On Huynen's Decomposition of a Kennaugh Matrix

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***Abstract*—For some special case, Huynen's decomposition cannot be used to extract a desired target from an average Kennaugh matrix. In this paper, the authors modify Huynen's method for overcoming its disadvantage, based on a simple transform of a Kennaugh matrix. Using an example, the effectiveness of the modified method is validated.**

***Index Terms*—Decomposition, Kennaugh matrix, radar polarimetry, scattering matrix.**

I. Introduction

The problem of target decompositions has been studied extensively in the past, notably by Huynen [1], Cloude, Pottier, Krogager, Czyz, Holm, Barnes, Lueneburg *et al.* [2]–[10]. Huynen's decomposition of a Kennaugh matrix (or a Stokes reflection matrix named by Huynen [1]) is one of basic decompositions [2]. This method is to extract a coherent scattering matrix (pure single target) from an averaged Kennaugh matrix. Huynen's approach is very simple and it has clear physical meaning [1], [2], [5]. However, Huynen's decomposition is very sensitive to noise for some special case [7].

In this letter, Huynen's decomposition is modified, based on a transformation of a Kennaugh matrix. The proposed method is very simple for calculation. Using an example, the authors demonstrate the effectiveness of the modified decomposition.

II. Huynen's Decomposition

In a polarization basis [e.g., the horizontal–vertical (H-V) basis] for the monostatic radar case, if the reciprocity theorem holds, a point target has a symmetric scattering matrix which contains five independent parameters if the absolute phase is not considered. For the incoherent polarized wave case, however, a Kennaugh matrix in general contains nine parameters. In this case, Huynen [1] proposed a method for extracting a single target from an averaged Kennaugh matrix (denoted by $[\overline{K}]$). Mathematically, Huynen's method is described as

$$[\overline{K}] = [K_0] + [K_n] \tag{1}$$

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where $[K_0]$ is the Kennaugh matrix of the extracted target, corresponding to a scattering matrix; $[K_n]$ is the remainder term, and it is regarded as noise contained in $[\overline{K}]$. Let

$$[\overline{K}] = \begin{bmatrix} A_0 + B_0 & C & H & F \\ C & A_0 + B & E & G \\ H & E & A_0 - B & D \\ F & G & D & -A_0 + B_0 \end{bmatrix}. \tag{2}$$

Then $[K_0]$ is given by

$$[K_0] = \begin{bmatrix} A_0 + B_0^s & C & H & F^s \\ C & A_0 + B^s & E^s & G \\ H & E^s & A_0 - B^s & D \\ F^s & G & D & B_0^s - A_0 \end{bmatrix} \tag{3}$$

where the parameters A_0, C, H, G, and D are give by (2), and the other parameters are chosen to satisfy the constraint that $[K_0]$ corresponds to a coherent scattering matrix. It leads to the following equations [1], [8]:

$$2A_0\left(B_0^s + B^s\right) = C^2 + D^2 \tag{4a}$$
$$2A_0\left(B_0^s - B^s\right) = H^2 + G^2 \tag{4b}$$
$$2A_0 E^s = CH - DG \tag{4c}$$
$$2A_0 F^s = CG + DH. \tag{4d}$$

Obviously, if A_0 is very small, the parameters B_0^s, B^s, E^s, and F^s are very sensitive to the averaged Kennaugh matrix $[\overline{K}]$, hence the matrix $[K_0]$ obtained by the above method may not be the desired Kennaugh matrix (refer to the example of Section IV in this letter). Furthermore, Huynen's approach cannot be employed when $A_0 = 0$. Therefore, it is necessary to modify Huynen's decomposition for small A_0 case.

III. Modified Decomposition

First, let us consider the coherent case. After introducing a transformation, we will propose the modified decomposition later for the incoherent case.

Let

$$[S] = \begin{bmatrix} s_{HH} & s_{HV} \\ s_{VH} & s_{VV} \end{bmatrix} \tag{5}$$

denote the symmetric scattering matrix of a target for the monostatic radar case, where $s_{HV} = s_{VH}$. Now, we introduce a transformation defined by

$$[T(S)] \equiv \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} [S] \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \equiv \begin{bmatrix} s_{HH} & is_{HV} \\ is_{VH} & -s_{VV} \end{bmatrix} \tag{6a}$$

where $i^2 = -1$. If we rewrite (6a) as the following form

$$[T(S)] \equiv -i \begin{bmatrix} \exp\left(-\frac{\pi}{4}i\right) & 0 \\ 0 & \exp\left(\frac{\pi}{4}i\right) \end{bmatrix} \times [S] \begin{bmatrix} \exp\left(-\frac{\pi}{4}i\right) & 0 \\ 0 & \exp\left(\frac{\pi}{4}i\right) \end{bmatrix} \equiv -i[U]^*[S][U]^H \equiv \begin{bmatrix} s_{HH} & i\, s_{HV} \\ i\, s_{VH} & -s_{VV} \end{bmatrix} \quad (6b)$$

then

$$[U] = \begin{bmatrix} \exp((\pi/4)i) & 0 \\ 0 & \exp(-(\pi/4)i) \end{bmatrix}$$

belongs to the SU(2) matrix group, where superscript * denotes the conjugate and the superscript H denotes the Hermitian conjugate. So (6) may be regarded as a transformation of the Sinclair matrix from the polarization basis $[1,\ 0]^t$ and $[0,\ 1]^t$ to the basis $[\exp((\pi/4)i),\ 0]^t$ and $[0,\ \exp(-(\pi/4)i)]^t$ if we omit an absolute phase $-(\pi/2)$. After the transformation (6a), the trace of a scattering matrix is changed. Although there exist other transformations which can be used to change the trace of a scattering matrix, the transformation (6a) is very simple and so is its reverse transformation. Sequentially, one easily obtains the corresponding transformation and its reverse transformation on the Kannaugh matrix. It will lead the modified decomposition of a Kennaugh to be simple [see (10a), (11), and (12)].

Let $[K(S)]$ and $[K(T(S))]$ denote the Kennaugh matrices of $[S]$ and $[T(S)]$, respectively. Then it is easy to prove that

$$[K(T(S))] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} [K(S)] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \quad (7)$$

From the Kennaugh matrices $[K(S)]$ and $[K(T(S))]$, one easily derives the following.

1) The parameter A_0 associated with $[K(S)]$ or $[S]$ is

$$A_0(S) \equiv A_0(M(S)) = \frac{1}{4}|s_{HH} + s_{VV}|^2. \quad (8a)$$

2) The parameter A_0 associated with $[K(T(S))]$ or $[T(S)]$ is

$$A_0(T(S)) \equiv A_0(M(T(S))) \frac{1}{4}|s_{HH} - s_{VV}|^2. \quad (8b)$$

For small $A_0(S)$ case, we conclude from (8a) and (8b) that $A_0(T(S))$ is much larger then $A_0(S)$ when $|s_{HH}|$ is not small. If $|s_{HH}|$ is small, the scattering matrix $[S]$ is rotated $\pi/4$ to a new matrix $[R] = [S(\pi/4)]$

$$[R] = \left[S\left(\frac{\pi}{4}\right)\right] = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} [S] \begin{bmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} \equiv \begin{bmatrix} r_{HH} & r_{HV} \\ r_{VH} & r_{VV} \end{bmatrix}. \quad (9a)$$

The corresponding Kennaugh matrix is

$$[K(R)] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [K(S)] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (9b)$$

It is straightforward to obtain that $r_{HH} = 1/2(s_{HH} - 2s_{HV} + s_{VV})$. If $A_0(S)$ is very small and $|s_{HH}|$ is also small, then the parameter $A_0(T(R))$ associated with $[T(R)]$ must be much larger than $A_0(S)$.

According to the above analysis for the cohorent case, we propose the following modified Huynen's approach:

Step 1) If A_0 associated with the Kennaugh matrix $[\overline{K}]$ is not small, e.g., $A_0 > m_{00}/10$, where m_{00} is the first row and first column element of $[\overline{K}]$, then use Huynen's approach; otherwise use the following steps.

Step 2) When $A_0 \leq m_{00}/10$, define

$$[T_1] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} [\overline{K}] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (10a)$$

$$[T_2] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} [\overline{KR}] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (10b)$$

where

$$[\overline{KR}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [\overline{K}] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Step 3) If $A_0(T_1)$ is greater than or equal to $A_0(T_2)$, apply Huynen's decomposition to $[T_1]$ and denote

$$[T_1] = [T_1(K)]_0 + [T_1(K)]_n. \quad (11)$$

Then the modified decomposition is

$$[\overline{K}] = [R1]^{-1}[T_1(K)]_0[R1] + [R1]^{-1}[T_1(K)]_n[R1] \equiv [K_0] + [K_n] \quad (12)$$

where $[R1]^{-1} = [R1]^t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

If $A_0(T_1)$ is less then $A_0(T_2)$, apply Huynen's approach to $[T_2]$ and denote

$$[T_2] = [T_2(K)]_0 + [T_2(K)]_n. \quad (13)$$

Then the modified decomposition is

$$[\overline{K}] = [R2][R1]^{-1}[T_2(K)]_0[R1][R2]^{-1} + [R2][R1]^{-1} \times [T_2(K)]_n[R1][R2]^{-1} \equiv [K_0] + [K_n] \quad (14)$$

where

$$[R2]^{-1} = [R2]^t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

IV. Example

Let us consider the following averaged Kennaugh matrix [7]:

$$[\overline{K}] = \begin{bmatrix} 1.020\,25 & 0.009\,75 & 0.002 & -0.199 \\ 0.009\,75 & 0.980\,25 & -0.002 & -0.001 \\ 0.002 & -0.002 & -0.97 & -0.02 \\ -0.199 & -0.001 & -0.02 & 1.01 \end{bmatrix}.$$

Using Huynen's method, one easily decomposes the above Kennaugh matrix $[\overline{K}]$ into two parts. The extracted Kennaugh matrix $[K]_{0H}$ corresponding to a single target is

$$[K]_{0H} = \begin{bmatrix} 0.029\,52 & 0.009\,75 & 0.002 & -0.004\,85 \\ 0.009\,75 & 0.029\,03 & 0.000\,05 & -0.001 \\ 0.002 & 0.000\,05 & -0.018\,78 & -0.02 \\ -0.004\,85 & -0.001 & -0.02 & 0.019\,27 \end{bmatrix}.$$

Obviously, it is not the desired Kennaugh matrix. If we use the modified approach for decomposing the matrix $[\overline{K}]$, then the extracted Kennaugh matrix is

$$[K]_{0MH} = \begin{bmatrix} 1.0052 & 0.0098 & 0.002 & -0.199 \\ 0.0098 & 0.9853 & -0.002 & 0.001 \\ 0.002 & -0.002 & -0.9850 & -0.02 \\ -0.199 & 0.001 & -0.02 & 1.0049 \end{bmatrix}.$$

The scattering matrices corresponding to $[K]_{0H}$ and $[K]_{0MH}$ are

$$[S]_{0H} = \begin{bmatrix} 0.1976 & 0.0049 + 0.0148i \\ 0.0049 + 0.0148i & -0.0963 + 0.1012i \end{bmatrix}$$

and

$$[S]_{0MH} = \begin{bmatrix} 1.0025 & 0.0985i \\ 0.0985i & -0.9927 + 0.0199i \end{bmatrix}$$

respectively. Note that $[\overline{K}]$ is the averaged Kennaugh matrix corresponding to the following random scattering matrix for a special case:

$$[S]_r = \begin{bmatrix} 1 + \Delta sn_1 & 0.1i + \Delta sn_2 \\ 0.1i + \Delta sn_2 & -0.99 + 0.02i + \Delta sn_3 \end{bmatrix}$$

where Δsn_k ($k = 1, 2, 3$) denotes the complex noises, characterized with zero-mean, Gaussian distributions and some other properties [7]. Obviously, $[S]_{0MH}$ is very close to $[S]_r$ but $[S]_{0H}$ is not, demonstrating the effectiveness of the modified decomposition.

If the average Kennaugh matrix $[\overline{K}]$ is translated to a covariance matrix, then we can also employ Cloude's method [2] and Holm' method [9] for target decomposition. Cloude's method is to decompose a covariance matrix to three components (rank-1 matrices) associated with three egienvalues and three eigenvectors of the covariance matrix. Then the extracted matrix is obtained from the main component associated with the maximum eigenvalue and the corresponding eigenvector. Holm–Barnes' method is a little similar to Cloude's method, also based on eigenvector decomposition. But the extracted matrix by Holm–Barnes' method is obtained from the component consisting of the difference between the maximum eigenvalue and the second maximum eigenvalue of the covariance matrix and the eigenvector associated with the maximum eigenvalue. Using the eigenvector based decomposition, we easily extract the following two scattering matrices corresponding to both the method:

$$[S]_{\mathrm{CLOUDE}} = \begin{bmatrix} 1.0027 & 0.1007\mathrm{i} \\ 0.1007\mathrm{i} & -0.9927 + 0.0200\mathrm{i} \end{bmatrix}$$
$$[S]_{\mathrm{HOLM-BARNES}} = \begin{bmatrix} 0.9979 & 0.1002\mathrm{i} \\ 0.1002\mathrm{i} & -0.9880 + 0.0200\mathrm{i} \end{bmatrix}.$$

Obviously, both the extracted scattering matrices are almost the same as that by the proposed method.

After an average Kennaugh matrix is translated to a covariance matrix, we may use Barnes' decomposition which is based on the orthogonality of a space [10]. Three scattering matrices are easily obtained. The first scattering matrix is

$$[S]_{\mathrm{Barnes-1}} = \begin{bmatrix} 0.1389 - 0.1398\mathrm{i} & 0.0140 + 0.0070\mathrm{i} \\ 0.0140 + 0.0070\mathrm{i} & 0.0033 + 0.1397\mathrm{i} \end{bmatrix}.$$

It is the same as the extracted matrix by Huynen's decomposition if an absolute phase is disregarded. The other matrices by Barnes' decomposition are

$$[S]_{\mathrm{Barnes-2}} = \begin{bmatrix} 0.9983 & -0.0082 - 0.8128\mathrm{i} \\ -0.0082 - 0.8128\mathrm{i} & -0.9884 + 0.0199\mathrm{i} \end{bmatrix}$$

$$[S]_{\mathrm{Barnes-3}} = \begin{bmatrix} 0.9963 & -0.0001 + 0.0880\mathrm{i} \\ -0.0001 + 0.0880\mathrm{i} & -0.9864 + 0.0204\mathrm{i} \end{bmatrix}$$

where the absolute phases of both the matrices are omitted. From the three extracted matrices, on easily finds that $[S]_{\mathrm{Barnes}^{-3}}$ is close to $[S]_r$.

V. Conclusion

The modified Huynen decomposition has been proposed in this letter. When A_0 associated with an averaged Kennaugh matrix is small, Huynen's approach cannot been employed for extracting a desired Kennaugh matrix. This letter transformed the averaged Kennaugh matrix into a new "Kennaugh matrix" and then applied Huynen's method to decompose the new matrix into two parts. The desired decomposition is finally obtained by an inverse transformation. The proposed method is very simple for calculation. In addition, we also used Cloude's method, Holm's method, and Barnes' method for decomposition. Form the extracted matrices by these methods, we conclude that the proposed method is correct.

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