Numerical Methods for Solving the Optimal Problem of Contrast Enhancement

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***Abstract*—This paper proposes two effective numerical methods to solve the optimal problem of contrast enhancement for the coherent and incoherent cases, respectively. For the coherent case, the objective function of the optimal problem is transformed into a bilinear form. Then a numerical method is presented by using the idea of the Sequential Unconstrained Minimization Technique (SUMT). For the incoherent case, a cross-iterative method is proposed for solving the optimal problem of contrast enhancement, based on the formula of the optimal contrast polarization state in the matched-polarized channel. Both the proposed methods are convergent and straightforward for programming. In addition, the proposed methods can be used for the bistatic radar case because in this paper, it is unnecessary to restrict the symmetry of the scattering matrix and the Kennaugh matrix. For showing the effectiveness of the proposed methods, we give three examples. The results of the calculation are completely identical with other papers', showing the validity of the proposed methods. In these three examples, if the receiving polarization state is independent of the transmitting's, the power ratios may be much larger than those in the cases of the co-polarized, the cross-polarized, and the matched-polarized channels, showing the importance of the considered models. In addition, the computation costs are estimated by these illustrative examples, illustrating that the proposed methods are very effective.**

***Index Terms*—Polarimetric contrast, polarization, radar polarimetry, scattering matrix.**

I. Introduction

The problem of optimization of polarimetric contrast enhancement (OPCE) (i.e., how to choose the optimal polarization states for enhancing the desired target versus undesired target: clutter), has been attracting attention in recent years [1]–[21]. For the OPCE problem, what we can control is the polarization states of the transmitter and the receiver. In the three special polarized radar channels (the co-polarized, the cross-polarized, and the matched-polarized channels), the independent variable is the polarization state of the transmitter only because the polarization state of the receiver depends on that of the transmitter. If there are two variable polarization states (i.e., one is the polarization state of the transmitter and the other is that of the receiver), a larger power ratio may be expected because there are two independent variables in the considered problem. For the incoherent case, Ioannidis *et al.* [13] studied the OPCE problem by using the method of Lagrangian multipliers. But they could not provided a simple method to obtain the optimal polarization states of the transmitter and the receiver. Later, Cadzow [16], and Swartz *et al.* [17] presented a simple procedure for solving the OPCE problem. Recently, the validity of this procedure was shown by Kong *et al.* [18] and Mott and Boerner [19]. However, the procedure proposed by Cadzow [16] cannot be used for the bistatic case, because it was assumed that the reciprocity theorem is true. For the coherent case on the other hand, Yang and Lin [14], [15] proposed a new mathematical model, and presented a method to solve this problem, but the method was inconvenient for programming.

In this paper, we propose two effective numerical methods to solve the OPCE problem for the coherent and incoherent cases, based on the idea of the Sequential Unconstrained Minimization Technique (SUMT) [25] and the formula of the optimal contrast polarization state in the matched-polarized channel [12]. Both the proposed methods are convergent and very simple for programming. In our mathematical models, the polarization states of the transmitter and the receiver are independent variables, making it possible to expect a higher contrast. For showing the advantages of the proposed methods and the considered models, we give three examples. The results of calculation are completely identical with [13]–[15], validating the proposed methods. In these examples, the power ratio corresponding to two independent polarization states is much larger than those in the cases of the copolarized, the cross-polarized and the matched-polarized channels, showing the importance of the considered models. Finally, we compare the computation cost by the illustrated examples, showing the effectiveness of the proposed methods. In addition, the proposed methods can be used to the bistatic radar case because in this paper, it is unnecessary to restrict the symmetry of the scattering matrix and the Kennaugh matrix.

II. Optimal Problem for the Coherent Case

For the coherent case, a radar target corresponds to a scattering matrix. Let $[S_1]$ and $[S_2]$ denote the scattering matrices of the desired target and undesired target (clutter), and let $\boldsymbol{a}$ and $\boldsymbol{b}$ denote the polarization states of the transmitting and receiving antennas, respectively. Then there exist infinite pairs of $\boldsymbol{a}$ and $\boldsymbol{b}$, such that the received power from the clutter equals zero [14], [15]. So, the mathematical model of the OPCE problem is

$$\text{maximize} \quad \|[S_1]\boldsymbol{a} \cdot \boldsymbol{b}\|^2 \tag{1a}$$
$$\text{subject to} \quad \|[S_2]\boldsymbol{a} \cdot \boldsymbol{b}\|^2 = 0 \tag{1b}$$
$$\|\boldsymbol{a}\|^2 = 1 \tag{1c}$$
$$\|\boldsymbol{b}\|^2 = 1. \tag{1d}$$

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In [14], [15], a method was presented to solve the above problem, but that method was inconvenient for programming. So, it is necessary to propose a practical method.

Now let us consider the same problem in a different way. Assume that $[K_1]$ and $[K_2]$ are the Kennaugh matrices of $[S_1]$ and $[S_2]$, $\boldsymbol{g}$ and $\boldsymbol{h}$ are the Stokes vectors of $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively. Then (1a)–(1d) can be transformed to the following problem:

$$\text{minimize} \quad (-[K_1]\boldsymbol{g}\cdot\boldsymbol{h}) \tag{2a}$$
$$\text{subject to:} \quad [K_2]\boldsymbol{g}\cdot\boldsymbol{h} = 0 \tag{2b}$$
$$g_1^2+g_2^2+g_3^2 = 1 \tag{2c}$$
$$h_1^2+h_2^2+h_3^2 = 1. \tag{2d}$$

Note that for an arbitrary pair of $\boldsymbol{g}$ and $\boldsymbol{h}$, $[K_2]\boldsymbol{g}\cdot\boldsymbol{h}$ is always nonnegative because it denotes the received power from the clutter. Therefore, if k is a very large constant, (1a)–(1d) or (2a)–(2d) is also equivalent to the following problem:

$$\text{Minimize} \quad (-[K_1]\boldsymbol{g}\cdot\boldsymbol{h} + k[K_2]\boldsymbol{g}\cdot\boldsymbol{h}) \tag{3a}$$
$$\text{subject to:} \quad g_1^2+g_2^2+g_3^2 = 1 \tag{3b}$$
$$h_1^2+h_2^2+h_3^2 = 1. \tag{3c}$$

The SUMT is a numerical method to use some sequences for transforming a constrained optimization into an unconstrained optimization [25]. Using this idea, we propose the following numerical method (denoted as Method 1) to solve the problem (3a)–(3c).

Step 1) Give an initial vector $\boldsymbol{g}_0$ and an initial value k_0.

Step 2) Calculate the Kennaugh matrices $[K_1]$ and $[K_2]$.

Step 3) Let $\boldsymbol{g}_n = \boldsymbol{g}_n^0 = (1, g_1^{n,0}, g_2^{n,0}, g_3^{n,0})^{\mathrm{t}}$ and calculate the matrix $[Q_n] = -[K_1] + k_n[K_2] \equiv (q_{ij}^n)_{4\times4}$, where the superscript $^{\mathrm{t}}$ denotes transpose. After $\boldsymbol{g}_n^l = (1, g_1^{n,l}, g_2^{n,l}, g_2^{n,l}, g_2^{n,l})^{\mathrm{t}}$ is obtained, then

$$\begin{aligned} -[K_1]\boldsymbol{g}_n^l\cdot\boldsymbol{h} + k_n[K_2]\boldsymbol{g}_n^l\cdot\boldsymbol{h} &= [Q_n]\boldsymbol{g}_n^l\cdot\boldsymbol{h} \\ &= \left(\sum_{j=2}^{4} q_{1j}g_{j-1}^{n,l} + q_{11}\right)h_2 + \left(\sum_{j=2}^{4} q_{2j}g_{j-1}^{n,l} + q_{21}\right)h_1 \\ &+ \left(\sum_{j=2}^{4} q_{3j}g_{j-1}^{n,l} + q_{31}\right) + \left(\sum_{j=2}^{4} q_{4j}g_{j-1}^{n,l} + q_{41}\right)h_3. \end{aligned} \tag{4}$$

Obviously, the above expression will be minimal if and only if

$$h_i = -w_i/w \equiv h_i^{n,l}, \qquad (i = 1, 2, 3,) \tag{5}$$

where

$$w_i = \sum_{j=2}^{4} q_{i+1j}g_{j-1}^{n,l} + q_{i+11}, \qquad (i = 1, 2, 3)$$
$$w = \sqrt{w_1^2+w_2^2+w_3^2}.$$

Step 4) After $\boldsymbol{h}_n^l = (1, h_1^{n,l}, h_2^{n,l}, h_3^{n,l})^{\mathrm{t}}$ is obtained, $\boldsymbol{g}_n^{l+1} = (1, g_1^{n,l+1}, g_2^{n,l+1}, g_3^{n,l+1})^{\mathrm{t}}$ is obtained as follows by a similar method:

$$g_i^{n,l+1} = -v_i/v, \qquad (i = 1, 2, 3) \tag{6}$$

where

$$v_i = \sum_{j=2}^{4} q_{ji+1}h_{j-1}^{n,l} + q_{1i+1}, \qquad (i = 1, 2, 3)$$
$$v = \sqrt{v_1^2+v_2^2+v_3^2}.$$

Step 5) If $\sum_{i=1}^{3} |g_i^{n,l+1} - g_i^{n,l}| \le \epsilon_n$ and $\sum_{i=1}^{3} |h_i^{n,l+1} - h_i^{n,l}| \le \epsilon_n$, let $\boldsymbol{g}_{n+1} \equiv \boldsymbol{g}_n^{l+1}, \boldsymbol{h}_{n+1} \equiv \boldsymbol{h}_n^{l+1}$ and let $k_{n+l} = 2k_n$.

Step 6) If k_{n+1} is very large (e. g., if $n \ge 10$), then $\boldsymbol{g}_{n+1}$ and $\boldsymbol{h}_{n+1}$ can be regarded as the optimal solution of the problem (2).

Notes:

1) Let

$$[S_1] = \begin{bmatrix} s1_1 & s1_2 \\ s1_3 & s1_4 \end{bmatrix} \quad \text{and} \quad [S_2] = \begin{bmatrix} s2_1 & s2_2 \\ s2_3 & s2_4 \end{bmatrix}.$$

According to our experience, we suggest selecting k_0 as

$$k_0 = \frac{10\sum_{i=1}^{4}|s1_i|^2}{\sum_{i=1}^{4}|s2_i|^2}$$

.

2) In the above algorithm, it is better to let $\epsilon_n = \epsilon_0/2^n$ for reducing the numbers of iteration.

3) Mathematically, it is better to select a starting vector being very close to the optimum solution. But in practice, it is impossible to do so because we do not know where the optimum solution is. In the following example, the optimum solution will be obtained, although a starting vector is not close to the optimum solution. To avoid the loss of some optimum solution(s), we advise selecting several starting points for obtaining the optimum polarization states by the comparison of the computing results. For example, we select the starting point $\boldsymbol{g}_0$ as follows: $(1, \pm1, 0, 0)^{\mathrm{t}}$, $(1, 0, \pm1, 0)^{\mathrm{t}}$, or $(1, 0, 0 \pm1)^{\mathrm{t}}$.

The above numerical method (Method 1) is very easy for programming. The procedure for Method 1 is shown in Fig. 1.

Example 1: Let us use the scattering matrices of [14] for checking the validity of Method 1. The scattering matrices of the target and clutter are

$$[S_1] = \begin{bmatrix} 2 & 2j \\ 0.5j & 0.5 \end{bmatrix}$$

and

$$[S_2] = \begin{bmatrix} 0.5005 + 0.4995j & 0.4995 + 0.5005j \\ -0.5005 + 0.4995j & 0.4995 - 0.5005j \end{bmatrix}$$

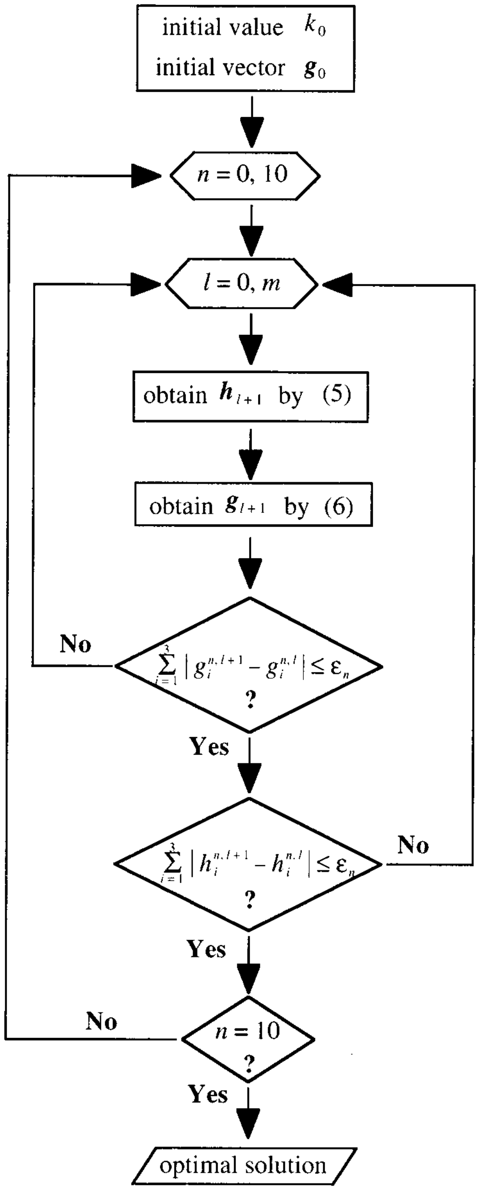

Fig. 1. Flowchart of Method 1.

respectively. Letting $\epsilon_n = 0.01/2^n$, $(n = 1, 2, \cdots, 10)$ and selecting some initial vectors shown in Table I, then by Method 1 we obtain the optimal polarization states of the transmitting antenna and the receiving antenna as

$$\boldsymbol{g}_n = (1, 0.000\,00, -0.695\,91, -0.718\,13)^t$$

and

$$\boldsymbol{h}_m = (1, 0.695\,10, 0.718\,91, 0.000\,00)^t$$

respectively. Under these optimal polarization states, the corresponding received powers of the target and clutter are 5.335 24 and 0.000 00, respectively. This result is identical with [14], [15], showing the validity of Method 1. Table I shows the numbers of the iterations for different initial vectors. Note that one iteration by Method 1 needs only 30 multiplication operations, six division operations and two square roots. From the numbers

TABLE I
RESULTS OF CALCULATION FOR DIFFERENT INITIAL VECTORS

initial vectors	numbers of iteration	received powers	
		target	clutter
$g_0 = (1, 1, 0, 0)'$	145	5.33526	0.00000
$g_0 = (1, -1, 0, 0)'$	145	5.33526	0.00000
$g_0 = (1, 0, 1, 0)'$	166	5.33524	0.00000
$g_0 = (1, 0, -1, 0)'$	147	5.33524	0.00000
$g_0 = (1, 0, 0, 1)'$	168	5.33525	0.00000
$g_0 = (1, 0, 0, -1)'$	148	5.33525	0.00000

of various operations and the numbers of the iterations for different initial vectors in Table I, one knows that Method 1 is very effective.

Now let us compare the above results with those by using the copol or the cross-pol nulls of the clutter for contrast enhancement. From [12], it is easy to obtain the following copol nulls of the clutter:

$$\boldsymbol{g}_{n1} = (1, 0.577\,35, -0.576\,67, 0.57\,803)^t$$

and

$$\boldsymbol{g}_{n2} = (1, -0.577\,35, 0.578\,01, -0.576\,69)^t.$$

The corresponding received powers from the desired target are 1.413 38 and 1.417 38, respectively. (For the above clutter, the cross-pol nulls do not exist.) Obviously, the results given by Table I are better than those by using the copol nulls of the clutter, which means that the model (1a)–(1d) is more important for contrast enhancement.

III. Optimal Problem for the Incoherent Case

A. Optimal Contrast Polarization State in the Matched-Polarized Channel

In this section, we will give the formula of the optimal polarization state for contrast enhancement in the matched-polarized channel, because the result will be used in the next section. Here, let us consider the following model [1]:

$$\text{maximize} \quad \left(\frac{\boldsymbol{a}^H[G_1]\boldsymbol{a}}{\boldsymbol{a}^H[G_2]\boldsymbol{a}}\right) \tag{7}$$

where $[G_1]$ and $[G_2]$ denote the Graves matrices [22] of the desired target and undesired target, respectively. The vector $\boldsymbol{a}$ denotes the transmitting polarization state ($\|\boldsymbol{a}\| = 1$) and the superscript ^{H} denotes conjugate transpose

Let $[G] = \begin{bmatrix} \sigma_1 & \sigma_2 \\ \sigma_3 & \sigma_4 \end{bmatrix}$, then we have

$$\boldsymbol{a}^H[G]\boldsymbol{a} = \tfrac{1}{2}(\sigma_1 + \sigma_4) + \tfrac{1}{2}(\sigma_1 - \sigma_4)g_1 + \tfrac{1}{2}\,(\sigma_2 + \sigma_3)g_2 + j\,\tfrac{1}{2}(\sigma_2 - \sigma_3)g_3 \tag{8}$$

where $g_1^2 + g_2^2 + g_3^2 = 1$. So, we rewrite (7) as the following form:

$$\text{maximize} \quad \left(\frac{A_0 + A_1 g_1 + A_2 g_2 + A_3 g_3}{B_0 + B_1 g_1 + B_2 g_2 + B_3 g_3} \right) \tag{9}$$

where A_i and B_i are obtained from (8). For the problem (9), we know from [12] that the optimal polarization state is

$$g_i = \frac{A_i - r_m B_i}{\sqrt{\sum_{i=1}^{3} (A_i - r_m B_i)^2}} \qquad (i = 1, 2, 3) \tag{10}$$

where

$$\begin{aligned} r_m &= \frac{z_{12} + \sqrt{z_{12}^2 - z_1 z_2}}{z_2}, \\ z_1 &= A_0^2 - A_1^2 - A_2^2 - A_3^2, \\ z_2 &= B_0^2 - B_1^2 - B_2^2 - B_3^2, \\ z_{12} &= A_0 B_0 - A_1 B_1 - A_2 B_2 - A_3 B_3. \end{aligned}$$

Note that the above formula (10) will play an important role in the next section. In the Appendix, one can find the proof of (10).

B. Algorithm for the Incoherent Case

Let

$$[K]_a = \begin{bmatrix} k_{00}^a & k_{01}^a & k_{02}^a & k_{03}^a \\ k_{10}^a & k_{11}^a & k_{12}^a & k_{13}^a \\ k_{20}^a & k_{21}^a & k_{22}^a & k_{23}^a \\ k_{30}^a & k_{31}^a & k_{32}^a & k_{33}^a \end{bmatrix} \tag{11a}$$

and

$$[K]_b = \begin{bmatrix} k_{00}^b & k_{01}^b & k_{02}^b & k_{03}^b \\ k_{10}^b & k_{11}^b & k_{12}^b & k_{13}^b \\ k_{20}^b & k_{21}^b & k_{22}^b & k_{23}^b \\ k_{30}^b & k_{31}^b & k_{32}^b & k_{33}^b \end{bmatrix} \tag{11b}$$

denote the Kennaugh matrices of the target and clutter, respectively. In the coherent case, the received power of the clutter may equal zero by using some polarization states. Since an effective algorithm has been presented in Section II for obtaining the optimal contrast polarization states of the transmitting antenna and receiving antenna for the coherent case, now we only need to study the case of $P_b = \boldsymbol{h}^{\rm t}[K]_b\, \boldsymbol{g} > 0$. Let us consider the following mathematical model:

$$\text{maximize} \quad \left(\frac{\boldsymbol{h}^{\rm t}[K]_a \boldsymbol{g}}{\boldsymbol{h}^{\rm t}[K]_b \boldsymbol{g}} \right) \tag{12a}$$

$$\text{subject to} \quad g_1^2 + g_2^2 + g_3^2 = 1 \tag{12b}$$

$$h_1^2 + h_2^2 + h_3^2 = 1. \tag{12c}$$

It should be noted that there is no restriction on the symmetry of the Kennaugh matrices $[K]_{a/b}$, which means that the result can be applied to the bistatic case.

For solving the above problem, we propose the following numerical method (Method 2).

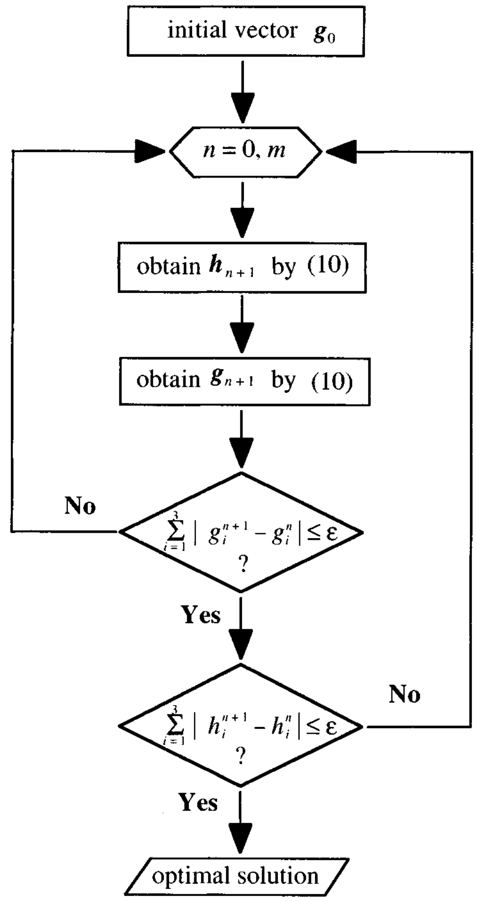

Fig. 2. Flowchart of Method 2.

Step 1) Select an initial vector $\boldsymbol{g}_0$.

Step 2) After $\boldsymbol{g}_n$ is obtained, consider the problem

$$\text{maximize} \quad \left(\frac{(\boldsymbol{h})^{\rm t}[K]_a \boldsymbol{g}_n}{(\boldsymbol{h})^{\rm t}[K]_b \boldsymbol{g}_n} \right). \tag{13}$$

Since (13) and (9) are of the same form, the optimal solution $\boldsymbol{h}_{n+1} = (1, h_1^{n+1}, h_2^{n+1}, h_3^{n+1})^{\rm t}$ is obtained by the formula (10).

Step 3) After $\boldsymbol{h}_{n+1}$ is obtained, consider the problem

$$\left(\frac{(\boldsymbol{h}_{n+1})^{\rm t}[K]_a \boldsymbol{g}}{(\boldsymbol{h}_{n+1})^{\rm t}[K]_b \boldsymbol{g}} \right). \tag{14}$$

Its solution $\boldsymbol{g}_{n+1} = (1, g_1^{n+1}, g_2^{n+1}, g_3^{n+1})^{\rm t}$ is obtained by using the formula (10).

Step 4) If $\sum_{i=1}^{3} |g_i^{n+1} - g_i^n| \le \sum$ and $\sum_{i=1}^{3} |h_i^{n+1} - h_i^n| \le \epsilon$, $\boldsymbol{g}_{n+1}$ and $\boldsymbol{h}_{n+1}$ are regarded as the optimal solution.

Obviously, Method 2 is simple for programming. Fig. 2 shows the procedure for the above algorithm.

Example 2: To verify Method 2, in this example, we use the following Kennaugh matrices, which were first given by Ioannidis *et al.* [13]. The first is the target

$$[K]_a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.63 & 0.22 & 0.43 \\ 0 & 0.22 & 0.37 & -0.75 \\ 0 & 0.43 & -0.75 & 0 \end{bmatrix}$$

TABLE II
RESULTS OF CALCULATION FOR EXAMPLE 2

initial vetors	iteration numbers	optimal polarization states	power ratio
$g_0 = (1, 1, 0, 0)'$	8	$g_m = (1, 0.430, -0.753, -0.498)'$ $h_m = (1, -0.430, 0.753, 0.498)'$	2.505
$g_0 = (1, -1, 0, 0)'$	8	$g_m = (1, -0.430, 0.753, 0.498)'$ $h_m = (1, 0.430, -0.753, -0.498)'$	2.505
$g_0 = (1, 0, 1, 0)'$	7	$g_m = (1, -0.430, 0.753, 0.498)'$ $h_m = (1, 0.430, -0.753, -0.498)'$	2.505
$g_0 = (1, 0, -1, 0)'$	7	$g_m = (1, 0.430, -0.753, -0.498)'$ $h_m = (1, -0.430, 0.753, 0.498)'$	2.505
$g_0 = (1, 0, 0, 1)'$	8	$g_m = (1, -0.430, 0.753, 0.498)'$ $h_m = (1, 0.430, -0.753, -0.498)'$	2.505
$g_0 = (1, 0, 0, -1)'$	8	$g_m = (1, 0.430, -0.753, -0.498)'$ $h_m = (1, -0.430, 0.753, 0.498)'$	2.505

and the second is the clutter

$$[K]_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Letting $\epsilon = 0.000\,01$ and selecting some initial vectors as shown in Table II, then by Method 2, we obtain the optimal polarization states of the transmitting antenna and the receiving antenna as

$$\begin{aligned} \boldsymbol{g}_m &= (1, 0.430\,48, -0.752\,64, -0.498\,21)^{\mathrm{t}}, \\ \boldsymbol{h}_m &= (1, -0.430\,48, 0.752\,64, 0.498\,21)^{\mathrm{t}} \end{aligned}$$

or

$$\begin{aligned} \boldsymbol{g}_m &= (1, -0.430\,48, 0.752\,64, 0.498\,21)^{\mathrm{t}}, \\ \boldsymbol{h}_m &= (1, 0.430\,48, -0.752\,64, -0.498\,21)^{\mathrm{t}}. \end{aligned}$$

This result is identical with [13], showing the validity of Method 2. Table II shows the numbers of iteration and the results of calculation for different initial vectors. Note that one iteration by our method needs only to use 88 multiplication operations, eight division operations and four square roots. From the numbers of various operations, one knows that Method 2 is very effective.

Example 3: Consider the following Kennaugh matrices, which satisfy the Fry-Kattawar inequalities and the Barakat condition, both necessary constraints on the Kennaugh matrix elements [24], [26], [27]

The target

$$[K]_a = \begin{bmatrix} 1.38 & 0.75 & 0 & 0.75 \\ 0.75 & 1.12 & 0 & 0.25 \\ 0 & 0 & -0.87 & 0 \\ 0.75 & 0.25 & 0 & 1.13 \end{bmatrix}$$

TABLE III
RESULTS OF CALCULATION FOR EXAMPLE 3

initial vetors	iteration numbers	optimal polarization states	power ratio
$g_0 = (1, 1, 0, 0)'$	5	$g_m = (1, -0.090, 0.995, 0.035)'$ $h_m = (1, -0.987, -0.148, 0.066)'$	8.454
$g_0 = (1, -1, 0, 0)'$	4	$g_m = (1, -0.987, -0.148, 0.066)'$ $h_m = (1, -0.090, 0.995, 0.035)'$	8.454
$g_0 = (1, 0, 1, 0)'$	3	$g_m = (1, -0.090, 0.995, 0.035)'$ $h_m = (1, -0.987, -0.148, 0.066)'$	8.454
$g_0 = (1, 0, -1, 0)'$	4	$g_m = (1, -0.987, -0.148, 0.066)'$ $h_m = (1, -0.090, 0.995, 0.035)'$	8.454
$g_0 = (1, 0, 0, 1)'$	5	$g_m = (1, -0.987, -0.148, 0.066)'$ $h_m = (1, -0.090, 0.995, 0.035)'$	8.454
$g_0 = (1, 0, 0, -1)'$	5	$g_m = (1, -0.090, 0.995, 0.035)'$ $h_m = (1, -0.987, -0.148, 0.066)'$	8.454

TABLE IV
OPTIMAL POLARIZATION STATES AND THE MAXIMUM POWER RATIOS IN THE THREE CHANNELS

channel	optimum polarization state	power ratio
co - pol	$g_m = (1, -0.661, 0.550, 0.511)'$	2.558
x - pol	$g_m = (1, 0, \pm 1, 0)'$	1.500
m - pol	$g_m = (1, -0.443, 0.815, 0.372)'$	1.095

and the clutter

$$[K]_b = \begin{bmatrix} 3.1 & 1.5 & -1.5 & 0 \\ 1.5 & 1.4 & 0 & 0 \\ -1.5 & 0 & 1.6 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}.$$

Letting $\epsilon = 0.000\,01$ and selecting some initial vector as shown in Table III, then by Method 2, we obtain the optimal polarization states of the transmitting antenna and the receiving antenna as

$$\begin{aligned} \boldsymbol{g}_m &= (1, -0.090\,10, 0.995\,31, 0.035\,08)^{\mathrm{t}}, \\ \boldsymbol{h}_m &= (1, -0.986\,69, -0.148\,41, 0.066\,41)^{\mathrm{t}} \end{aligned}$$

or

$$\begin{aligned} \boldsymbol{g}_m &= (1, -0.986\,69, -0.148\,41, 0.066\,41)^{\mathrm{t}}, \\ \boldsymbol{h}_m &= (1, -0.090\,10, 0.995\,31, 0.035\,08)^{\mathrm{t}}. \end{aligned}$$

Under these polarization states, the corresponding maximum power ratio is 8.453 85. Table III provides the numbers of iteration and the results of calculation for different initial vectors, illustrating that Method 2 is very effective.

Table IV shows the optimal polarization states and the maximum power ratios in the cases of the co-polarized, the cross-polarized, and the matched-polarized channels. From this table, we find that the maximum power ratios are not so big in the cases of three channels, which means that they do not provide significant change in the contrast. Therefore, it is better to use the model (12) for the OPCE problem.

IV. Conclusion

In this paper, we proposed two effective numerical methods for obtaining the optimal contrast polarization states of the transmitter and the receiver for the coherent and incoherent case based on the formula of the optimal contrast polarization state in the matched-polarized channel [12] and the idea of the SUMT [25]. Both the proposed algorithms are convergent and straightforward for programming. In addition, the proposed methods can be applied to the bistatic radar case, because the scattering matrix and the Kennaugh matrix may be asymmetric. For illustrating the effectiveness of the proposed methods, we provided three examples. The results of calculation were completely identical with [13]–[15], showing the validity of the proposed methods. In these examples, if the receiving polarization state is independent of the transmitting's, the power ratios may be much larger than those in the cases of the co-polarized, the cross-polarized, and the matched-polarized channels, illustrating the importance of the considered models. Finally, we compared the computation cost by the illustrated examples, showing that the proposed methods are very effective.

Appendix
Proof of the Formula (10)

Let

$$r = \frac{A_0 + A_1 g_1 + A_2 g_2 + A_3 g_3}{B_0 + B_1 g_1 + B_2 g_2 + B_3 g_3} \tag{A.1}$$

and let r_m be the maximum of the above expression. Then we have for any $\boldsymbol{g}$ that

$$r_m \geq \frac{A_0 + A_1 g_1 + A_2 g_2 + A_3 g_3}{B_0 + B_1 g_1 + B_2 g_2 + B_3 g_3} \tag{A.2}$$

or

$$A_0 - r_m B_0 + (A_1 - r_m B_1) g_1 + (A_2 - r_m B_2) g_2 + (A_3 - r_m B_3) g_3 \leq 0. \tag{A.3}$$

The problem here is to find $\boldsymbol{g}$ such that the left-hand side of (A.3) will be maximal. Obviously, $A_0 - r_m B_0 + (A_1 - r_m B_1) g_1 + (A_2 - r_m B_2) g_2 + (A_3 - r_m B_3) g_3$ becomes zero if and only if

$$g_i = \frac{A_i - r_m B_i}{\sqrt{\sum_{i=1}^{3} (A_i - r_m B_i)^2}} \qquad (i = 1, 2, 3) \tag{A.4}$$

which yields

$$A_0 - r_m B_0 + \sqrt{\sum_{i=1}^{3} (A_i - r_m B_i)^2} = 0. \tag{A.5}$$

From this equation, we have

$$r_m = \frac{z_{12} + \sqrt{z_{12}^2 - z_1 z_2}}{z_2} \tag{A.6}$$

where

$$\begin{aligned} z_1 &= A_0^2 - A_1^2 - A_2^2 - A_3^2, \\ z_2 &= B_0^2 - B_1^2 - B_2^2 - B_3^2, \\ z_{12} &= A_0 B_0 - A_1 B_1 - A_2 B_2 - A_3 B_3. \end{aligned}$$

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