

# Real time displacement measurement in sinusoidal phase modulating interferometry

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A new signal processing system for real time displacement measurement in sinusoidal phase modulating interferometry is described. Although sinusoidal phase modulating interferometry is effective in measuring with high accuracy the displacement of an object, conventional signal processing takes a long time. In this method, detection of the object's displacement is easily achieved by sampling the interference signal at those times that satisfy certain conditions and by processing the sampled signals with electric circuits in real time. The delay time of this signal processing system is  $<45 \mu\text{s}$ .

## I. Introduction

As the technology of precise manufacturing advances, it becomes more important to measure and control with high accuracy microscopic movements of a machine tool. Optical interferometric techniques are used for measuring microscopic movements, and, among these techniques, the heterodyne interferometric method is most widely used.<sup>1-3</sup> This interferometric method, however, needs a frequency shifter such as Bragg cells to obtain two beams of different frequencies; thus the optical system is complete.<sup>4,5</sup>

We have proposed interferometry using sinusoidal phase modulated light which is easily obtained using a sinusoidally vibrating mirror. Using this sinusoidal phase modulating interferometry, we have measured an object's displacement with great accuracy.<sup>6-8</sup> In this method, however, it takes a long time to obtain the object's displacement from an interference signal, because it is necessary to calculate a Fourier transform of the signal with a computer. So we proposed a real time method to measure displacement without using the computer<sup>9</sup>; but it has a short delay time, about a few hundred milliseconds, because the phase modulated signal is heterodyne demodulated by a phase locked loop (PLL). This delay time makes it hard to control the microscopic movement of a machine tool with the measured displacement signals.

In this paper we describe a new signal processing system without computer or heterodyne demodulation to detect the object's displacement from the sinusoidal phase modulated interference signal. Detection of the object's displacement is achieved by sampling the interference signal at times that satisfy certain conditions. Moreover, this method does not involve any division process when we obtain an object's displacement from quadratic signals. A time delay at the measurement mainly depends on the conversion time of an A-D converter and the calculation time of the microprocessor, and it is much shorter than that of the conventional method.

In Sec. II, the principles of the real time measurement and of this system are described. In Sec. III, the signal processing system using a microprocessor is described. Experimental results are given in Sec. IV.

## II. Principles

### A. Signal Detection from the Optical System

The configuration of an interferometer for real time displacement measurement is shown in Fig. 1. In this system, the Twyman-Green interferometer is constructed, and a laser diode (LD) is used as the light source. The injection current of the laser diode, produced by the laser diode modulator (LM), consists of dc bias current  $I_0$  and modulation current  $I_m(t)$ . The modulation current changes the wavelength of the laser diode by the amount of  $\beta I_m(t)$ . The central wavelength  $\lambda_0$  is determined by the current  $I_0$ . The current  $I_m(t)$  is represented by

$$I_m(t) = a \cos \omega_c t. \quad (1)$$

A beam reflected by a mirror  $M$  is the reference wave. The displacement of the object is represented by  $r(t)$ .

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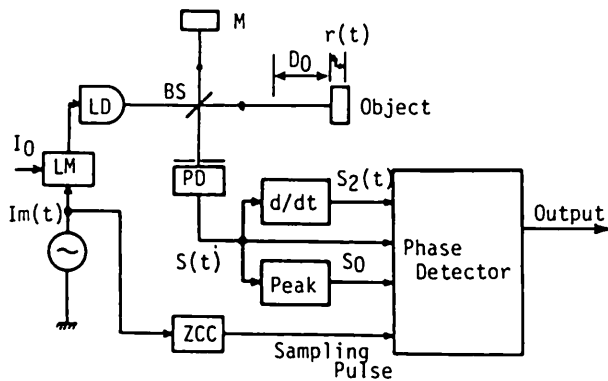


Fig. 1. Real time displacement measurement system with synchronous sampling of the sinusoidal phase modulated interference signal.

The optical path difference between the two arms of the interferometer is  $2[D_0 + r(t)]$ . Then the ac component of the interference signal detected with a photodetector (PD) is given by

$$S(t) = S_0 \cos[z \cos \omega_c t + \alpha(t)], \quad (2)$$

where  $z = 4\pi\alpha\beta D_0/\lambda_0^2$ , and phase  $\alpha(t)$  is written as

$$\alpha(t) = (4\pi/\lambda_0)[D_0 + r(t)]. \quad (3)$$

If the phase  $\alpha(t)$  is obtained in real time from the signal  $S(t)$ , the object's displacement is measured in real time.

We now describe the principle of the signal processing to obtain the phase  $\alpha(t)$ . Dividing the signal  $S(t)$  by the amplitude  $S_0$  gives

$$S_1(t) = \cos[z \cos \omega_c t + \alpha(t)]. \quad (4)$$

Sampling  $S_1(t)$  at times  $t_m$ , which satisfy the condition of  $\cos \omega_c t_m = 0$ , we obtain

$$S_1(t_m) = \cos[\alpha(t_m)], \quad (5)$$

where  $t_m = 2m\pi + \pi/2$  or  $2m\pi + 3\pi/2$  and  $m = 0, \pm 1, \pm 2, \dots$ . On the other hand, differentiating Eq. (2), we obtain

$$S_2(t) = S_0[-z\omega_c \sin \omega_c t + \{d\alpha(t)/dt\}] \times \{-\sin[z \cos \omega_c t + \alpha(t)]\}. \quad (6)$$

Sampling  $S_2(t)$  at the same times  $t_m$  when  $S_1(t)$  is sampled, we obtain

$$S_2(t_m) = \pm \epsilon S_0 \sin[\alpha(t_m)], \quad (7)$$

where

$$\epsilon = z\omega_c - \{d\alpha(t)/dt\}. \quad (8)$$

The plus and minus signs are taken at times  $t_m = 2m\pi + \pi/2$  and  $t_m = 2m\pi + 3\pi/2$ , respectively. Assuming that the displacement of the object is a sinusoidal vibration such as

$$r(t) = b \cos \omega_d t, \quad (9)$$

the minimum value of Eq. (8) is

$$\epsilon_{\min} = z\omega_c - 4\pi b\omega_d/\lambda_0. \quad (10)$$

Figure 2 shows the values of  $\epsilon_{\min}$  as a function of vibra-

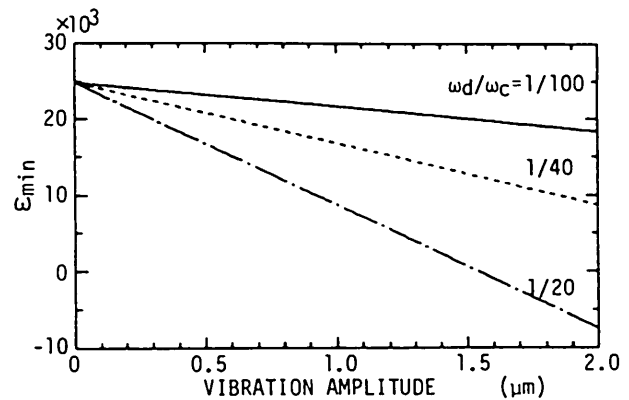


Fig. 2. Values of  $\epsilon_{\min}$  as a function of the vibration amplitude  $b$  for various values of the ratio  $\omega_d/\omega_c$ .

tion amplitude  $b$  for various values of the ratio  $\omega_d/\omega_c$ . It is evident that  $\epsilon_{\min}$  does not have a minus value when the modulation frequency is much larger than the vibration frequency. Thus we can determine the sign of  $\sin \alpha(t_m)$  from Eq. (7). Therefore, we can decide the phase  $\alpha(t_m)$  from the value of  $\cos \alpha(t_m)$  given by Eq. (5) and the sign of  $\sin \alpha(t_m)$  given by Eq. (7). But the phase  $\alpha(t_m)$  is defined in the region between  $-\pi$  and  $+\pi$ , so we must make a correction for the phase  $\alpha(t_m)$  when it takes a value out of that region. The method of deciding the phase in a wider region (more than  $2\pi$ ) is described in Sec. II.B.

#### B. Decision of the Phase $\alpha(t)$

First, we prepare two different phases for  $\cos \alpha(t)$  corresponding to the sign of  $\sin \alpha(t)$  as shown in Table I. When the sign of  $\sin \alpha(t)$  is plus, the value of  $\alpha(t)$  is  $P$ , which is obtained from the arccosine conversion table defined in the region between zero and  $\pi$ . Although the value of  $\alpha(t)$  is  $-P$  when the sign of  $\sin \alpha(t)$  is minus, we assign  $-P + \pi$  for the value of  $\alpha(t)$ . Although the detected phase  $\alpha(t)$  involves an offset  $\pi$ , the offset can be ignored because a relative displacement is required.

Table I. Phase Obtained from  $\cos \alpha$

Sign of $\sin \alpha(t)$	Phase $\alpha(t)$
+	$P$
-	$-P + \pi$

Table II. Corrections for the Phase  $\alpha$  to Obtain a Continuous Value

Sign of $\cos \alpha(t)$	Change of sign of $\sin \alpha(t)$	Corrective value
+	- $\rightarrow$ +	$+\pi$
	+ $\rightarrow$ -	$-\pi$
-	- $\rightarrow$ +	$-\pi$
	+ $\rightarrow$ -	$+\pi$

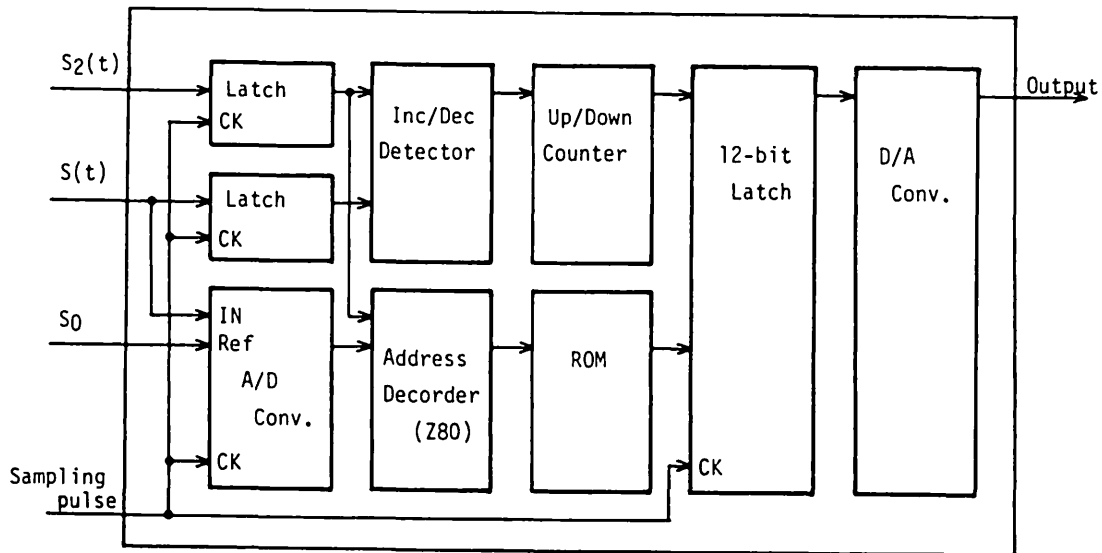


Fig. 3. Block diagram of a phase detector.

Next, to obtain the continuous phase  $\alpha(t)$  in a wide region (more than  $2\pi$ ), we must make a correction for the phase  $\alpha(t)$  when the sign of  $\sin\alpha(t)$  changes. The correction is achieved as follows: When the sign of  $\cos\alpha(t)$  stays plus, and the sign of  $\sin\alpha(t)$  changes from minus to plus,  $\pi$  is added to the phase  $\alpha(t)$ . Conversely, if the sign of  $\sin\alpha(t)$  changes from plus to minus,  $-\pi$  is added to the phase  $\alpha(t)$ . When the sign of  $\cos\alpha(t)$  stays minus, similar operations are done as shown in Table II.

### III. Signal Processing System

As shown in Fig. 1,  $S_2(t)$  is obtained by differentiating  $S(t)$  using a differentiator. The amplitude  $S_0$  of the  $S(t)$  is obtained using a peak-hold circuit. A sampling pulse, which samples  $S(t)$  and  $S_2(t)$ , is generated from the modulation current  $I_m(t)$  using a zero-cross circuit.

A block diagram of the phase detector is shown in Fig. 3. The  $S(t)$  is fed to the analog input of the 8-bit A-D converter, and the amplitude  $S_0$  of the  $S(t)$  is fed to the reference input. Then the  $S(t)$  is sampled and converted to  $S(t_m)/S_0$ . Moreover,  $S(t)$  and  $S_2(t)$  are led to the latch circuits, and their signs are sampled. Output data  $A$  of the A-D converter and sign data of the  $S_2(t_m)$  are led to the address decoder, which is used on a microprocessor Z80. If the sign of  $S_2(t_m)$  is plus, the output data of the address decoder are  $A$ . If the sign is minus, it is  $-A + 255$ . A table of arccosines has been written in ROM. The data of the address 0 are 0(rad), and the data of the address 255 are  $\pi$ (rad). The ROM is accessed by the microprocessor with the address which is generated by the address decoder. We obtain the 8-bit data  $a_1$  as a lower bit of the phase  $\alpha(t_m)$ , corresponding to Table I.

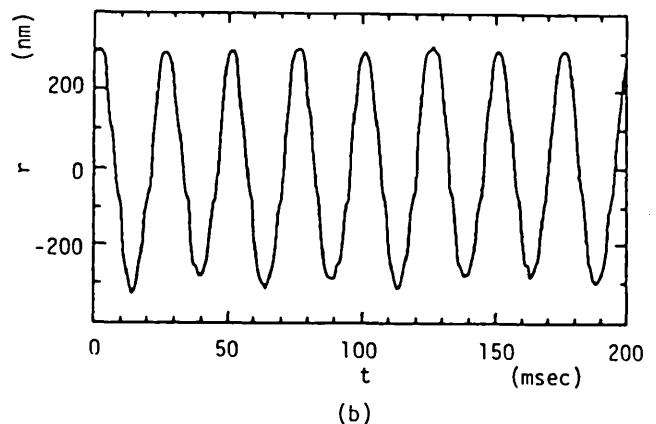
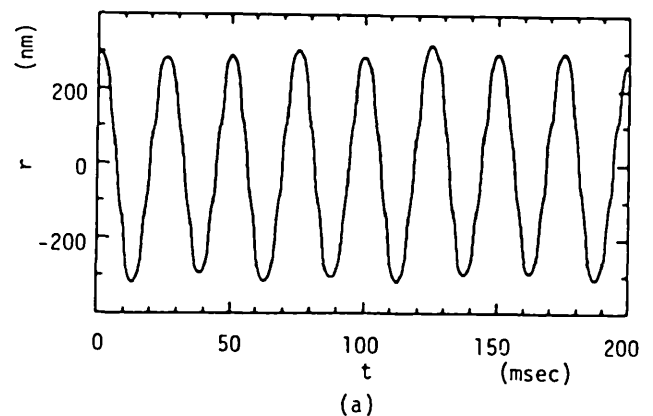


Fig. 4. Sinusoidal vibrations measured through (a) the computer processing and (b) the real time signal processor proposed here.

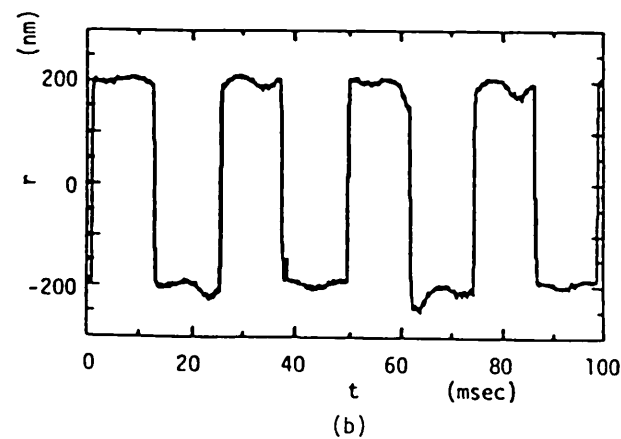
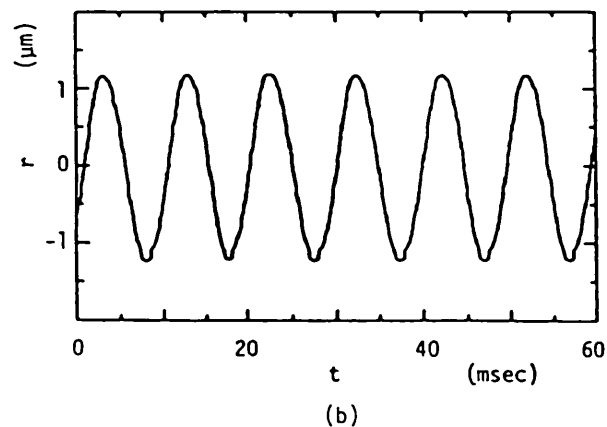
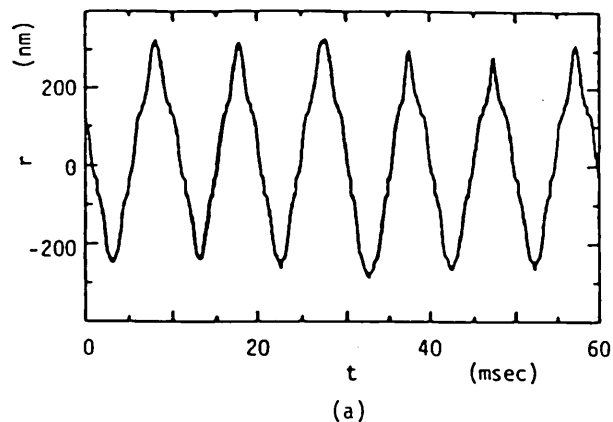
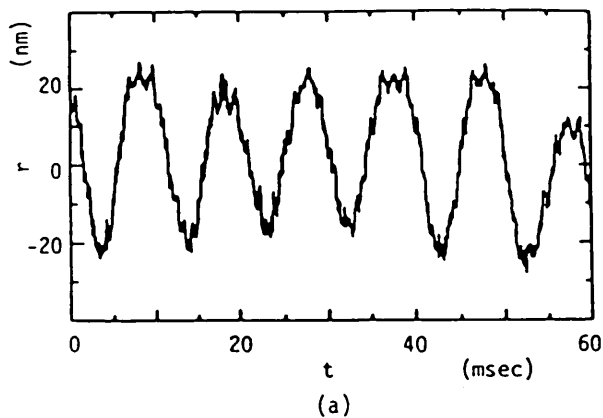


Fig. 5. Measured sinusoidal vibrations of  $f = 100$  Hz. The amplitude of vibrations are (a) 20 nm and (b) 1.2  $\mu\text{m}$ .

Fig. 6. Measured (a) triangle vibration of  $f = 100$  Hz and (b) rectangular vibration of  $f = 40$  Hz.

The signs of  $\cos\alpha(t_m)$  and  $\sin\alpha(t_m)$  are led to an increment-decrement detector to detect an increase or decrease of  $\alpha(t_m)$  by  $\pi$  as shown in Table II. The number of changes by  $\pi$  is counted with a 4-bit up-down counter. Then we obtain 4-bit data  $a_2$  as a higher bit of the phase  $\alpha(t_m)$ . Finally, we obtain 12-bit digital data as  $\alpha(t_m)$ , which is expressed by

$$\alpha(t_m) = a_1 + a_2\pi, \quad (11)$$

where  $0 \leq a_1 \leq \pi$  and  $-8 \leq a_2 \leq 7$ . We also obtain  $\alpha(t_m)$  as analog data by using a 12-bit D-A converter.

Delay time incurred in signal processing is mostly dependent on the conversion time of the A-D converter and running time of the microprocessor program. The total delay time of this signal processing system is  $\sim 45 \mu\text{s}$ .

#### IV. Experiments

The experimental setup is shown in Fig. 1. The light source is a laser diode with 5-mW maximum output power and a wavelength of 780 nm. The object is a mirror mounted on a piezoelectric transducer. The amplitude of the sinusoidal phase modulation  $z$  is 1.2, and the optical path difference  $2D_0$  is 20 cm.

First, a 50-Hz sinusoidal vibration was applied to the object. The frequency of the sinusoidal phase modulation was 1 kHz. The displacement measured by conventional processing with a computer is shown in Fig. 4(a); Fig. 4(b) shows the displacement measured in real time by the method described in this paper. The two measurements agree well and indicate that the method proposed here can measure the displacement in real time with the same accuracy as obtained by the conventional method.

Next, we investigated the measurable range of this method. The 100-Hz sinusoidal vibrations were applied to the object. The sinusoidal phase modulation frequency was 20 kHz. The measured results for a small amplitude vibration are shown in Fig. 5(a). The accuracy of the phase detection depends on the bit numbers on an A-D converter. The maximum displacement which can be measured in this experimental system is shown in Fig. 5(b). The measurable amplitude becomes larger by increasing the total bit number.

Finally, the triangle vibration of  $f = 100$  Hz and rectangular vibration of  $f = 40$  Hz were measured as shown in Figs. 6(a) and (b), respectively. The sinusoidal phase modulation frequencies are as in Fig. 5. The

rapid displacements contained in triangular and rectangular vibration can be measured exactly.

## V. Conclusions

We have proposed real time displacement measurement in a sinusoidal phase modulating interferometer. The interference signal is sampled synchronously with the sinusoidal phase modulation signal. In this method, the carrier component in the interference signal is removed by sampling the signal at specified times. We have shown that it is possible to measure the object's displacement in real time by using a simple hardware circuit. The time delay is  $<45 \mu\text{s}$ ; it is short enough to control movement of the object.

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## ***NIST Work Cited By Nobel Committee***

When the Royal Swedish Academy of Sciences awarded the Nobel Prize in Physics on Oct. 12, 1989, for research related to atomic clocks, they cited work at NIST's Boulder, Colo., laboratories. Norman F. Ramsey of Harvard University received the prize for discovering the theoretical basis for current cesium atomic clocks; Hans G. Dehmelt of the University of Washington and Wolfgang Paul of the University of Bonn shared the Nobel for developing the ion trap technique which makes it possible to study a single electron or a single ion with extreme precision. In referring to the latter work, the Nobel committee said "this opened the way to a new kind of spectroscopy, which has been further refined and applied particularly at the National Institute of Standards and Technology ... in Boulder, Colorado." Researchers in NIST's Time and Frequency Division are doing **research based on this technique which could some day result in atomic clocks that will neither gain nor lose a second in 10 billion years**, or roughly the age of the universe. Professor Ramsey also has close ties with NIST. He has been a member of the NIST statutory Visiting Committee and in the 1986-87 academic year he was a visiting fellow at the Joint Institute for Laboratory Astrophysics (JILA) and is currently a Fellow-Adjoint at JILA. JILA is a cooperative research effort between NIST and the University of Colorado.