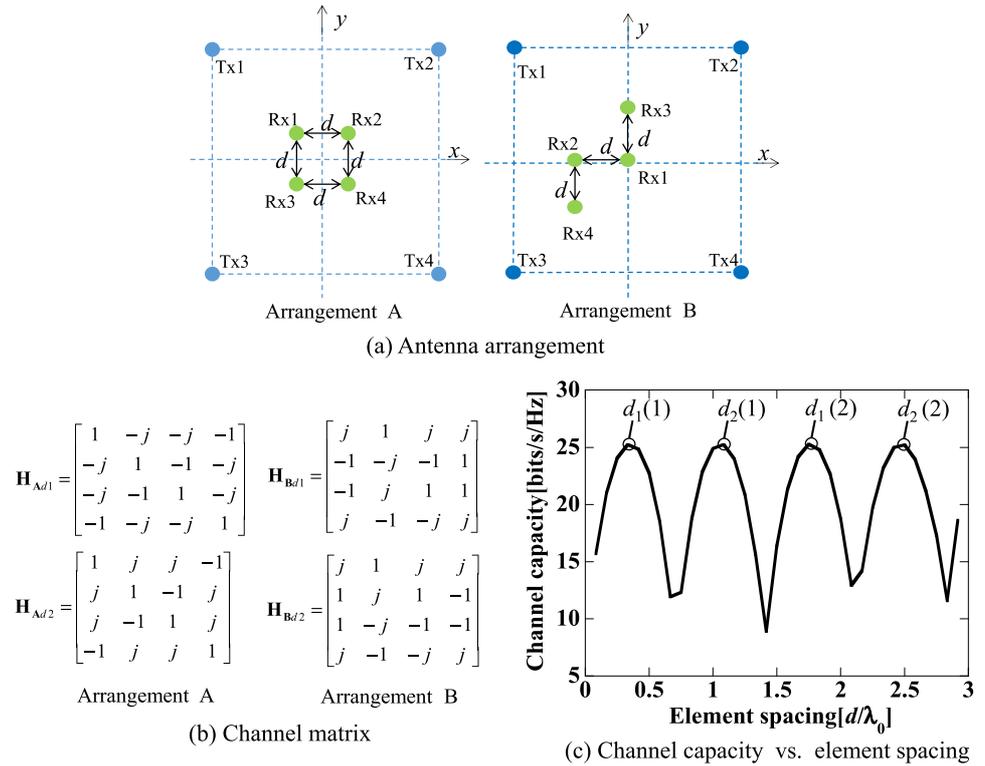


Fig. 1. Proposed antenna arrangement.

regarding Tx2, 3, and 4 can be determined with a similar way. When the proper element spacing ( $d = d_2(n) = \sqrt{2}(3/4 + (n - 1))\lambda_0$   $n$ : natural number,  $\lambda_0$ : wavelength) with a square antenna arrangement is selected,  $\mathbf{H}_{Ad2}$  is obtained.

On the other hand, the channel matrixes,  $\mathbf{H}_{Bd1}$  and  $\mathbf{H}_{Bd2}$  in Arrangement B of Fig. 1(b) cannot be obtained. When considering only phase differences among Rx1 to Rx4, channel matrixes for  $d_1(n)$  and  $d_2(n)$  on the Arrangement B ( $\mathbf{H}_{Bd1}^{(b)}$  and  $\mathbf{H}_{Bd2}^{(b)}$ ) are denoted as

$$\mathbf{H}_{Bd1}^{(b)} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ j & -j & j & -j \\ j & j & -j & -j \\ 1 & -1 & -1 & 1 \end{bmatrix}, \quad \mathbf{H}_{Bd2}^{(b)} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -j & j & -j & j \\ -j & -j & j & j \\ 1 & -1 & -1 & 1 \end{bmatrix}. \quad (1)$$

To solve the problem on the Arrangement B, the phase rotation of 90 degrees is given for Tx1, 3 and 4, respectively.  $\mathbf{H}_{Bd1}^{(b)}$  and  $\mathbf{H}_{Bd2}^{(b)}$  are changed to  $\mathbf{H}_{Bd1}$  and  $\mathbf{H}_{Bd2}$  in Fig. 1(b) by this phase rotation.

Fig. 1(c) shows the channel capacity versus the element spacing when considering only direct wave. The SNR at the receiver side is 25 dB when  $d$  is  $1.77\lambda_0$  ( $d_1(2)$  in Fig. 1(c)). Note that Arrangement A and B show the same channel capacity with each other. As can be seen in Fig. 1(c), the channel capacity is maximized with a fixed interval of element spacing. Hence, it is shown that the proposed antenna arrangements obtain the optimal condition if we use the Arrangement A and B with the proper element spacing.

### 3 Effectiveness of proposed antenna arrangement

#### 3.1 Simulation conditions

In the previous section, only direct wave is considered. On the other hand, it is reasonable to assume the multipath environment, especially, in indoor scenario where the MIMO transmission is applied. We conducted the ray-trace simulation when considering Arrangement A and B in Fig. 1 (a). The room with  $10 \times 10 \times 3\text{m}^3$  is assumed. Tx's and Rx's are located at the height of 1.5 m above the floor. The four transmit antennas are located at the four corners of the room, respectively. The frequency is 2.5 GHz. A ray-launching method as the ray-tracing simulation is adopted. The material for the room is concrete. The SNR at the receiver side is set to be 25 dB when the reflection is not considered and  $d$  is  $1.77\lambda_0$ . The number of reflections is five when using the multipath waves, because the delay spread is not increased even if the number of reflections exceeds five.

In an actual office room, the terminal station cannot be located at the ideal position. In order to avoid such a specific condition, the receive antennas are assumed to be located inside the area of  $10\lambda_0 \times 10\lambda_0$  from the center of the room. Moreover, the channel capacity is evaluated when the receive antennas are moved with the interval of  $\lambda_0/4$  on  $X$  and  $Y$  axis inside the area of  $10\lambda_0 \times 10\lambda_0$ . Total evaluated points were 1681 and the cumulative density function (CDF) of channel capacity is evaluated by the results on each evaluated point. Moreover, we compare the capacity of an omni-directional antenna with that of a directional antenna for the transmit antennas. When using the directional antennas, the half power beam width (HPBW) is set to be 90 degrees, and the main beam of transmit antennas are directed toward the center of the room. Beam pattern is used by  $\cos\theta$ . The sidelobe level is  $-15\text{dB}$  respect to the maximum power. To compare the effect on only radiation pattern in this study, the antenna gain is not considered: the gains of main beam direction are the same between the omni-directional and directional antennas.

#### 3.2 Channel capacity evaluation

Fig. 2 (a) shows the channel capacity when the number of reflections,  $N_R$  is zero and five, respectively. Only direct wave is considered when  $N_R = 0$ . Arrangement A in Fig. 1 (a) is used. The element spacing,  $d$  is  $1.77\lambda_0$  ( $d_1(2)$  in Fig. 1 (c)). The channel capacity by the omni-directional antennas is the same with that by the directional antennas, because the unitary condition regarding the channel matrix is maintained when  $N_R = 0$ . In addition, the variation of channel capacity due to the change of receivers' position is less than 2.5 bit/s/Hz.

When the omni-directional antennas are used with  $N_R = 5$ , the range of the channel capacity is from 19 to 26 bits/s/Hz when the range from 5 to 95% on the CDF is considered. On the other hand, when using the directional antennas, the range of the channel capacity is from 22 to 25 bits/s/Hz. Therefore, it is verified that the degradation on the channel capacity by the

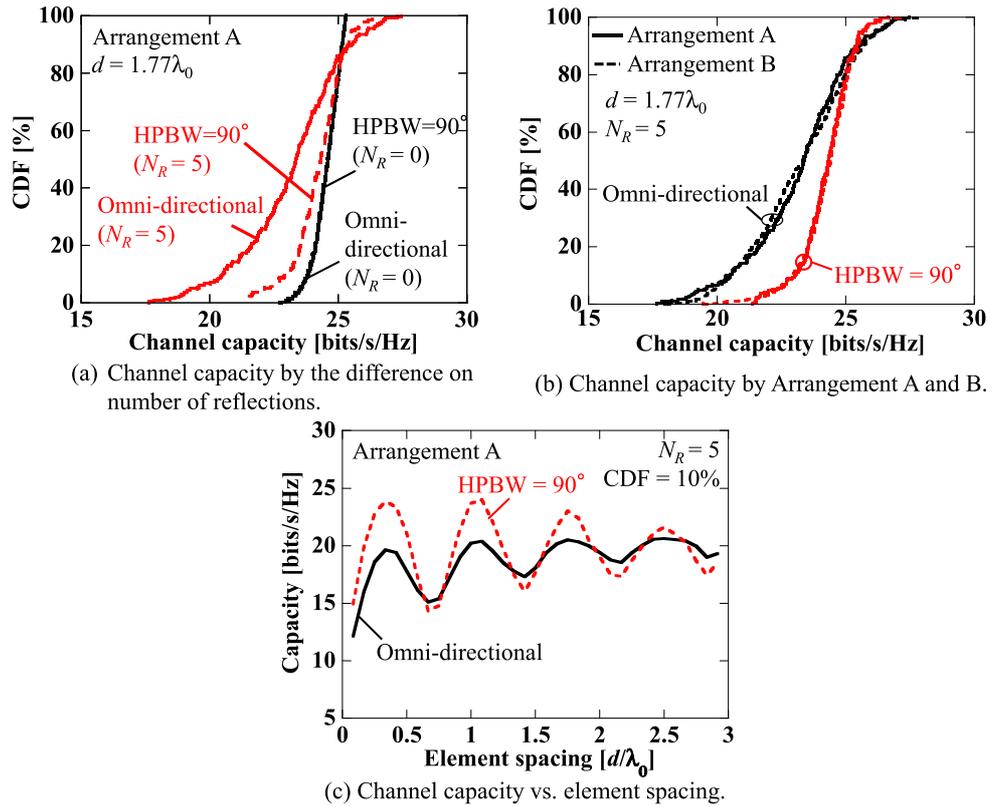


Fig. 2. Results on channel capacity.

directional antennas is much smaller than that by the omni-directional antennas when the multipath environment is considered.

Next, we focus on the comparison between the Arrangement A and B and results are shown in Fig. 2 (b).  $d$  is  $1.77\lambda_0$  and  $N_R$  is five. As can be seen in Fig. 2 (b), the capacity by Arrangement A is the same with that by Arrangement B regardless of the beam width. We confirmed that the same characteristics on the channel capacity between Arrangement A and B are obtained regardless of the number of reflections,  $N_R$ . Since Arrangement A can be easily implemented compared to Arrangement B, only Arrangement A is evaluated hereafter.

Fig. 2 (c) shows the channel capacity versus element spacing when  $N_R$  is five and CDF = 10%. As can be seen in Fig. 2 (c), the channel capacity at the element spacings which are not the optimal conditions is greatly decreased regardless of the beam width even in the multipath environment. Although Arrangement A itself in Fig. 1 (a) is not new antenna arrangement, the optimal condition on the element spacing shown in Sect. 2 is effective regardless of the beam width and this condition can be applicable for the multipath scenarios. Since the channel capacity by the directional antennas is smaller than that by the omni-directional antennas when considering the worst cases as shown in Fig. 2 (c), it is verified that the selection of the optimal antenna spacing is more important when using the directional antennas.

### 3.3 Spatial correlation evaluation

To consider the results of Fig. 2 (a), we evaluate spatial correlation between receive antennas. When the receive antenna # $a$  and # $b$  ( $a \neq b, a, b = 1 \sim 4$ ) are considered, the spatial correlation between the receive antenna # $a$  and # $b$ ,  $\rho_{a-b}$  is denoted as

$$\rho_{a-b} = \frac{\left| \sum_{j=1}^M h_{aj}^* h_{bj} \right|}{\sqrt{\sum_{j=1}^M |h_{aj}|^2} \sqrt{\sum_{j=1}^M |h_{bj}|^2}}, \quad (2)$$

where  $h_{aj}$  and  $h_{bj}$  denote the channel responses for  $a$ -th and  $b$ -th receive antennas when  $i$ -th transmit antenna is used.  $M$  is the number of transmit and receive antennas, respectively and  $M = 4$  in this letter. The results when the element spacing is smallest (Rx1-2) and largest (Rx1-4) for Arrangement A in Fig. 1 (a) are plotted in Fig. 3 when  $N_R$  is five. As can be seen in Fig. 3,  $\rho_{1-4}$  is distributed in the range of low values of 0 to 0.5 when using the directional antennas. Therefore, even in the multipath environment, the higher channel capacity is obtained by the directional antennas than the omni-directional antennas.

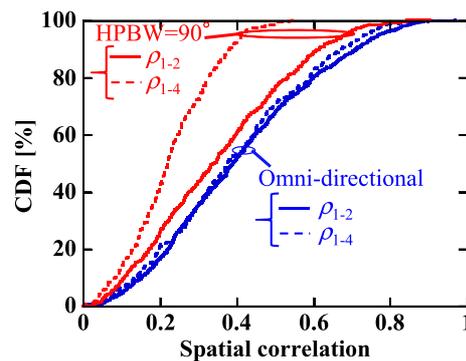


Fig. 3. CDF of spatial correlation.

## 4 Conclusion

In this letter, we have proposed two types of novel antenna arrangements for  $4 \times 4$  MIMO system whose channel matrixes are unitary matrix, and verified that the selection of the optimal antenna spacing is important for LOS-MIMO scenario even in the multipath environment. We conducted the ray-tracing simulation in the indoor environment to clarify the effectiveness of the proposed antenna arrangement. It is shown that the degradation on the channel capacity is small by the combination between the proposed antenna arrangement and directional antennas when the multipath environment is considered: when the range from 5 to 95% on the CDF is considered, the ranges on the channel capacity by the directional and omni-directional antennas are 3 and 7 bits/s/Hz, respectively. In addition, it is verified that the use of directional antennas reduces the spatial correlation.

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