The Characteristic Polarization States and the Equi-Power Curves

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***Abstract*—Characteristic polarization state theory is restudied for the symmetric coherent Sinclair scattering matrix case. First, the geometric relations of the characteristic polarization states on the Poincaré sphere are derived. Based on these relations, simple formulas are given for all of the characteristic polarization states of this Sinclair matrix in Stokes vector form. From the formulation, it is clear that the CO-POL Nulls are fundamental characteristic polarization states for the symmetric coherent Sinclair scattering matrix case, in that the others can straightforwardly be obtained from the Stokes vectors of the CO-POL Nulls. For further study of the characteristic polarization state and the distribution of the received powers on the Poincaré sphere, we introduce the concept of the equi-power curve. It is defined as the curve on the Poincaré sphere on which the received powers in some defined channel have the same value. We deal with the characteristics of the equi-power curves for various special cases. In addition, we show how the characteristic polarization states are generated by the equi-power curves. It is demonstrated that the characteristic polarization states can usually be regarded as the points of contact of the Poincaré sphere and a conicoid representing a power-related quadratic form. This leads to a new method to introduce the characteristic polarization states. The new method provides a geometric interpretation for visualization of changes in polarized states on the Poincaré sphere.**

***Index Terms*—Polarization, radar polarimetry, scattering matrix.**

I. Introduction

THE problem with the characteristic polarization states was first considered by Kennaugh [1], and later was studied and redeveloped by Huynen [2], Boerner *et al.* [3]–[11], Van Zyl [12], [13], Mott [14], Lüneburg and Cloude [15], and Yamaguchi *et al.* [16]. This problem involves finding the polarization states for which the radar receives minimum/maximum power. In the symmetric coherent Sinclair scattering matrix case, it is known that there usually exist, in total, five pairs of characteristic polarization states: two CO-POL Nulls for which the radar receives zero power in the co-pol channel; one CO-POL Max for which the radar receives maximum power in the co-pol channel; and one CO-POL Saddle for which the radar receives critical power in the co-pol channel, two X-POL Nulls for which the radar receives zero power in the cross-pol channel, two X-POL Saddles for which the radar receives critical power in the cross-pol channel, and two X-POL Maxs for which the radar receives maximum power in the cross-pol channel. However, the CO-POL Saddle and the CO-POL Max are identical with the X-POL Nulls; and, therefore, there exist, in total, eight different physical characteristic polarization states for the symmetric coherent Sinclair scattering matrix case. On the Poincaré sphere, the CO-POL Nulls, the CO-POL Max, and the CO-POL Saddle form an interesting pattern, called the *Huynen fork* [2].

Up to now, although there have been several approaches applied for determining the characteristic polarization states, we still desire to find a set of "final more rigorous and complete formulations" for the characteristic polarization states of a coherent symmetric scattering matrix [7]. On the other hand, the Poincaré sphere and the Stokes vector are frequently used in radar polarimetry because the former is a useful graphical aid for the visualization of polarization effects. Therefore, it is important to find a set of formulas or a simple method to obtain the Stokes vectors of the characteristic polarization states in a more direct approach.

This paper consists of two parts. In the first part, we initially study the geometric relations of the characteristic polarization states on the Poincaré sphere for the symmetric coherent Sinclair scattering matrix case. Then, based on these relations, a simple formula is proposed to obtain all characteristic polarization states (in the Stokes vector form) in a closed comprehensive presentation. From this formulation, we observe that the CO-POL Nulls are the fundamental characteristic polarization states for the symmetric Sinclair scattering matrix case, in that the other characteristic polarization states can be obtained straightforwardly from the expressions of the CO-POL Nulls by using the proposed method. Finally, an example is given. The calculation results are identical with those given by other optimization approaches [4], [5], [7], [11], [16], [19], [20], corroborating the validity of the proposed method.

In the second part of this paper, the theory of the equi-power curves for three special channel cases are investigated systematically. Here, the equi-power curve is defined as the curve on

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the Poincaré sphere on which the received powers in some defined channel have the same value. Kennaugh [1] and Huynen [2] considered the similar problem in the co-pol channel case. Mott [14] further pursued the problem, based on the concept of the polarization match factor, but he did not use this concept for studying the problem on the characteristic polarization states. In this paper, we deal with the characteristics of the equi-power curves which are important for analyzing the distribution of the received powers on the Poincaré sphere. Besides, we show how the characteristic polarization states are generated by the equi-power curves. In this way, we provide a new method for introducing the characteristic polarization states, which is quite different from Kennaugh's method [1] (based on the voltage equation), Huynen's method [2] (based on the power equation), Boerner's method [4], [5] (based on the polarization ratio), and Yamaguchi's method [16] (based on the eigenvalue problem).

II. The Characteristic Polarization States in the CO-POL Channel and an Eigenvalue Problem

A. The Characteristic Polarization States in the Co-Pol Channel

Let

$$[S] = \begin{bmatrix} s_1 & s_2 \\ s_2 & s_3 \end{bmatrix} \tag{1}$$

denote *a symmetric scattering matrix* of a radar target, and let

$$[K] = \begin{bmatrix} k_{00} & k_{01} & k_{02} & k_{03} \\ k_{01} & k_{11} & k_{12} & k_{13} \\ k_{02} & k_{12} & k_{22} & k_{23} \\ k_{03} & k_{13} & k_{23} & k_{33} \end{bmatrix} \tag{2}$$

denote the corresponding Kennaugh matrix which is defined in the Appendix. If $v = \sqrt{k_{01}^2 + k_{02}^2 + k_{03}^2} \neq 0$, then the characteristic polarization states in the co-pol channel are given by [19] and [20]:

1) the CO-POL Max:

$$\boldsymbol{g}_m^c = (1, k_{01}/v, k_{02}/v, k_{03}/v)^t \tag{3}$$

2) the CO-POL Saddle:

$$\boldsymbol{g}_s^c = (1, -k_{01}/v, -k_{02}/v, -k_{03}/v)^t \tag{4}$$

3) for the case of $s_3 \neq 0$, the CO-POL Nulls:

$$g_1 = \frac{|s_3|^2 - |\lambda_{1,2} + s_2|^2}{|s_3|^2 + |\lambda_{1,2} + s_2|^2} \tag{5a}$$

$$g_2 = \frac{-2\,\mathrm{Re}(s_3^*(\lambda_{1,2} + s_2))}{|s_3|^2 + |\lambda_{1,2} + s_2|^2} \tag{5b}$$

$$g_3 = \frac{-2\,\mathrm{Im}(s_3^*(\lambda_{1,2} + s_2))}{|s_3|^2 + |\lambda_{1,2} + s_2|^2} \tag{5c}$$

for the case of $s_1 \neq 0$, the CO-POL Nulls:

$$g_1 = \frac{|\lambda_{1,2} - s_2|^2 - |s_1|^2}{|s_1|^2 + |\lambda_{1,2} - s_2|^2} \tag{6a}$$

$$g_2 = \frac{2\,\mathrm{Re}(s_1^*(\lambda_{1,2} - s_2))}{|s_1|^2 + |\lambda_{1,2} - s_2|^2} \tag{6b}$$

$$g_3 = \frac{-2\,\mathrm{Im}(s_1^*(\lambda_{1,2} - s_2))}{|s_1|^2 + |\lambda_{1,2} - s_2|^2}. \tag{6c}$$

In the above equations, the superscripts t and $*$ denote the transposition of a vector and the complex conjugation, respectively; g_i $(i = 1, 2, 3)$ denotes the components of the sub-Stokes vectors of the CO-POL Nulls $\boldsymbol{g}_{n1}^c$ and $\boldsymbol{g}_{n2}^c; and \lambda_{1,2}$ is given by

$$\lambda_{1,2} = \pm\sqrt{s_2^2 - s_1 s_3}. \tag{7}$$

B. An Eigenvalue Problem

In the cross-pol channel, the received power is expressed as

$$P_x = \frac{1}{2}\boldsymbol{g}^t[K]_x\boldsymbol{g} \tag{8}$$

where $\boldsymbol{g} = (1, g_1, g_2, g_3)^t$ denotes the Stokes vector of the polarization state of the transmitter (without loss of generality, we assume that $g_1^2 + g_2^2 + g_3^2 = 1$ for all the Stokes vectors in this paper); and $[K]_x$ is the modified Kennaugh matrix, given by

$$[K]_x = \begin{bmatrix} k_{00} & k_{01} & k_{02} & k_{03} \\ -k_{01} & -k_{11} & -k_{12} & -k_{13} \\ -k_{02} & -k_{12} & -k_{22} & -k_{23} \\ -k_{03} & -k_{13} & -k_{23} & -k_{33} \end{bmatrix}. \tag{9}$$

Let

$$\boldsymbol{X} = [g_1, g_2, g_3]^t$$

and

$$[J_k] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{12} & k_{22} & k_{23} \\ k_{13} & k_{23} & k_{33} \end{bmatrix}. \tag{10}$$

Then (8) can be rewritten as

$$P_x = \frac{1}{2}(k_{00} - \boldsymbol{X}^t[J_k]\boldsymbol{X}). \tag{11}$$

By solving the following eigenvalue problem:

$$[J_k]\boldsymbol{X} = \lambda\boldsymbol{X} \tag{12}$$

we obtain three linearly independent eigenvectors $\boldsymbol{X}_i (i = 1, 2, 3)$. It is easy to prove that $\pm\boldsymbol{X}_i (i = 1, 2, 3)$ are the X-POL Maxs, the X-POL Saddles, and the X-POL Nulls [16]. According to the definition of the X-POL Nulls, we know from (11) that the eigenvalue corresponding to the X-POL Nulls is

$$\lambda_n = k_{00}. \tag{13}$$

Denote the eigenvalues corresponding to the X-POL Maxs and the X-POL Saddles as λ_m and λ_s, respectively. Then we know from algebraic theory that

$$\lambda_n + \lambda_m + \lambda_s = \mathrm{trace}[J_k] = k_{11} + k_{22} + k_{33} \tag{14a}$$

$$\lambda_n\lambda_m\lambda_s = \det[J_k]. \tag{14b}$$

Since $k_{11} + k_{22} + k_{33} = k_{00}$, we know from (13) and (14) that

$$\lambda_s + \lambda_m = 0 \tag{15a}$$

$$\lambda_s\lambda_m = \frac{\det[J_k]}{k_{00}}. \tag{15b}$$

Note that the received power corresponding to the X-POL Max is greater then or equal to that corresponding to the X-POL

Saddle. Therefore, we have from (11) that $\lambda_s \geq \lambda_m$. This result, together with (15a) and (15b), lead to

$$\lambda_s = \sqrt{\frac{\det[J_k]}{-k_{00}}} \geq 0 \tag{16a}$$

$$\lambda_m = -\sqrt{\frac{\det[J_k]}{-k_{00}}} \leq 0. \tag{16b}$$

In this way, all eigenvalues of the matrix $[J_k]$ are obtained. We will make use of the above results later.

III. New Approach for Obtaining the Characteristic Polarization States

There exist some relations in the characteristic polarization states of the symmetric coherent Sinclair scattering matrix. Kennaugh [1] first noticed these relations. Later, Huynen tried to prove the relations by using his parameters and deducing the famous *polarization fork* concept [2]. Boerner *et al.* extended this work, based on the polarization ratio [4], [5]. This section tries to derive the geometric relations of the characteristic polarization states on the Poincaré sphere for the symmetric coherent Sinclair scattering matrix case. Then, based on these results and the formula of the CO-POL Nulls, a new approach is proposed to obtain all the characteristic polarization states in the Stokes vector form.

On the Poincaré sphere, let $\boldsymbol{G}_c(m)$, $\boldsymbol{G}_c(s)$, and $\boldsymbol{G}_c^{1,2}(n)$ denote the CO-POL Max, the CO-POL Saddle, and the CO-POL Nulls, respectively; and let $\boldsymbol{G}_x^{1,2}(m)$, $\boldsymbol{G}_x^{1,2}(s)$, and $\boldsymbol{G}_x^{1,2}(n)$ denote the X-POL Maxs, the X-POL Saddles, and the X-POL Nulls, respectively. Here, $\boldsymbol{G}_c(m)$, $\boldsymbol{G}_c(s)$, $\boldsymbol{G}_c^{1,2}(n)$, $\boldsymbol{G}_x^{1,2}(m)$, $\boldsymbol{G}_x^{1,2}(s)$, and $\boldsymbol{G}_x^{1,2}(n)$ are expressed in the form of the sub-Stokes vectors and their tips are denoted as $G_c(m)$, $G_c(s)$, $G_c^{1,2}(n)$, $G_x^{1,2}(m)$, $G_x^{1,2}(s)$, and $G_x^{1,2}(n)$, respectively (see Fig. 1).

In this section, we assume that $\boldsymbol{G}_c^2(n) \neq \pm\boldsymbol{G}_c^1(n)$ The problem for the cases of $\boldsymbol{G}_c^1(n) = \pm\boldsymbol{G}_c^2(n)$ will be studied in Section V. Under the assumption of $\boldsymbol{G}_c^2(n) \neq \pm\boldsymbol{G}_c^1(n)$, the geometric relations of the characteristic polarization states are expressed as

- R1) $\boldsymbol{G}_x^1(n) = \boldsymbol{G}_c(m), \boldsymbol{G}_x^2(n) = \boldsymbol{G}_c(s)$;
- R2) $\boldsymbol{G}_c^1(n)$ and $\boldsymbol{G}_c^2(n)$ are symmetric about the segment $\overline{G_c(m)G_c(s)}$;
- R3) $\boldsymbol{G}_x^{1,2}(m)$ is parallel to the segment $\overline{G_c^1(n)G_c^2(n)}$;
- R4) $\boldsymbol{G}_x^{1,2}(s)$ is perpendicular to the segment $\overline{G_c^1(n)G_c^2(n)}$ as well as to the segment $\overline{G_x^1(n)G_x^2(n)}$.

Now let us prove the above relations.

Proof of (R1): Let $\boldsymbol{a}$ denote the X-POL Nulls, then $[S]\boldsymbol{a} \cdot \boldsymbol{a}^\perp = 0$, which yields Kennaugh's pseudoeigenvalue equation

$$[S]\boldsymbol{a} = \lambda \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \boldsymbol{a}^\perp = \lambda \boldsymbol{a}^*$$

where $\boldsymbol{a}^\perp$ is defined as $\boldsymbol{a}^\perp = [\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}]\boldsymbol{a}^*$. This leads to

$$[S]^H[S]\boldsymbol{a} = |\lambda|^2\boldsymbol{a}. \tag{17a}$$

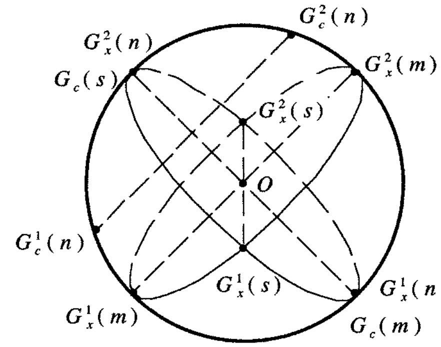

Fig. 1. Characteristic polarization states on the Poincaré sphere.

The eigenvectors of this problem are the solutions of the following problem:

$$\text{optimize } \boldsymbol{a}^H[S]^H[S]\boldsymbol{a} \tag{17b}$$

or

$$\text{optimize } k_{00} + k_{01}g_1 + k_{02}g_2 + k_{03}g_3 \tag{17c}$$

where $g_i(i = 1, 2, 3)$ denotes the components of the sub-Stokes vector of $\boldsymbol{a}$. Obviously, the solutions of (17c) are identical with the CO-POL Max and the CO-POL Saddle. #

Proof of (R2): From the assumption $\boldsymbol{G}_c^2(n) \neq \pm\boldsymbol{G}_c^1(n)$, it is easy to prove that $|s_1| + |s_3| \neq 0$. For the case of $s_3 \neq 0$, we have from (7) that $\lambda_1 = -\lambda_2$ and $\lambda_1^2 = \lambda_2^2 = s_2^2 - s_1 s_3$. Using this result and (5), we can prove that

$$\boldsymbol{G}_c^1(n) + \boldsymbol{G}_c^2(n) = \frac{-2|s_3|^2 v}{(|s_3|^2 + |\lambda_1 + s_2|^2)(|s_3|^2 + |\lambda_2 + s_2|^2)}\boldsymbol{G}_c(m). \tag{18a}$$

For the case of $s_1 \neq 0$, we have from (6) that

$$\boldsymbol{G}_c^1(n) + \boldsymbol{G}_c^2(n) = \frac{-2|s_1|^2 v}{(|s_1|^2 + |\lambda_1 - s_2|^2)(|s_1|^2 + |\lambda_2 - s_2|^2)}\boldsymbol{G}_c(m). \tag{18b}$$

Note that $\boldsymbol{G}_c^1(n)$ and $\boldsymbol{G}_c^2(n)$ have the same amplitude. From the above equations, we deduce that (R2) holds. #

Proof of (R3): Let

$$\boldsymbol{v}^t = [k_{01}, k_{02}, k_{03}]. \tag{19}$$

Then using (10) and applying the Lagrangian multiplier method to the problem

$$\begin{cases} \text{minimize } \frac{1}{2}\boldsymbol{g}^t[K]_c\boldsymbol{g} \\ \text{subject to: } g_1^2 + g_2^2 + g_3^2 = 1 \end{cases}$$

we can prove that there exist two real numbers, η_1 and η_2, such that

$$[J_k]\boldsymbol{G}_c^1(n) + v = \boldsymbol{\eta}_1\boldsymbol{G}_c^1(n) \tag{20a}$$

$$[J_k]\boldsymbol{G}_c^2(n) + v = \boldsymbol{\eta}_2\boldsymbol{G}_c^2(n). \tag{20b}$$

According to the definition of the CO-POL Nulls, we have

$$[J_k]\boldsymbol{G}_c^{1,2}(n) \cdot \boldsymbol{G}_c^{1,2}(n) + 2v \cdot \boldsymbol{G}_c^{1,2}(n) + k_{00} = 0.$$

Substituting (20) into the above equation, we can easily obtain $\eta_1 = -k_{00} - \boldsymbol{v} \cdot \boldsymbol{G}_c^1(n)$ and $\eta_2 = -k_{00} - \boldsymbol{v} \cdot \boldsymbol{G}_c^2(n)$. Note that $\boldsymbol{v} = v\boldsymbol{G}_c(m)$, where $v = \|\boldsymbol{v}\| = \sqrt{k_{01}^2 + k_{02}^2 + k_{03}^2}$. From (R2), we deduce that $\eta_1 = \eta_2$, denoted as η. Therefore, we have from (20a) and (20b) that

$$[J_k]\left(\boldsymbol{G}_c^1(n) - \boldsymbol{G}_c^2(n)\right) = \eta\left(\boldsymbol{G}_c^1(n) - \boldsymbol{G}_c^2(n)\right). \tag{21}$$

Note that $\eta = -k_{00} - \boldsymbol{v}\cdot\boldsymbol{G}_c^{1,2}(n) \leq -k_{00} + \|\boldsymbol{v}\| \leq 0$. Comparing (21) with (12) and (16), we can deduce that $\eta = \lambda_m$, which means that (R3) is true. #

Proof of (R4): Since $[J_k]$ is a real symmetric matrix and it does not have multiple eigenvalues if $\boldsymbol{G}_c^2(n) \neq \pm\boldsymbol{G}_c^1(n)$, therefore $\boldsymbol{G}_x^{1,2}(s)$, $\boldsymbol{G}_x^{1,2}(m)$, and $\boldsymbol{G}_x^{1,2}(n)$, the eigenvectors of the matrix $[J_k]$, are orthogonal. From this orthogonality and the results (R1-3), we deduce that (R4) is true. #

Based on the above relations and the formulas (5) and (6), the characteristic polarization states of a symmetric coherent Sinclair scattering matrix can be obtained by the following simple method.

1) *The CO-POL Nulls* $\boldsymbol{G}_c^{1,2}(n)$ are given by (5) and (6).
2) *The CO-POL Max* $\boldsymbol{G}_c(m)$ is

$$\boldsymbol{G}_c(m) = -\left(\boldsymbol{G}_2^1(n) + \boldsymbol{G}_c^2(n)\right) / \left\|\boldsymbol{G}_c^1(n) + \boldsymbol{G}_c^2(n)\right\|. \tag{22}$$

3) *The CO-POL Saddle* $\boldsymbol{G}_c(s)$ is

$$\boldsymbol{G}_c(s) = \left(\boldsymbol{G}_c^1(n) + \boldsymbol{G}_c^2(n)\right) / \left\|\boldsymbol{G}_c^1(n) + \boldsymbol{G}_c^2(n)\right\|. \tag{23}$$

4) *The X-POL Nulls* $\boldsymbol{G}_x^{1,2}(n)$ are

$$\boldsymbol{G}_x^1(n) = \boldsymbol{G}_c(m), \quad \boldsymbol{G}_x^2(n) = \boldsymbol{G}_c(s). \tag{24}$$

5) *The X-POL Maxs* $\boldsymbol{G}_x^{1,2}(m)$ are

$$\boldsymbol{G}_x^1(m), \boldsymbol{G}_x^2(m) = \pm\, \underline{\boldsymbol{G}_c^1(n)\boldsymbol{G}_c^2(n)} \Big/ \left\|\underline{\boldsymbol{G}_c^1(n)\boldsymbol{G}_c^2(n)}\right\| \tag{25}$$

where $\underline{G_c^1(n)G_c^2(n)}$ denotes the vector from the point $G_c^1(n)$ to the point $G_c^2(n)$ and "=" means that both sides of the above equation have the same components.

6) *The X-POL Saddles* $\boldsymbol{G}_x^{1,2}(s)$ are

$$\boldsymbol{G}_x^1(s), \boldsymbol{G}_x^2(s) = \pm\left(\boldsymbol{G}_c^1(n) \times \boldsymbol{G}_c^2(n)\right) / \left\|\boldsymbol{G}_c^1(n) \times \boldsymbol{G}_c^2(n)\right\| \tag{26}$$

where "×" denotes the vector product. By the above method, we can easily obtain all the characteristic polarization states in the Stokes vector form for the case of $\boldsymbol{G}_c^1(n) \neq \pm\boldsymbol{G}_c^2(n)$. Let us use an example for showing the validity of the proposed method.

Example: Let $[S] = \begin{bmatrix} 2i & 0.5 \\ 0.5 & -1 \end{bmatrix}$ be a scattering matrix of a radar target. According to (5), we obtain the CO-POL Nulls $\boldsymbol{G}_c^1(n) = (-(1/3), (\sqrt{7}/3), -(1/3))^t$ and $\boldsymbol{G}_c^2(n) = (-(1/3), -(\sqrt{7}/3), -(1/3))^t$. Then, from (22) to (26), the following results can be obtained easily:

$$\text{CO-POL Max: } \boldsymbol{G}_c(m) = \left(\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)^t$$

$$\text{CO-POL Saddle: } \boldsymbol{G}_c(s) = \left(-\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2}\right)^t$$

$$\text{X-POL Maxs: } \boldsymbol{G}_x^1(m) = (0,1,0)^t \text{ and } \boldsymbol{G}_x^2(m) = (0,-1,0)^t$$

$$\text{X-POL Saddles: } \boldsymbol{G}_x^1(s) = \left(\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2}\right)^t$$

and

$$\boldsymbol{G}_x^2(s) = \left(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)^t$$

$$\text{X-POL Nulls: } \boldsymbol{G}_x^1(n) = \left(\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)^t$$

and

$$\boldsymbol{G}_x^2(n) = \left(-\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2}\right)^t.$$

These results are identical to those found by using other optimization approaches [4], [5], [7], [11], [16], [19], [20], corroborating the validity of the proposed method.

IV. Equi-Power Curves of the Received Power

In this section, we also assume that $\boldsymbol{G}_c^1(n) \neq \pm\boldsymbol{G}_c^2(n)$. The problem for the cases of $\boldsymbol{G}_c^1(n) \neq \pm\boldsymbol{G}_c^2(n)$ will be studied in Section V.

A. The Co-Pol Channel Case

In the co-pol channel, let C be an arbitrary constant and $0 \leq C \leq \max P_c$. Then,

$$\begin{cases} P_c = \frac{1}{2}\boldsymbol{g}^t[K]_c\boldsymbol{g} = C \\ g_1^2 + g_2^2 + g_3^2 = 1 \end{cases} \tag{27}$$

represents a curve on the Poincaré sphere. We call this curve *the equi-power curve* in the co-pol channel because the received powers on this curve have the same value. Now let us consider the shape of this equi-power curve. Using (10), we can express the received power in the co-pol channel as

$$\begin{aligned} P_c &= \frac{1}{2}\boldsymbol{g}^t[K]_c\boldsymbol{g} = \frac{1}{2}([J_k]\boldsymbol{X}\cdot\boldsymbol{X} + 2\boldsymbol{v}\cdot\boldsymbol{X} + k_{00}) \\ &= \frac{1}{2}([J_k]_+\boldsymbol{X}\cdot\boldsymbol{X} + 2\boldsymbol{v}\cdot\boldsymbol{X}) \end{aligned} \tag{28}$$

where $[J_k]_+$ is given by

$$[J_k]_+ = \begin{bmatrix} k_{11}+k_{00} & k_{12} & k_{13} \\ k_{12} & k_{22}+k_{00} & k_{23} \\ k_{13} & k_{23} & k_{33}+k_{00} \end{bmatrix}. \tag{29}$$

According to algebra theory, $P_c = (1/2)([J_k]_+\boldsymbol{X}\cdot\boldsymbol{X} + 2\boldsymbol{v}\cdot\boldsymbol{X}) = C$ represents a conicoid, the shape of which is determined by the matrix $[J_k]_+$. From (13), (16), and the proof of (R3), the eigenvalues of the matrix $[J_k]_+$ are

$$\lambda_1^+ = 2k_{00} > 0 \tag{30a}$$
$$\lambda_2^+ = -\boldsymbol{v}\cdot\boldsymbol{G}_c^2(n) > 0 \tag{30b}$$
$$\lambda_3^+ = 2k_{00} + \boldsymbol{v}\cdot\boldsymbol{G}_c^2(n) > 0. \tag{30c}$$

Therefore, $P_c = (1/2)([J_k]_+ \boldsymbol{X}\cdot\boldsymbol{X} + 2\boldsymbol{v}\cdot\boldsymbol{X}) = C$ represents *an ellipsoid*, and the equi-power curve (27) is the intersection of the ellipsoid and the Poincaré sphere. Fig. 2(b) and (e) shows the equi-power curves in the co-pol channel on the Poincaré sphere. Letting C vary from 0 to $\max P_c$, we can find the process by which the characteristic polarization states in the co-pol channel are derived, which is quite different from the others [1], [2], [4], [5], [16].

When $C = 0$, the ellipsoid is tangent to the Poincaré sphere at two points: the CO-POL Nulls [see Fig. 2(a)]; when $C = \max P_c$, the ellipsoid is tangent to the Poincaré sphere at another point: the CO-POL Max [see Fig. 2(d)]; when $C = 2k_{00} - \max P_c$, the ellipsoid is tangent to the Poincaré sphere at the opposite point of the CO-POL Max: the CO-POL Saddle [see Fig. 2(c)]. Furthermore, *the characteristic polarization states in the co-pol channel can be regarded as the points of contact between the special ellipsoids and the Poincaré sphere*.

Note that the center and main directions of the ellipsoid are independent of the constant C. Therefore, for any constant $C(0 \le C \le \max P_c)$, the corresponding ellipsoid has three fixed main directions (the same as $\boldsymbol{G}_x(s)$, $\boldsymbol{G}_x(m)$, and $\boldsymbol{G}_x(n)$) and a fixed center. According to analytic geometry, we know that the center $\boldsymbol{c}$ of the ellipsoid $P_c = (1/2)([J_k]_+ \boldsymbol{X}\cdot\boldsymbol{X} + 2\boldsymbol{v}\cdot\boldsymbol{X}) = C$ is determined by

$$[J_k]_+ \boldsymbol{c} = -\boldsymbol{v}.$$

B. The Cross-Pol Channel Case

In the cross-pol channel, let C be an arbitrary constant and $0 \le P \le \max P_x$. Then,

$$\begin{cases} P_x = \frac{1}{2}\boldsymbol{g}^t[K]_x\boldsymbol{g} = C \\ g_1^2 + g_2^2 + g_3^2 = 1 \end{cases} \tag{31}$$

represents a curve on the Poincaré sphere. We call this curve *the equi-power curve in the cross-pol channel*. From (11), the received power in the cross-pol channel can also be expressed as

$$P_x = \frac{1}{2}(k_{00} - [J_k]\boldsymbol{X}\cdot\boldsymbol{X}) = \frac{1}{2}[J_k]_- \boldsymbol{X}\cdot\boldsymbol{X}$$

where

$$[J_k]_- = -\begin{bmatrix} k_{11}-k_{00} & k_{12} & k_{13} \\ k_{12} & k_{22}-k_{00} & k_{23} \\ k_{13} & k_{23} & k_{33}-k_{00} \end{bmatrix}. \tag{32}$$

From (13), (16), (29), and (30), we know that the eigenvalues of the matrix $[J_k]_-$ are

$$\lambda_1^- = 0 \tag{33a}$$
$$\lambda_2^- = 2k_{00} + \boldsymbol{v}\cdot\boldsymbol{G}_c^2(n) > 0 \tag{33b}$$
$$\lambda_3^- = -\boldsymbol{v}\cdot\boldsymbol{G}_c^2(n) > 0. \tag{33c}$$

Therefore, $P_x = (1/2)[J_k]_- \boldsymbol{X}\cdot\boldsymbol{X} = C$ represents *an elliptic cylinder* and the equi-power curve in the cross-pol channel is the intersection of the elliptic cylinder and the Poincaré sphere [see Fig. 3(b) and (e)]. When $C = 0$, the cylinder degenerates into a line and the equi-power curve degenerates into two points: the X-POL Nulls [see Fig. 3(a)]. When $C = \max P_x$, the equi-power curve degenerates into two other points: the X-POL Maxs [see Fig. 3(d)]. When $C = 2k_{00} - \max P_x$, the cylinder

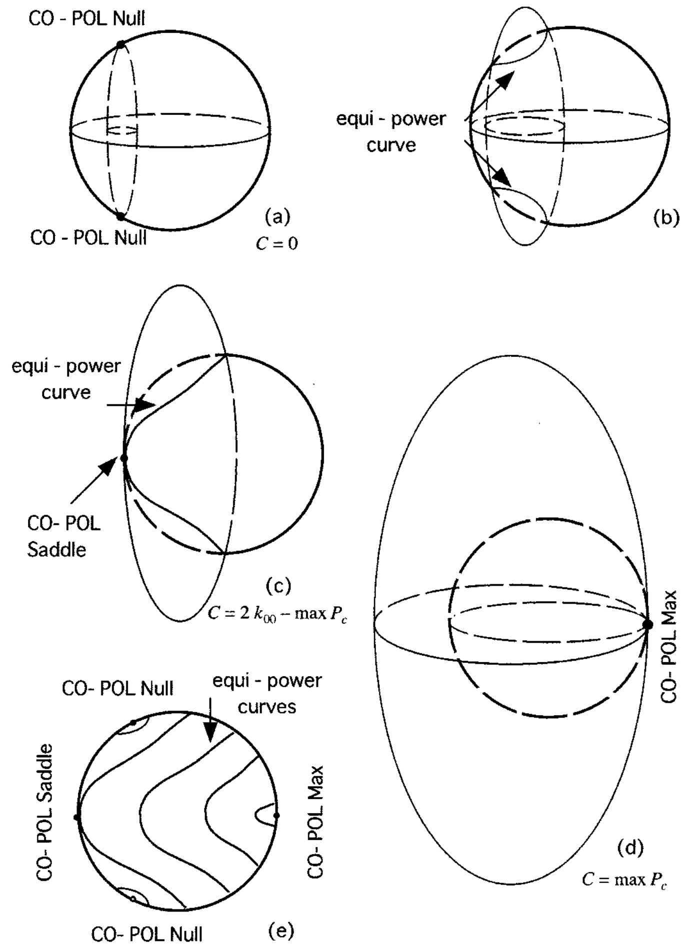

Fig. 2. Equi-power curves and the characteristic polarization states in the co-pol channel.

is tangent to the Poincaré sphere at two other points: the X-POL Saddles [see Fig. 3(c)]. In general, the elliptic cylinder intersects the Poincaré sphere, forming the equi-power curve in the cross-pol channel [see Fig. 3(b) and (e)]. Therefore, we can use the above equi-power curves for deriving the characteristic polarization states in the cross-pol channel.

It should be pointed out that for an arbitrary constant C, the central line of the elliptic cylinder $P_x = (1/2)[J_k]_- \boldsymbol{X}\cdot\boldsymbol{X} = C$ is $\boldsymbol{X} = \boldsymbol{v}t$, and that the X-POL Nulls are located at this line, where $\boldsymbol{v}$ is given by (19).

C. The Matched-Pol Channel Case

The received power in the matched-pol channel is

$$P_m = k_{00} + k_{01}g_1 + k_{02}g_2 + k_{03}g_3. \tag{34}$$

If $v = \sqrt{k_{01}^2 + k_{02}^2 + k_{03}^2} = 0$, then the received power is a constant k_{00}. Let us consider the case of $v \ne 0$. For an arbitrary constant $C(\min P_m \le C \le \max P_m)$

$$\begin{cases} P_m = k_{00} + k_{01}g_1 + k_{02}g_2 + k_{03}g_3 = C \\ g_1^2 + g_2^2 + g_3^2 = 1 \end{cases} \tag{35}$$

represents a circle, called *the equi-power circle in the matched-pol channel* [see Fig. 4(b) and (d)]. This equi-power circle is formed by using the plane $P_m = C$ to cut the Poincaré sphere. When $C = \min P_m$, the plane $P_m = C$ is tangent to the Poincaré sphere at one point: the *M-POL Min* [see Fig. 4(a)]; when $C = \max P_m$, the

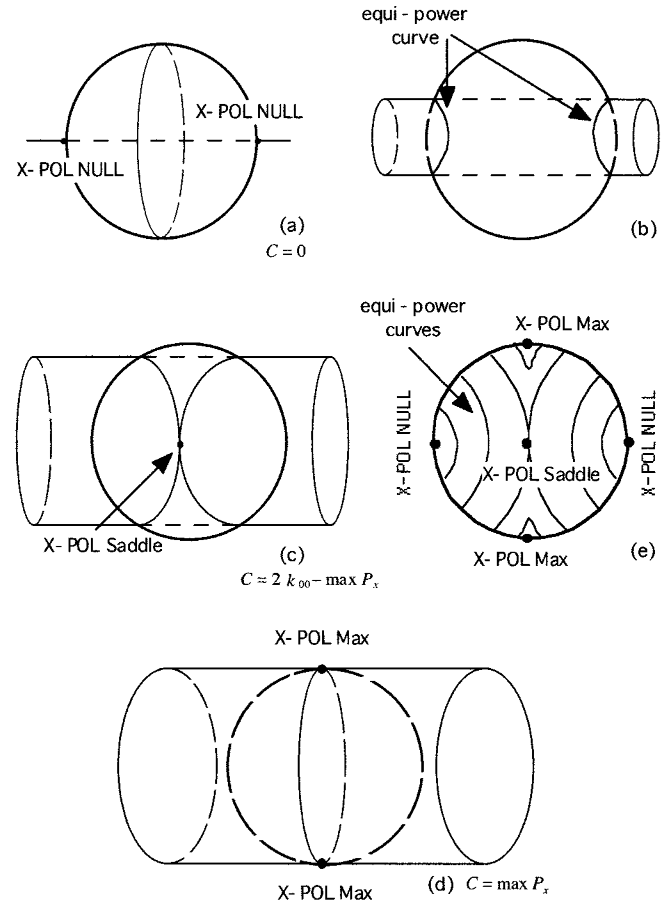

Fig. 3. Equi-power curves and the characteristic polarization states in the cross-pol channel.

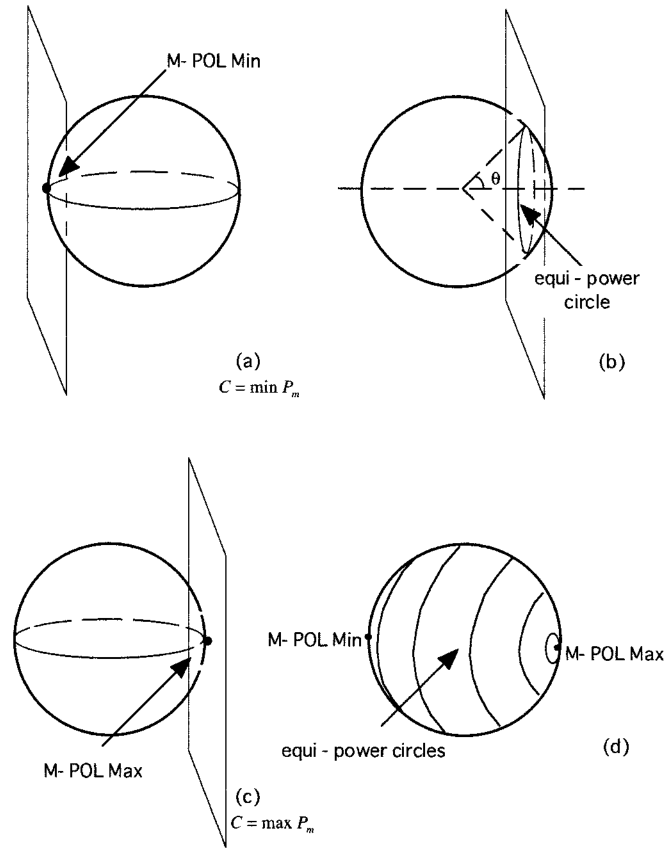

Fig. 4. Characteristic polarization states and the equi-power curves in the matched-pol channel.

plane $P_m = C$ is tangent to the Poincaré sphere at another point: the *M-POL Max* [see Fig. 4(c)]. If $\min P_m \leq C \leq \max P_m$, the plane intersects the Poincaré sphere, forming the equi-power circle [see Fig. 4(b) and (d)].

Let $C = k_{00} + v\cos\theta$, then θ has a concrete geometric meaning, shown in Fig. 4(b). Obviously, the range of θ is $0 \leq \theta \leq \pi$. $\theta = 0$ and $\theta = \pi$ correspond to the M-POL Max and the M-POL Min, respectively.

V. The Characteristic Polarization States and the Equi-Power Curves for Two Special Cases

In this section, we study the characteristic polarization states and the equi-power curves for two special cases: $\boldsymbol{G}_c^2(n) = \boldsymbol{G}_c^1(n)$ and $\boldsymbol{G}_c^2(n) = -\boldsymbol{G}_c^1(n)$.

A. Case of $\boldsymbol{G}_c^2(n) = -\boldsymbol{G}_c^1(n)$

First, let us prove the following conclusion:

$v = \sqrt{k_{01}^2 + k_{02}^2 + k_{03}^2} = 0$ holds if and only if $\boldsymbol{G}_c^2(n) = -\boldsymbol{G}_c^1(n)$.

Proof: From (3) and (19), we know that $\boldsymbol{v} = v\boldsymbol{G}_c(m)$. If $s_1 = s_3 = 0$, it is easy to check that $\boldsymbol{G}_c^2(n) = -\boldsymbol{G}_c^1(n)$ and $\boldsymbol{v} = 0$; and if $s_3 \neq 0$, or $s_1 = 0$, we can observe from (18) that $\boldsymbol{G}_c^2(n) = -\boldsymbol{G}_c^1(n)$. Summing up all cases, we deduce that the above conclusion is true. #

From $v = \sqrt{k_{01}^2 + k_{02}^2 + k_{03}^2} = 0$ and the expression of the Kennaugh matrix [14], we know that the scattering matrix for the case of $\boldsymbol{G}_c^2(n) = -\boldsymbol{G}_c^1(n)$ has the following form:

$$[S] = A\begin{bmatrix} r_1 & r_2 e^{\theta j} \\ r_2 e^{\theta j} & -r_1 e^{2\theta j} \end{bmatrix} \tag{36}$$

where A is a complex number, and r_1, r_2, and θ are real numbers. Obviously, sphere targets (or plates) and diplanes belong to this kind of targets.

Note that for the case of $s_1 = s_3 = 0$, the scattering matrix of the target is $[S] = s_2\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. The CO- POL Nulls of this target (diplane) are given by

$$g_1 = \pm 1, \quad g_2 = 0, \quad g_3 = 0. \tag{37}$$

In the co-pol channel, (30a)–(30c) become $\lambda_1^+ = 2k_{00}$, $\lambda_2^+ = 0$, and $\lambda_3^+ = 2k_{00}$, respectively. Therefore, the conicoid $P_c = (1/2)([J_k]_+\boldsymbol{X} \cdot \boldsymbol{X} + 2\boldsymbol{v} \cdot \boldsymbol{X} = C$ is a cylinder. Letting C be expressed by different constants, we then obtain the corresponding equi-power curves in the co-pol channel, consisting of a set of circles [see Fig. 5]. When $C = 0$, the cylinder degenerates into a line through two points of the Poincaré sphere: the CO-POL Nulls [see Fig. 5(a)]; when $C = \max P_c$, the equi-power curve is a circle, having the same center as the Poincaré sphere. We denote this circle as the *CO-POL Max Circle* [Fig. 5(c)]. Because $\overline{G_c^1(n)G_c^2(n)}$ is orthogonal to the plane in which the CO-POL Max

Circle lies, the equation of the plane is $\boldsymbol{G}_c^1(n) \cdot \boldsymbol{X} = 0$. Therefore, the equation of the CO-POL Max Circle can be written as

$$\begin{cases} \boldsymbol{G}_c^1(n) \cdot \boldsymbol{X} = 0 \\ g_1^2 + g_2^2 + g_3^2 = 1. \end{cases} \tag{38}$$

According to the above analysis, we conclude the following conclusion.

For the case of $\boldsymbol{G}_c^2(n) = -\boldsymbol{G}_c^1(n)$, the characteristic polarization states in the co-pol channel become: a pair of CO-POL Nulls and a CO-POL Max Circle. The former is determined by (5), (6), or (37), and the latter is determined by (38).

Now let us consider *the cross-pol channel case*. When $\boldsymbol{G}_c^2(n) = -\boldsymbol{G}_c^1(n)$, we know from (33) that the eigenvalues of the matrix $[J_k]_-$ are $\lambda_1^- = 0$, $\lambda_2^- = 2k_{00}$, and $\lambda_3^- = 0$. Therefore, $P_x = (1/2)[J_k]_- \boldsymbol{X} \cdot \boldsymbol{X} = C$ degenerates into two planes. When $C = 0$, the two planes coincide, intersecting the Poincaré sphere in a circle [see Fig. 6(a)]. We call this circle the *X-POL Null Circle* [see Fig. 6(a)]. Obviously, this circle is the same as the CO-POL Max Circle [see Fig. 5(c)]. When $C = k_{00}$, $P_x (1/2)[J_k]_- \boldsymbol{X} \cdot \boldsymbol{X} = k_{00}$ represents two planes, tangent to the Poincaré sphere at two points: the X-POL Maxs [see Fig. 6(c)], which are the same as the CO-POL Nulls [see Fig. 5(a)]. Fig. 6 shows the equi-power curves in the cross-pol channel.

According to the above analysis, we obtain the following conclusion.

For the case of $\boldsymbol{G}_c^2(n) = -\boldsymbol{G}_c^1(n)$, the characteristic polarization states in the cross-pol channel become an X-POL Null Circle and a pair of X-POL Maxs. The equation of the X-POL Null Circle is given by (36). The X-POL Maxs are the same as the CO-POL Nulls which are determined by (5), (6), or (37).

B. *Case of* $\boldsymbol{G}_c^2(n) = \boldsymbol{G}_c^1(n)$

From (5), (6), and (7), it is not difficult to prove that $\boldsymbol{G}_c^2(n) = \boldsymbol{G}_c^1(n)$ *is equivalent to* $s_2^2 - s_1 s_3 = 0$. *Therefore, the rank of the scattering matrices corresponding to the case of* $\boldsymbol{G}_c^2(n) = \boldsymbol{G}_c^1(n)$ *is 1*. For this reason, we call these kind of targets *rank-1 targets*. Wires and helixes are the typical targets of rank-1.

Using the above method, we can also obtain the equi-power curves in the co-pol and cross-pol channels, consisting of a set of circles. It is easy to obtain the following conclusion.

For the case of $\boldsymbol{G}_c^2(n) = \boldsymbol{G}_c^1(n)$, the characteristic polarization states in the co-pol channel are two identical CO-POL Nulls and one CO-POL Max. Both of them can be obtained by the proposed method in Section IV.

For the case of $\boldsymbol{G}_c^2(n) = \boldsymbol{G}_c^1(n)$, the characteristic polarization states in the cross-pol channel become: one pair of X-POL Nulls and one X-POL Max Circle. The former is given by

$$\boldsymbol{G}_x^{1,2}(n) = \pm(v_{1/v}, v_{2/v}, v_{3/v})^t \tag{39}$$

and the latter is given by

$$\begin{cases} \boldsymbol{v} \cdot \boldsymbol{X} = 0 \\ g_1^2 + g_2^2 + g_3^2 = 1 \end{cases}. \tag{40}$$

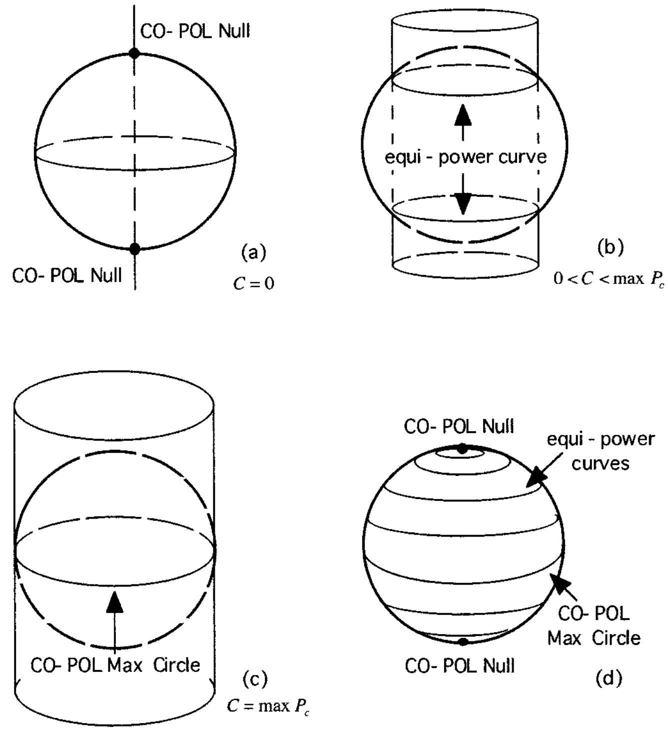

Fig. 5. Equi-power curves in the case of the co-pol channel when $\boldsymbol{G}_c^2(n) = -\boldsymbol{G}_c^1(n)$.

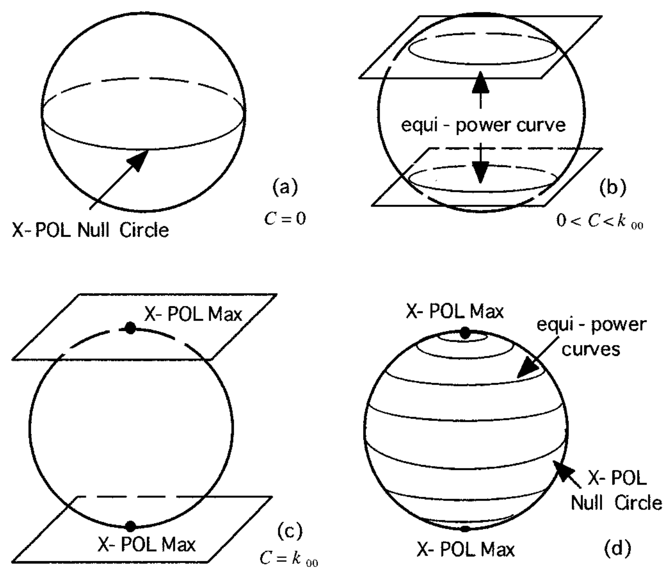

Fig. 6. Equi-power curves in the case of the cross-pol channel when $\boldsymbol{G}_c^2(n) = -\boldsymbol{G}_c^1(n)$.

VI. Summary

This paper has restudied the Kennaugh characteristic polarization state theory for the symmetric coherent Sinclair scattering matrix case. First, we proved some results about the geometric relations of the characteristic polarization states on the Poincaré sphere by mathematical methods. Based on these relations, a very simple method has been proposed for obtaining all the characteristic polarization states in the Stokes vector form. From the proposed method, we observe that the CO-POL Nulls are the fundamental characteristic polarization states for the symmetric coherent Sinclair scattering matrix case, in that the others can be obtained easily from the Stokes vectors of the CO-POL Nulls. By a concrete example, the validity of the proposed method has been shown.

In Section II, we obtained the eigenvalues of an important matrix, based on algebraic theory. Using this result, we have studied the problem of the equi-power curves on the Poincaré sphere. In Sections IV and V, we showed the equi-power curves on the Poincaré sphere for three special channel cases; and we also showed how the characteristic polarization states are generated by using the equi-power curves. For the case of $\boldsymbol{G}_c^2(n) \neq \pm\boldsymbol{G}_c^1(n)$ (i.e., the CO-POL Nulls are not the same, and they do not have the opposite position on the Poincaré sphere), we observe that all the characteristic polarization states can be regarded as the points of contact of the Poincaré sphere and some conicoids for some special constants C. However, the characteristic polarization states in the cases of $\boldsymbol{G}_c^2(n) = \pm\boldsymbol{G}_c^1(n)$ are different from those in the case of $\boldsymbol{G}_c^2(n) \neq \pm\boldsymbol{G}_c^1(n)$. For the case of $\boldsymbol{G}_c^2(n) = \boldsymbol{G}_c^1(n)$, there exists a circle, named the X-POL Max Circle, for which the radar receives maximum power in the cross-pol channel; and for the case of $\boldsymbol{G}_c^2(n) = -\boldsymbol{G}_c^1(n)$, there also exists a circle, named the X-POL Null Circle or the CO-POL Max Circle, for which the radar receives no echo in the cross-pol channel and receives maximum power in the co-pol channel. The equations of the X-POL Max Circle and the X-POL Null Circle/CO-POL Max Circle have been presented in this paper.

The proposed method to derive the characteristic polarization states in this paper is quite different from Kennaugh's method [1] (based on the voltage equation), Huynen's method [2] (based on the power equation and voltage equation), Boerner's method [4], [5] (based on the polarization ratio), and Yamaguchi's method [16] (based on the eigenvalue problem). This proposed method provides us with a geometric interpretation or visualization of changes in polarized states on the Poincaré sphere. In addition, the concept of the equi-power curves also affords a useful tool for analyzing the distribution of the received powers on the Poincaré sphere.

APPENDIX
DEFINITION OF THE KENNAUGH MATRIX

For a symmetric scattering matrix $[S] = \begin{bmatrix} s_1 & s_2 \\ s_2 & s_3 \end{bmatrix}$, the elements of the Kennaugh matrix are given by

$$
\begin{aligned}
k_{00} &= \frac{1}{2}(|s_1|^2 + 2|s_2|^2 + |s_3|^2)\\
k_{01} &= k_{10} = \frac{1}{2}(|s_1|^2 - |s_3|^2)\\
k_{02} &= k_{20} = \mathrm{Re}(s_1 s_2^* + s_2 s_3^*)\\
k_{03} &= k_{30} = \mathrm{Im}(s_1 s_2^* + s_2 s_3^*)\\
k_{11} &= \frac{1}{2}(|s_1|^2 - 2|s_2|^2 + |s_3|^2)\\
k_{12} &= k_{21} = \mathrm{Re}(s_1 s_2^* - s_2 s_3^*)\\
k_{13} &= k_{31} = \mathrm{Im}(s_1 s_2^* - s_2 s_3^*)\\
k_{22} &= \mathrm{Re}(s_1 s_3^*) + |s|_2^2\\
k_{23} &= k_{32} = \mathrm{Im}(s_1 s_3^*)\\
k_{33} &= -\mathrm{Re}\,(s_1 s_3^*) + |s|_2^2.
\end{aligned}
$$

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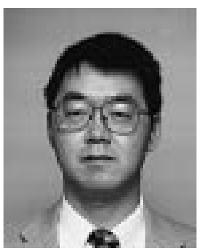

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