Simple-to-Evaluate Error Probabilities for Impulse Radio UWB Multiple Access Systems with Pulse-Based Polarity Randomization

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***Abstract*— Simple-to-evaluate and accurate bit error probabilities are presented for impulse radio ultra-wideband multiple access systems that use time hopping sequences with pulse-based polarity randomization and binary phase shift keying modulation. Simplified improved Gaussian approximation is used. It is shown that despite having the same processing gain, which is the multiplication of the number of frames per bit and chips per frame, different combinations of the parameters result in different performances. The theoretical results are validated by Monte Carlo simulations.**

***Index Terms*— Ultra-wideband, multiaccess communication, error analysis, simplified improved Gaussian approximation.**

I. Introduction

RECENTLY, time hopping impulse radio (TH-IR) ultra-wideband (UWB) multiple access communications have received considerable attention [1]−[3]. Though initially unipolar communication combined with pulse position modulation (PPM) was focused in [1]−[3], a system with pulse-based polarity randomization combined with binary phase shift keying (BPSK) was recently proposed in [4]. Approximate closed form error probabilities were given for the system in [4] that were based on the improved Gaussian approximation (IGA) previously introduced in [5]. However, the computational complexity of the method is quite intensive which increases with the total number of users. In this letter, we present a simple error analysis for the system that facilitates to obtain simple-to-evaluate and accurate error probabilities based on Holtzman's simplified improved Gaussian approximation (SIGA) [6].

Previously, the SIGA was extensively used in literature to obtain expressions for error probabilities in DS-CDMA (see [7], [8] and the references therein). However, whether the method can be applied for TH-IR UWB multiple access system has not yet been investigated. In this letter, we extend the method for a TH-IR UWB multiple access system that uses pulse-based polarity randomization. We show that the method is also acceptably accurate for the TH-IR UWB multiple access system.

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II. System Model

The TH-IR UWB system under consideration with pulse-based polarity randomization and BPSK modulation is similar to the one in [4]. There are K simultaneous users in the system. A typical representation of the signal from user k $(k = 1, 2, \ldots, K)$ has the form

$$s^{(k)}(t) = \sqrt{\frac{E_k}{N_f}} \sum_{j=-\infty}^{+\infty} d_j^{(k)} b_{\lfloor j/N_f \rfloor}^{(k)} w(t - jT_f - c_j^{(k)} T_c) \quad (1)$$

where t is time and $w(t)$ is the communication pulse with unit energy. The rest of the signal structure is described as follows:

- E_k is the bit energy of user k.
- N_f is the number of frames or pulses representing one information bit. Each frame has one pulse.
- T_f is the average pulse repetition time, also known as frame time. So, the bit duration is $T_b = N_f T_f$.
- T_c is the chip time. Each frame is divided in N_c $(N_c \geq 2)$ chips giving $T_f = N_c T_c$. The duration of the pulse $w(t)$ is considered to be equal to T_c.
- $\{b_i^{(k)}\}$ is the i-th bit of user k which is a random variable (RV) uniform on $\{+1, -1\}$. Here, $i = \lfloor j/N_f \rfloor$ and $\lfloor . \rfloor$ represents floor function.
- $\{d_j^{(k)}\}$ is the random polarity code which is also a RV uniform on $\{+1, -1\}$ and is periodic with period N_f.
- $\{c_j^{(k)}\}$ is the TH code which is a RV uniform on $\{0, 1, \ldots, N_c - 1\}$ and is periodic with period N_f.
- The processing gain of the system is $PG = N_f N_c$.

III. Multiple Access Interference Modeling

Considering an additive white Gaussian noise (AWGN) channel, the received signal can be given by

$$r(t) = \sum_{k=1}^{K} s^{(k)}(t - \tau_k) + n(t) \quad (2)$$

where $n(t)$ is AWGN noise with two sided power spectral density of $N_o/2$ and τ_k is the random delay of the signal received from user k relative to the desired user 1 ($i.e.$ $\tau_1 = 0$) which is uniformly distributed over $[0, T_b]$. Let $\tau_k = \Delta_k + \gamma_k T_f$ where, γ_k is a RV uniform on $\{0, 1, \ldots, N_f - 1\}$ and

$\Delta_k = \alpha_k + \beta_k T_c$ where, β_k is a RV uniform on $\{0, 1, \ldots, N_c - 1\}$ and $0 \leq \alpha_k < T_c$.

Considering a template of the form

$$s_{temp}^{(1)}(t) = \sum_{j=iN_f}^{(i+1)N_f-1} d_j^{(1)} w(t - jT_f - c_j^{(1)} T_c) \quad (3)$$

the decision statistics for a coherent correlation receiver while we are detecting the i-th bit of user 1 can be given by [4], [5]

$$y^{(1)} = \sqrt{E_1 N_f} b_i^{(1)} + \sum_{j=iN_f}^{(i+1)N_f-1} \sum_{k=2}^{K} \sqrt{\frac{E_k}{N_f}} a_j^{(k)} + n \quad (4)$$

where the right side of (4) has three parts, of which the first part is the desired signal component, the second part is the multiple access interference (MAI) component and the third part n is the AWGN component having variance of $\sigma_n^2 = N_o N_f/2$. Here, $a_j^{(k)}$ is the MAI component on the j-th frame of user 1 contributed by user k, given by

$$a_j^{(k)} = P_j^{(k)} \hat{R}_w(\alpha_k) \hat{\delta}_j(\Delta_k) + Q_j^{(k)} R_w(\alpha_k) \delta_j(\Delta_k) \quad (5)$$

where $\hat{R}_w(\alpha_k)$ and $R_w(\alpha_k)$ are continuous-time partial auto-correlation functions of the pulse $w(t)$ defined as $\hat{R}_w(\alpha_k) = \int_{\alpha_k}^{T_c} w(t) w(t - \alpha_k)\, dt$ and $R_w(\alpha_k) = \hat{R}_w(T_c - \alpha_k)$. The independent RVs $P_j^{(k)}$ and $Q_j^{(k)}$ are uniform on $\{+1, -1\}$, have zero mean and variances $E[(P_j^{(k)})^2] = E[(Q_j^{(k)})^2] = 1$ where $E[.]$ represents mean value. By defining $l_1 = c_j^{(1)} T_c$, $l_2 = \Delta_k + c_{j-\gamma_k}^{(k)} T_c$ and $l_3 = \Delta_k - (T_f - c_{j-\gamma_k-1}^{(k)} T_c)$, the functions $\hat{\delta}_j(\Delta_k) = 1$, if $0 \leq l_2 - l_1 < T_c$ or, $0 \leq l_3 - l_1 < T_c$; $\hat{\delta}_j(\Delta_k) = 0$, otherwise and $\delta_j(\Delta_k) = 1$ if $0 \leq l_1 - l_2 < T_c$ or, $0 \leq l_1 - l_3 < T_c$; $\delta_j(\Delta_k) = 0$, otherwise.

The MAI variance $\Psi = \sum_{k=2}^{K} Z^{(k)}$ is a RV which is a function of the delays $\Delta = [\Delta_1, \Delta_2, \ldots, \Delta_K]$ with $Z^{(k)}$ given by

$$Z^{(k)} = \sum_{j=iN_f}^{(i+1)N_f-1} Z_j^{(k)} \quad (6)$$

with

$$Z_j^{(k)} = \frac{E_k}{N_f} E\left[\left(a_j^{(k)}\right)^2 \mid \Delta_k\right] \quad (7)$$

where the expectation in (7) is taken over the RVs $P_j^{(k)}$ and $Q_j^{(k)}$ given Δ_k (and hence α_k).

Synchronous system. The system is frame synchronous if $\Delta_k = 0$ and bit synchronous if $\Delta_k = \gamma_k = 0$. In both cases, $\alpha_k = 0$ and $R_w(0) = 0$, $\hat{R}_w(0) = 1$. Here note that the $(j - \gamma_k)$th frame of user k collides with the jth frame of user 1. However, interference may occur only if the pulses in the respective frames coincide (or, collide). Hence, we get

$$Z_j^{(k)} = \begin{cases} \frac{E_k}{N_f}, & \text{coincidence} \\ 0, & \text{no coincidence} \end{cases} \quad (8)$$

Note that a coincidence in any frame occurs with a probability $p = 1/N_c$, the probability of no coincidence being $1 - p$. Because the position of pulse in any frame of any user is selected randomly and independently, we can write

$$Z^{(k)} = \frac{E_k}{N_f} J_s \quad (9)$$

where J_s is a RV binomially distributed over $\{0, 1, \ldots, N_f\}$. The density function of J_s is given by [9]

$$f_J(j) = \sum_{j=0}^{N_f} \binom{N_f}{j} p^j (1-p)^{N_f - j} \quad (10)$$

with $p = 1/N_c$ where $\binom{N_f}{j}$ is binomial coefficient.

Asynchronous system. In asynchronous system, α_k is a RV uniformly distributed over $[0, T_c]$. Hence, there will usually be partial coincidences. However, we note that there is only one pulse in average to interfere with an arbitrary pulse of the desired user. As a result, $Z_j^{(k)}$ can be approximately given by

$$Z_j^{(k)} = \begin{cases} \left(\frac{E_k}{N_f}\right)\left(\frac{\hat{R}_w^2(\alpha_k) + R_w^2(\alpha_k)}{2}\right), & \text{partial coincidence} \\ 0, & \text{no coincidence} \end{cases} \quad (11)$$

However, under the assumption stated above, note that $\hat{R}_w(\alpha_k)$ and $R_w(\alpha_k)$ over the same frame duration are mutually exclusive. Here a partial coincidence in any frame may occur with a probability $p = 2/N_c$ and the probability of no coincidence is $1 - p$ as before. Using the knowledge from the synchronous system, $Z^{(k)}$ can be approximately given by

$$Z^{(k)} = \left(\frac{E_k}{N_f}\right)\left(\frac{\hat{R}_w^2(\alpha_k) + R_w^2(\alpha_k)}{2}\right) J_{as} \quad (12)$$

where J_{as} is a RV binomially distributed over $\{0, 1, \ldots, N_f\}$. The density function of J_{as} is given by (10) with $p = 2/N_c$.

IV. Error Probabilities

If the mean μ and the standard deviation σ of the MAI variance Ψ are known, the simple-to-evaluate bit error probability (BEP) according to the SIGA can be given by [6]

$$P_e^{SIGA} = \frac{2}{3} Q\left(\sqrt{\frac{E_1 N_f}{\mu + \sigma_n^2}}\right) + \frac{1}{6} Q\left(\sqrt{\frac{E_1 N_f}{\mu + \sqrt{3}\sigma + \sigma_n^2}}\right) + \frac{1}{6} Q\left(\sqrt{\frac{E_1 N_f}{\mu - \sqrt{3}\sigma + \sigma_n^2}}\right) \quad (13)$$

where $Q(x) = (2\pi)^{-1/2} \int_x^\infty exp(-u^2/2)\, du$.

Synchronous system. By noting that $E[J_s] = N_f/N_c$ and $E[J_s^2] = N_f/N_c(N_f/N_c - 1/N_c + 1)$ [9], the mean of Ψ can be given by $\mu = \sum_{k=2}^{K} E[Z^{(k)}] = \sum_{k=2}^{K} E_k/N_c$ and the standard deviation, $\sigma = [\sum_{k=2}^{K}(E[(Z^{(k)})^2] - E[Z^{(k)}]^2)]^{1/2}$ can be given by

$$\sigma = \frac{1}{N_c}\left[\left(\frac{N_c - 1}{N_f}\right) \sum_{k=2}^{K} E_k^2\right]^{1/2} \quad (14)$$

Asynchronous system. For asynchronous system, $E[J_{as}] = 2N_f/N_c$, $E[J_{as}^2] = 2N_f/N_c(2N_f/N_c - 2/N_c + 1)$ [9] and the mean of Ψ is given by $\mu = \sum_{k=2}^{K} E[Z^{(k)}] = 2m_w \sum_{k=2}^{K} E_k/N_c$ where $m_w = \frac{1}{T_c}\int_0^{T_c} \hat{R}_w^2(\alpha) d\alpha = \frac{1}{T_c}\int_0^{T_c} R_w^2(\alpha) d\alpha$. The standard deviation takes the form

$$\sigma = \frac{1}{N_c}\left[\left(\left(\frac{2N_f + N_c - 2}{N_f}\right) w_w - 4m_w^2\right) \sum_{k=2}^{K} E_k^2\right]^{1/2} \quad (15)$$

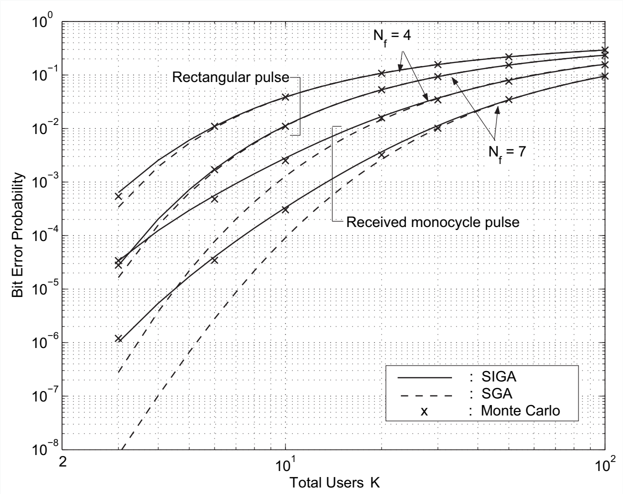

Fig. 1. Bit error probability versus total users K for $N_f = 4, 7$ and $N_c = 5$ at SNR, $E_b/N_o = 14$ dB. Asynchronous system with $E_k = E_b$, $k = 1, 2, \ldots, K$ is considered.

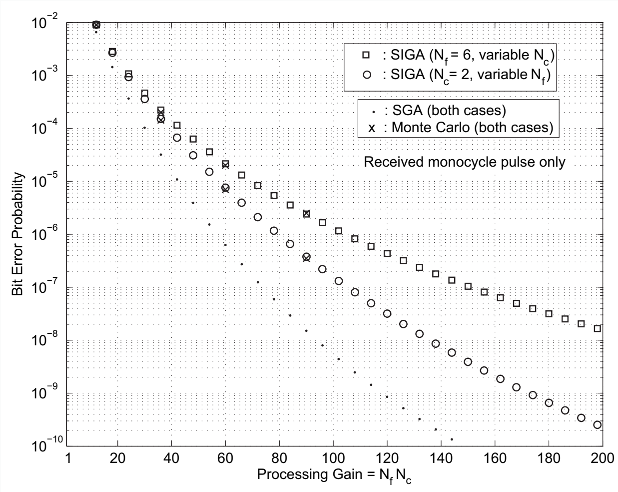

Fig. 2. Bit error probability versus processing gain $PG = N_f N_c$ for total users, $K = 10$, SNR, $E_b/N_o = 16$ dB with various N_f and N_c. Asynchronous system with $E_k = E_b$, $k = 1, 2, \ldots, K$ is considered.

where $w_w = \frac{1}{T_c}\int_0^{T_c} \hat{R}_w^4(\alpha)d\alpha = \frac{1}{T_c}\int_0^{T_c} R_w^4(\alpha)d\alpha$. Note that here we have used the fact $\frac{1}{T_c}\int_0^{T_c} \hat{R}_w^2(\alpha)R_w^2(\alpha)d\alpha = 0$.

The BEP based on the well-known standard Gaussian approximation (SGA) is given by the 1st Q-function of (13) without the multiplication factor 2/3.

V. Numerical Examples and Conclusion

In this section, the results presented in the previous section are evaluated by illustrative examples. Only asynchronous system with perfect power control (*i.e.* $E_k = E_b$, $k = 1, 2, \ldots, K$) is considered. Theoretical results from SIGA are verified by Monte Carlo simulations and are compared with those from SGA. Though the method can be used for any pulse shape $w(t) = v(t)/\sqrt{\mathcal{E}_v}$, we use two types of pulses namely 1) the received monocycle pulse where $v(t+T_c/2) = [1 - 4\pi(t/t_m)^2]exp[-2\pi(t/t_m)^2]$ with $t_m = 0.39T_c$ which was previously used for TH-IR UWB in [1], [4] and 2) rectangular pulse where $v(t) = 1,\ 0 \leq t < T_c$; $v(t) = 0$, otherwise. The energy of $v(t)$ is $\mathcal{E}_v = \int_{-\infty}^{+\infty} v^2(t)dt$. The partial autocorrelations of $w(t)$ are given by 1) $\hat{R}_w(\alpha) = (1/\mathcal{E}_v)[1 - 4\pi(\alpha/t_m)^2 + (4/3)\pi^2(\alpha/t_m)^4]exp[-\pi(\alpha/t_m)^2]$ and 2) $\hat{R}_w(\alpha) = (T_c - \alpha)/\mathcal{E}_v$ respectively and $R_w(\alpha) = \hat{R}_w(T_c - \alpha)$ in both cases. Fig. 1 shows the BEP versus total users K for $N_f = 4$ and 7 with $N_c = 5$ and signal-to-noise ratio (SNR), $E_b/N_o = 14$ dB. The results from SIGA closely match with those from simulation for both pulses and can be considered to be reliable approximations. The SGA, on the other hand, is optimistic for small to medium number of users. Note that the MAI variance converges to a Gaussian RV for large number of users [5] and hence the SGA is accurate in that region.

Fig. 2 presents the BEP versus processing gain, $PG = N_f N_c$ for $K = 10$ and $E_b/N_o = 16$ dB considering only monocycle pulse. As expected, performance improvement is seen with increase in the PG. The effects of N_f and N_c on the multiple access performance looks similar from the view point of SGA. However, except in small PG, the actual system performance becomes different for different combinations of N_f and N_c despite having the same PG. The PG is increased by 1) increasing N_c keeping N_f fixed at 6 and 2) increasing N_f keeping N_c fixed at 2. As seen, better performance is obtained in the second case. It is evident that the SIGA accurately tracks this performance difference, whereas the SGA becomes increasingly optimistic. It can be concluded here that the use of long repetition code brings more performance improvement than decreasing signal duty-cycle $(= 1/N_c)$ to increase the PG.

The simple-to-evaluate SIGA previously used mainly for DS-CDMA is found to be reasonably accurate for TH-IR UWB multiple access system as well. This rediscovers the inherent similarity between UWB and CDMA.

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