

# Double sinusoidal phase-modulating laser diode interferometer for distance measurement

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Sinusoidal phase-modulating interferometry is used to detect exactly the sinusoidal phase variation of an interference signal even when the amplitude of the interference signal is varied by modulation of the injection current. We can easily provide a sinusoidal phase-modulating interferometer with a feedback control system that eliminates the phase fluctuations caused by mechanical vibrations. The methods using sinusoidal phase-modulating interferometry improve the resolution of distance measurements. Experimental results show that the thickness of gauge blocks is measured with a resolution of  $\sim 0.5 \mu\text{m}$ .

## 1. Introduction

A distance shorter than the optical wavelength can be measured by detecting the optical phase difference between the interfering lightwaves in an interferometer. When the distance is larger than the wavelength, ambiguities of an integer multiple of  $2\pi$  occur in the optical phase difference. To avoid these ambiguities, two or three different wavelengths are used. Since laser diodes provide different optical wavelengths inexpensively compared with other lasers, they are suitable as light sources in two-wavelength interferometry.<sup>1,2</sup> Two-wavelength interferometers produce an equivalent wavelength much larger than the original wavelength used, which increases the measuring range. However, it makes the interferometer more complicated to use the two light sources with different wavelengths.

The wavelength of laser diodes can be easily tuned by the injection current. It has been shown that periodic modulation of the wavelength of laser diodes offers interesting possibilities for measuring distances much greater than the wavelength without ambiguities.<sup>3-5</sup> In Ref. 3, the wavelength is modulated with a symmetric triangular wave, and the change in phase of the light reflected back from the target into the laser is measured by using the self-coupling effect of a laser to an external reflector. This method is effective in a

measuring range below 1.5 m with subcentimeter resolution.

In Refs. 4 and 5, the wavelength is modulated with a sinusoidal wave. An interference signal detected in Twyman-Green interferometers contains a sinusoidal phase variation produced by modulating the injection current with a sinusoidal wave signal. The amplitude of the phase variation is proportional to the optical path difference or the distance. In Ref. 4, to detect the phase variation a carrier frequency is introduced in the interference signal with two acoustic modulators. The resolution of this method is  $\sim 3 \mu\text{m}$  at a measuring range of a few centimeters. In Ref. 5, the carrier frequency is not given to the interference signal. The interference signal contains the sinusoidal phase variation, and it is the same signal as that in the sinusoidal phase-modulating (SPM) interferometers reported in Refs. 6 and 7. The amplitude of the sinusoidal phase variation is obtained with electric circuits such as filters and dividers. The resolution of this method is submillimeter over the range of hundreds of millimeters.

In this paper, a carrier signal is given to the interference signal by using a vibrating mirror, and SPM interferometry is applied to detect the sinusoidal phase variation. Since no acoustic modulators are required, in much the same way as the method proposed in Ref. 5, the optical setup is simpler than that reported in Ref. 4. In SPM interferometry, the phase variation is exactly obtained through signal processing of the interference signal with a computer even when there is amplitude variation of the interference signal caused by modulation of the injection current. So, we can increase the amplitude of the sinusoidal signal of the injection current so far as mode jump does not occur, which enables us to improve the resolution in distance

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measurements. In addition, by controlling the vibrating mirror with a feedback control system, we eliminate the phase fluctuation caused by mechanical vibrations of the optical devices. We can obtain an exact amplitude of the sinusoidal variation without suffering from external disturbances. These approaches provide an increase in measurement resolution, and we measure the thickness of gauge blocks with a resolution of  $\sim 0.5 \mu\text{m}$ .

## II. Principle

Figure 1 shows the basic setup of a double SPM interferometer for distance measurements. The injection current of the laser diode consists of a dc bias component and a sinusoidal modulation signal  $b \cos \omega_b t$ . The modulation of the injection current results in both an intensity modulation and a wavelength modulation, given by

$$\begin{aligned} g(t) &= g_0[1 + m \cos(\omega_b t)], \\ \lambda(t) &= \lambda_0 + \beta b \cos(\omega_b t), \end{aligned} \quad (1)$$

where  $g_0$  is the average intensity,  $m$  is the modulation depth of the intensity, and  $\beta$  is a constant of proportional relationship between the wavelength and the injection current. The light emitted from the laser diode is collimated by a lens and is split into a reference beam and an object beam with a beam splitter (BS). The reference beam is reflected by a mirror that is vibrated with a piezoelectric transducer (PZT). The vibration of the mirror is a sinusoidal motion  $a \cos(\omega_c t + \theta)$ . The optical path difference between the reference and object beams is  $\Delta l$ . The interference pattern detected with a photodiode (PD) is expressed by

$$S(t) = g(t) + g(t)\cos[z_c \cos(\omega_c t + \theta) + z_b \cos(\omega_b t) + \alpha], \quad (2)$$

where

$$\begin{aligned} z_c &= (4\pi/\lambda_0)a, \\ z_b &= (2\pi\beta b/\lambda_0^2)\Delta l, \\ \alpha &= (2\pi/\lambda_0)\Delta l. \end{aligned} \quad (3)$$

The interference signal  $S(t)$  contains two different SPM's. One,  $z_c \cos(\omega_c t + \theta)$ , is used as a carrier signal,

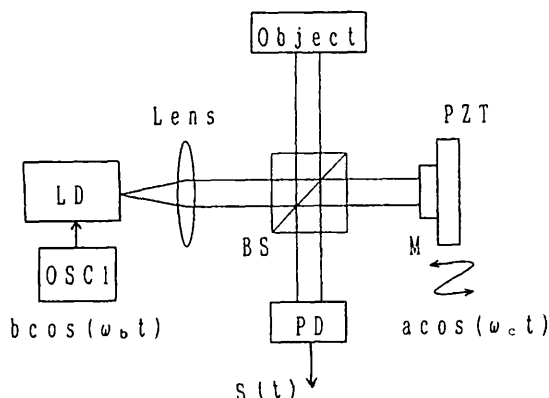


Fig. 1. Basic setup of a double SPM laser diode interferometer for distance measurements.

and the condition of  $\omega_c > 2\omega_b$  is necessary. The other is produced by modulating the injection current of the laser diode, and its modulation depth  $z_b$  is proportional to the optical phase difference  $\Delta l$ . We extract the value of  $z_b$  from the interference signal to measure the optical path difference or distance of the object.

We need the frequency components of signal  $S(t)$  between  $\omega_c/2$  and  $5\omega_c/2$  to obtain the phase variation  $z_b \cos(\omega_b t) + \alpha$ . Putting  $\Phi(t) = z_b \cos(\omega_b t) + \alpha$  and neglecting the first term  $g(t)$  in signal  $S(t)$ , the interference signal is rewritten as

$$S(t) = g(t)\cos[z_c \cos(\omega_c t + \theta) + \Phi(t)]. \quad (4)$$

First, we must measure amplitude  $z_c$  and argument  $\theta$  of the SPM before starting the distance measurements. To measure these values, we stop the modulation produced by the injection current, that is,  $g(t) = 1$  and  $\Phi(t) = \alpha$ . Then the interference signal is written as

$$S(t) = \cos[z_c \cos(\omega_c t + \theta) + \alpha]. \quad (5)$$

The method to obtain the values of  $z_c$  and  $\theta$  from the signal given by Eq. (5) has been reported in Ref. 6. If the Fourier transform of  $S(t)$  is represented by  $F(\omega)$ , these values are obtained from the components of  $F(\omega_c)$  and  $F(3\omega_c)$ .

Next, we extract the phase variation  $\Phi(t)$  from the signal given by Eq. (4) using the values of  $z_c$  and  $\theta$ . This method has been reported in Ref. 7 for  $g(t) = 1$ . The expansion of Eq. (4) is given by

$$\begin{aligned} S(t) &= |g(t)\cos[\Phi(t)]\{J_0(z_c) - 2J_2(z_c)\cos(2\omega_c t + 2\theta) + \dots\} \\ &\quad - |g(t)\sin[\Phi(t)]\{2J_1(z_c)\cos(\omega_c t + \theta) - \dots\}, \end{aligned} \quad (6)$$

where  $J_n(z_c)$  is the  $n$ th-order Bessel function. The following conditions must be satisfied:

$$\begin{aligned} \mathcal{F}\{g(t)\cos[\Phi(t)]\} &= 0, \\ \mathcal{F}\{g(t)\sin[\Phi(t)]\} &= 0, \quad |\omega| > \omega_c/2, \end{aligned} \quad (7)$$

where  $\mathcal{F}\{y\}$  is a Fourier transform of  $y$ . We designate the frequency components existing in the regions of  $\omega_c/2 < \omega < 3\omega_c/2$  and  $3\omega_c/2 < \omega < 5\omega_c/2$  by  $F_1(\omega)$  and  $F_2(\omega)$ , respectively. Then we have

$$\begin{aligned} F_1(\omega + \omega_c) &= -J_1(z_c)\exp(j\theta)\mathcal{F}\{g(t)\sin[\Phi(t)]\}, \\ F_2(\omega + 2\omega_c) &= -J_2(z_c)\exp(j2\theta)\mathcal{F}\{g(t)\cos[\Phi(t)]\}, \quad |\omega| < \omega_c/2. \end{aligned} \quad (8)$$

We can obtain the values in Eqs. (8) through Fourier transform of the detected interference signal. Using the values of  $z_c$  and  $\theta$ , we calculate the two functions for  $\omega$ :

$$\begin{aligned} -F_1(\omega + \omega_c)/J_1(z_c)\exp(j\theta), \\ -F_2(\omega + 2\omega_c)/J_2(z_c)\exp(j2\theta). \end{aligned} \quad (9)$$

Taking the inverse Fourier transform of functions (9), we can obtain the functions of  $g(t)\sin[\Phi(t)]$  and  $g(t)\cos[\Phi(t)]$  to calculate the argument  $\Phi(t)$ . Finally, we take the Fourier transform of  $\Phi(t)$  to obtain the value of  $z_b$ , which is the amplitude of the frequency component  $\omega_b$ . We used a computer to obtain the

value of  $z_b$  or the distance of an object from the interference signal.

### III. Elimination of Phase Fluctuations

Optical devices in an interferometer vibrate in response to external mechanical vibrations. This causes fluctuations in phase  $\alpha$  of the interference signal. These fluctuations are usually greater than amplitude  $z_b$  of the sinusoidal phase variation, so that the measurement accuracy in  $z_b$  decreases. We eliminate fluctuations in phase  $\alpha$  by using a feedback control system.

Figure 2 shows the experimental setup, where the feedback control system is provided for the basic setup shown in Fig. 1. The vibrating mirror attached to the PZT is moved to compensate the fluctuation in phase  $\alpha$  with the feedback control system. The feedback controller produces a control signal that is applied to the PZT. If the value of  $\alpha$  is nearly equal to  $\pi/4$ , both components,  $F_1(\omega)$  and  $F_2(\omega)$ , in Eqs. (8) do not become zero. Therefore, the value of  $\alpha$  is kept at  $\pi/4$  by the feedback control system.

We now explain how to produce the feedback signal from the interference signal. We obtain the following signals from the interference signal through heterodyne detection techniques:

$$\begin{aligned} J_2(z_c)g(t)\cos[\Phi(t)], \\ J_1(z_c)g(t)\sin[\Phi(t)]. \end{aligned} \quad (10)$$

These signals are fed to low-pass filters whose cutoff frequency is  $\omega_b/2\pi$ . The outputs of the filters are given by

$$\begin{aligned} h_1(t) &= J_2(z_c)J_0(z_b)\cos\alpha - mJ_2(z_c)J_1(z_b)\sin\alpha, \\ h_2(t) &= J_1(z_c)J_0(z_b)\sin\alpha + mJ_1(z_c)J_1(z_b)\cos\alpha. \end{aligned} \quad (11)$$

Since the value of  $J_1(z_b)$  is  $<0.02$  in the range of  $z_b < 0.05$ , we can neglect the second term in Eqs. (11). By amplifying the signals in Eqs. (11) by some factors and adding them, we obtain the following feedback signal:

$$J_1(z_c)h_1(t) - J_2(z_c)h_2(t) \propto \cos(\alpha + \pi/4). \quad (12)$$

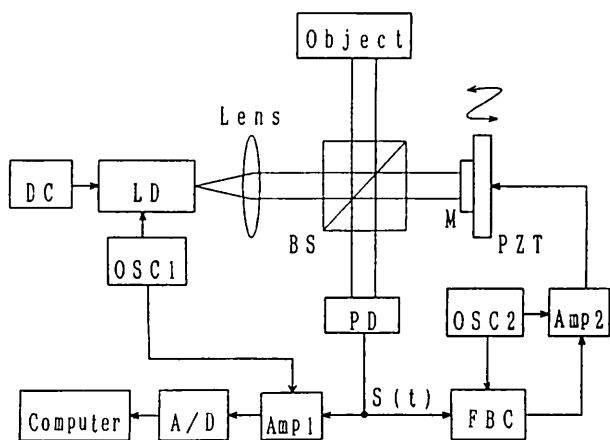


Fig. 2. Experimental setup where the feedback control system is provided to eliminate phase fluctuations caused by mechanical vibrations.

The control signal is produced by passing the feedback signal through both a proportional amplifier and an integrator. The control signal is added to the sinusoidal signal  $a \cos(\omega_c t)$  with the amplifier (Amp2) to be applied to the PZT. The feedback control that keeps phase  $\alpha$  at  $\pi/4$  increases the measurement accuracy in  $z_b$ .

### IV. Experiments

Figure 2 shows the experimental setup that has been built to investigate the principle described. We used a GaAlAs laser diode with maximum output power of 10 mW and a wavelength of  $\sim 780$  nm. The temperature of the laser was not controlled. The modulation efficiency  $\beta$  was  $\sim 0.006$  nm/mA. Oscillator 1 (OSC1) provides a sinusoidal wave signal with which the injection current is modulated. The frequency of modulation  $\omega_b/2\pi$  was 0.6 kHz and its amplitude  $b$  was 5 or 10 mA. When the value of  $b$  was increased by more than 10 mA, mode hopping occurred. Oscillator 2 (OSC2) and AMP2 produce the sinusoidal signal  $a \cos(\omega_c t)$ , where the frequency  $\omega_c/2\pi$  was 4.8 kHz. Amplitude  $z_c$  of the SPM with a vibrating mirror was  $\sim 2.6$  rad. The feedback controller produces the control signal using the signals  $\cos(\omega_c t)$  and  $\cos(2\omega_c t)$ , which are the outputs of OSC2.

The signal  $g(t)$  is eliminated from the detected interference signal in AMP1 using the output of OSC1,  $b \cos(\omega_b t)$ . The output signal of AMP1 is shown in Fig. 3(a). This signal is sampled with an analog-to-digital converter. The sampling frequency was  $4.8 \times 16$  kHz and the number of the sample point was 4096. The length of the sampled data to be processed in a micro-computer was 5.3 ms. The amplitude of the Fourier transform of the data is shown in Fig. 3(b). We obtained the  $\Phi(t)$  from the frequency components  $F_1(\omega)$  and  $F_2(\omega)$ . Figure 3(c) shows the  $\Phi(t)$  and its Fourier transform is shown in Fig. 3(d). When the feedback control did not work, the fluctuation in phase  $\alpha$  was about ten times as large as that shown in Fig. 3(c). The feedback control reduced the phase fluctuation so that there was no phase fluctuation effect on the measurement of  $z_b$ . The average value of  $\Phi(t)$  or phase  $\alpha$  was  $\sim \pi/4$ , as shown in Fig. 3(c). The frequency resolution in the Fourier transform of  $\Phi(t)$  was  $\sim 19$  Hz. The 0.6-kHz amplitude of the frequency component is the value of  $z_b$  shown in Fig. 3(d).

We used the top of the micrometer head as an object. We located the position of the micrometer head where the value of  $z_b$  was zero. The value of  $\Delta l$  was considered to be zero at this position. We move the object parallel to the optical axis at intervals of  $d \mu\text{m}$ . At each position we measure the  $z_b$  value to investigate the proportional relationship between the optical path difference  $\Delta l$  and the value of  $z_b$ . The experimental results for  $d = 1$  and  $5 \mu\text{m}$  are shown in Figs. 4 and 5, respectively. Figure 4 shows that the measurement resolution is  $\sim 0.5 \mu\text{m}$  in the  $\Delta l$  region below  $16 \mu\text{m}$ . Figure 5 shows that the measurement becomes impossible in the  $\Delta l$  region above  $70 \mu\text{m}$  at  $b = 10$  mA. Then

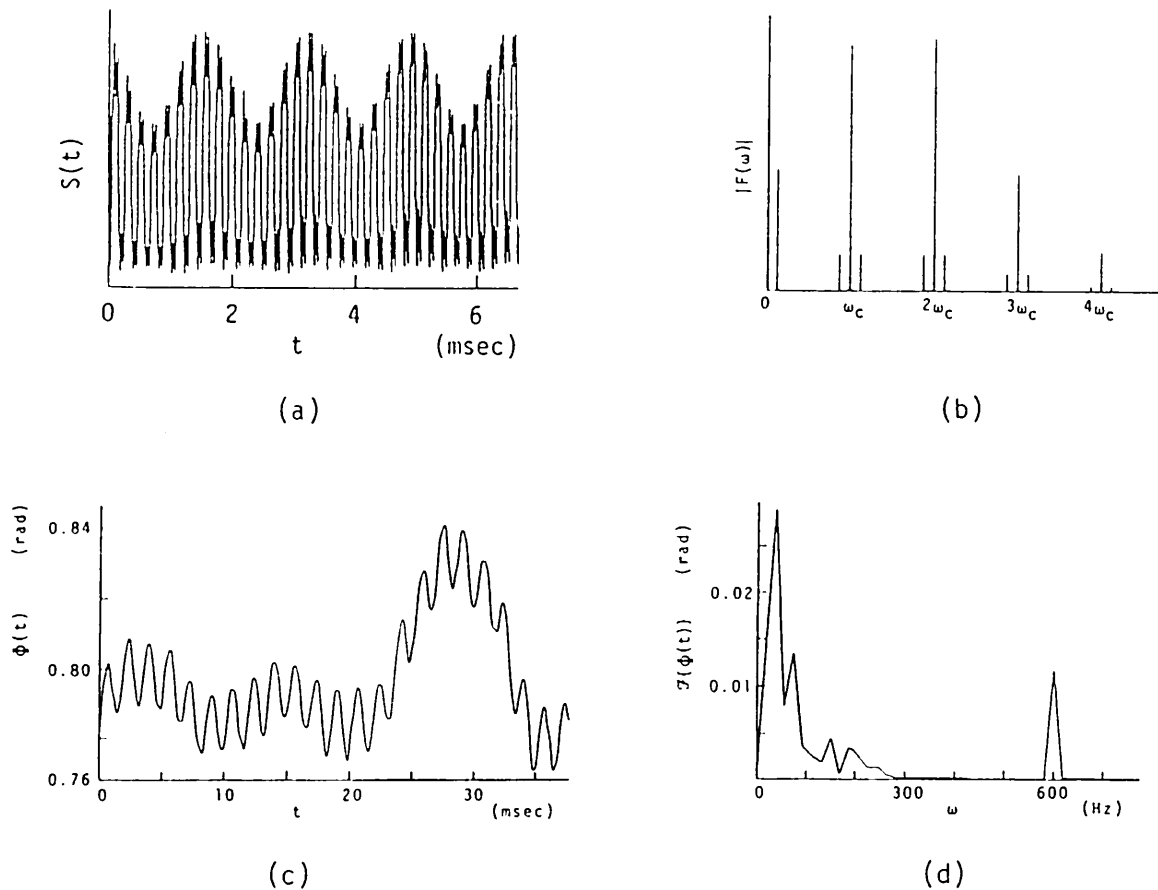


Fig. 3. Signal processing to obtain amplitude  $z_b$  of the sinusoidal phase variation: (a) detected signal  $S(t)$  given by Eq. (4); (b) amplitude of the Fourier transform of signal  $S(t)$ ; (c) phase variation  $\Phi(t)$  obtained from the frequency components of  $F_1(\omega)$  and  $F_2(\omega)$ ; (d) Fourier transform of the phase variation  $\phi(t)$ .

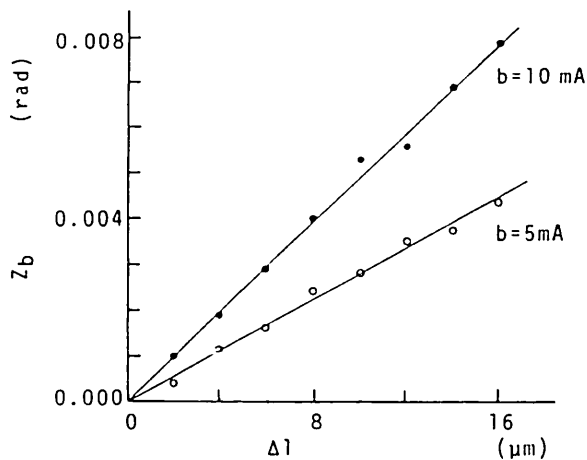


Fig. 4. Measured relationship between the optical path difference and the amplitude of the sinusoidal phase variation at  $b = 10$  mA.

the frequency components  $F_1(\omega)$  and  $F_2(\omega)$  overlap near the point of  $\omega = (3/2)\omega_c$ . The measured values of  $z_b$  are proportional to the optical path difference  $\Delta l$ . However, a proportional constant at  $b = 10$  mA is not exactly twice as large as that at  $b = 5$  mA. This error seems to result because the room temperature and the dc bias of the injection current were different in each measurement. These results tell us that the laser

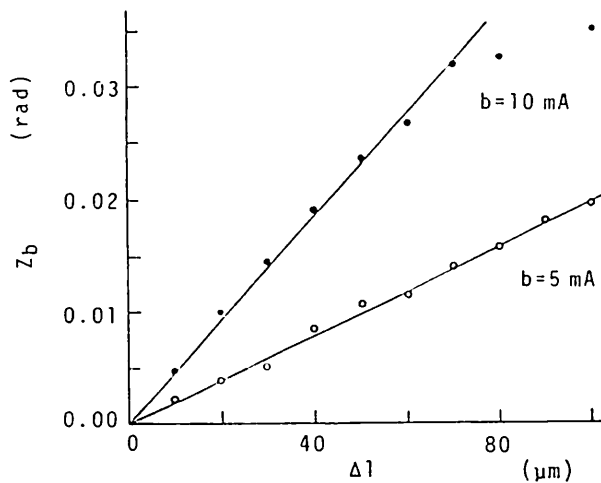


Fig. 5. Measured relationship between the optical path difference and the amplitude of the sinusoidal phase variation at  $b = 5$  mA.

temperature must be stabilized and the dc bias of the injection current must be kept constant.

#### V. Thickness Measurement of the Gauge Block

We measured the thickness of gauge blocks with the experimental setup shown in Fig. 6. The object wave is deflected by a right angle and illuminated on the

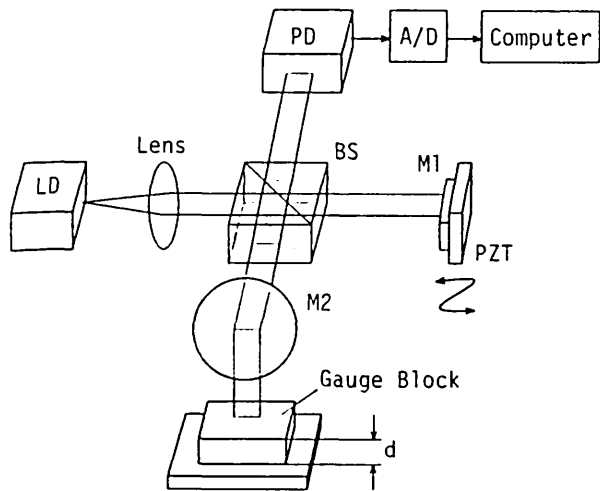


Fig. 6. Double SPM laser diode interferometer for thickness measurement of gauge blocks.

Table I. Thickness Measurement Results of Gauge Blocks

Gauge Block	$d_e$ ( $\mu\text{m}$ )	$Z_b$ ( $\times 10^{-3}$ rad)	Measured Thickness
A	1.0	0.56	0.57
B	2.0	2.15	2.2
C	5.0	5.10	5.2
D	8.0	7.72	7.0
E	10.0	9.84	10.2

surface of the gauge block that is set on a standard base. Thickness  $d$  of the gauge blocks was slightly larger than 1 mm, that is,  $d = 1000 + d_e \mu\text{m}$ . We measured thickness  $d_e$  compared to the gauge block of  $d = 1.0$  mm. First, we set the 1.0-mm gauge block on a standard base and located the position of the vibrating mirror where the value of  $z_b$  became zero. Next, we moved the vibrating mirror parallel to the optical axis, leaving some distance between the two, and measured a constant of the proportional relationship between the value of  $z_b$  and optical path difference  $\Delta l$ . The constant was  $0.00049 \text{ rad}/\mu\text{m}$ . Finally, we replaced the gauge block with the one to be measured, and we measured the value of  $z_b$  to obtain thickness  $d_e$ . The measured results are given in Table I. We measured five gauge blocks whose thicknesses are listed in the

second row of Table I. The measured value of  $z_b$  and the measured thickness are listed in the third and fourth rows, respectively. The results show that we can measure gauge block thickness with a measurement resolution of  $0.5 \mu\text{m}$ .

## VI. Conclusion

SPM interferometry was applied to the distance measurement using the light wavelength variation of a laser diode that produced the double SPM laser diode interferometer. In this interferometer we could increase the amplitude of the sinusoidal signal of the injection current to 10 mA, and we could easily construct the feedback control system that eliminates the phase fluctuations caused by mechanical vibrations. Thus, we measured the distance and the thickness of block gauges with a resolution of  $\sim 0.5 \mu\text{m}$ . However, a constant of the proportional relationship between the amplitude of the sinusoidal phase variation and the optical path difference changed in each measurement because of uncontrolled temperature and dc bias current of the laser diode. The stabilization of the proportional constant and real-time measurement using signal processing with electric circuits will be investigated in the future.

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