Radiation Efficiency Measurement Based on Wheeler Method
Using 90-degree 3dB Hybrid Coupler and Sliding Short

Hirofumi Iguchi and Nozomu Ishii
Graduate School of Science and Technology, Niigata University
8050, Ikarashi 2-nocho, Nishi-ku, Niigata, 950-2181, Japan
E-mail: nishii@eng.niigata-u.ac.jp

1. Introduction

Wheeler cap method is known as a simple method for measuring the radiation efficiency of small antennas [1]. In this method, the radiation efficiency can be evaluated by using reflection coefficients when the antenna under test is located in free space and is covered with the shield. Also, it is known that the uncertainty of the measurement for the reflection coefficient using the vector network analyzer (VNA) increases when its magnitude is close to 1. This is because all the input power will return to feeding port except for power lost in the antenna and the shield when the antenna is covered with the shield. Therefore, it is desired to develop an alternative method for measuring the reflection coefficient as its uncertainty should be less than the direct measurement using VNA [2].

The technique that used a hybrid coupler was proposed by Randus et al., as a method of reducing the uncertainty of the reflection coefficient measurement [3], [4]. It uses an amplified subtraction function of the 180-degree 3dB hybrid coupler when measuring the transmission coefficient with a reference standard (RS). However, it is available only when \( \Gamma_{RS} \approx \pm \Gamma_{DUT} \), where \( \Gamma_{RS} \) and \( \Gamma_{DUT} \) are reflection coefficients of RS and device under test (DUT), respectively. Also, a predictive value of \( \Gamma_{DUT} \) could be known before the measurement. To overcome the difficulty, we have proposed a method of measuring the reflection coefficient of arbitrary DUT by two-port transmission measurement using the hybrid coupler [5]. In this method, a RS that \( |\Gamma_{RS}| \) is constant but \( \angle \Gamma_{RS} \) can be varied is realized by a sliding short that is composed of a short connected with a variable-length coaxial line. The locus of the reflection coefficient of RS draws a circle on the reflection coefficient plane because the phase of \( \Gamma_{RS} \) can be changed by the sliding short. If well-chosen DUT and RS are connected into the hybrid coupler and transmission coefficient between unused two ports of the hybrid is measured, the locus of the transmission coefficient also draws a circle on the transmission coefficient plane. Then, the reflection coefficient of DUT, \( \Gamma_{DUT} \), can be obtained by determining the center and radius of the circles by means of the least square method.

In this paper, the proposed method using a 90-degree 3dB hybrid coupler is applied to the measurement of the radiation efficiency of a monopole antenna based on Wheeler cap method. And we show the uncertainty of the radiation efficiency measurement. Until now, the length of the sliding short is changed from 0mm to 75mm at intervals of 5mm, that is, the circle can be obtained by using 16 points on the reflection/transmission coefficient planes. In this paper, it is examined how the uncertainty is changed when the number of the points that are used to determine the circles is reduced.

2. Measurement Procedure

2.1 Reflection Coefficient Measurement Using Hybrid Coupler and Sliding Short

Available 90-degree 3dB hybrid couplers are not ideal so that we should pay attention to the insertion loss and phase difference due to the connectors and connecting cables. To compensate the effects, it is required to measure all \( S \) parameters of components included in the measurement system as shown in Fig. 1. Desired reflection coefficient, \( \Gamma_x \), can be determined according to the following procedure. Transmission coefficient \( T_{21} \) in the measurement system can be expressed as
Figure 1: Reflection coefficient measurement system using hybrid coupler

Figure 2: Plots of $\Gamma_{\text{ref}}$ and $T_{21}$ on reflection/transmission coefficient planes

$$T_{21} = \frac{F_1 + F_2 \Gamma_{\text{ref}}}{1 - F_3 \Gamma_{\text{ref}}}$$  \hspace{1cm} (1)

where $F_1, F_2$ and $F_3$ are a function of $\Gamma_x$ and can be algebraically calculated by using $S$ parameters of the measurement system. If the value of $\Gamma_{\text{ref}}$ is changed by moving the sliding short, $\Gamma_{\text{ref}}$ draws a circle on the reflection coefficient plane as well as $T_{21}$ draws a circle on the transmission coefficient plane as shown in Fig. 2. The center $z_{\text{ref}}$ and the radius $r_{\text{ref}}$ of the $\Gamma_{\text{ref}}$ circle, and the center $z_c$ and the radius $r_c$ of the $T_{21}$ circle are determined by means of the least square method. If $z_{\text{ref}}$ and $r_{\text{ref}}$ are estimated, then $z_c$ and $r_c$ can be determined by the following equations

$$z_c = \frac{(F_1 + F_2 z_{\text{ref}})(1 - F_3 z_{\text{ref}})^* + r_{\text{ref}}^2 F_2 F_3^*}{|1 - F_3 z_{\text{ref}}|^2 - r_{\text{ref}}^2 |F_3|^2}, \quad r_c = \frac{r_{\text{ref}}|F_1 F_3 + F_2|}{||1 - F_3 z_{\text{ref}}|^2 - r_{\text{ref}}^2 |F_3|^2||}. \hspace{1cm} (2)$$

Note that all variables except $\Gamma_x$ are known in (2).

2.2 Wheeler Cap Method

The above method can be applied to the radiation efficiency measurement based on the Wheeler cap method, where the efficiency $\eta$ can be estimated as

$$\eta = \frac{|\Gamma^s|^2 - |\Gamma^f|^2}{1 - |\Gamma^f|^2}, \hspace{1cm} (3)$$

where $\Gamma^f$ and $\Gamma^s$ are reflection coefficients when the antenna is located in free space and covered with the shield, respectively.

2.3 Measurement Setup

Our setup for the Wheeler efficiency measurement is shown in Fig. 1. A 90-degree 3dB hybrid coupler (Cernex, CHC0102U322T) is used. The antenna under test is a monopole antenna with a length of 40mm and the shield is a rectangular metallic cap of 150mm×150mm×75mm. A 3dB attenuator is inserted between a fixed short and a variable-length coaxial line (Hiros, RS HLS-JJ-1(40)), where its length can be changed from 0mm to 75mm at regular intervals. In this paper, three ways of measurement points are examined, that is, 4 points at intervals of 20mm, 8 points at intervals of 10mm, and 16 points at intervals of 5mm.

The measurement procedure is as follows: First, all $S$ parameters of the 4-port measurement system, $S_{ij}(i,j = 1,2,3,4)$, are measured by VNA. Next, $T_{21}$s are measured by changing the length of the
variable coaxial line when the antenna is located in free space. $\Gamma_{\text{ref}}$S are directly measured by VNA as the length of the variable coaxial line is changed. Then, $\Gamma_f$ can be estimated. Similarly, $\Gamma_s$ can be estimated when the antenna is covered with the shield. Therefore, the Wheeler efficiency, $\eta_x$, can be estimated. To confirm our measurement, $\eta_x$ is compared with $\eta_{\text{VNA}}$, which can be directly measured by VNA.

### 2.4 Monte Carlo Simulation for Uncertainty Evaluation

Measured quantities in measuring the radiation efficiency based on the Wheeler cap method using the hybrid coupler are $S$ parameters of the measurement system, $S_{ij}$, connectors and cables, the transmission coefficient, $T_{21}$, the reflection coefficient of the RS, $\Gamma_{\text{ref}}$. In this paper, the uncertainty for 4, 8 and 16 point selections to find the circle is examined, where the number of the measurement for 4, 8 and 16 point selections is 29, 41 and 65. By generating the normal random numbers for the magnitude and phase of measured reflection or transmission coefficients, we can estimate the uncertainty of the radiation efficiency. The average and standard deviation of the normal random numbers are assumed to correspond to the measured value and standard uncertainty which can be given by the spreadsheet provided by the manufacturer of VNA.

### 3. Measurement Results

Fig. 3 shows the radiation efficiency $\eta_{\text{90HD}}$ and $\eta_{\text{VNA}}$ for a monopole antenna with a length 40mm by the reflection coefficient measurement system using a 90-degree 3dB hybrid coupler and the direct measurement using VNA. It can be confirmed that the radiation efficiency using the hybrid coupler is independent of the number of selecting the length of the sliding short. Fig. 4 shows simulated standard uncertainty of the measurement for the radiation efficiency. In the Monte Carlo simulation, the reflection coefficients $|\Gamma_f|$, $|\Gamma_s|$ are not stable below 1.35GHz because they can be greater than 1 by generated random number. Also, the relative uncertainty of $\eta_{\text{90HD}}$, $u_r(\eta_{\text{90HD}})/\eta_{\text{90HD}}$, can be somewhat smaller than relative uncertainty of $\eta_{\text{VNA}}$, $u_r(\eta_{\text{VNA}})/\eta_{\text{VNA}}$, below 1.35GHz. Also, Table 1 shows the numerical comparison of the expanded uncertainty at 1.75GHz. Compared to the uncertainty of 4, 8 and 16 point selections, the uncertainty is smaller as the number of the selections is larger.

### 4. Conclusions

To reduce the uncertainty in measurement of the arbitrary reflection coefficient, we proposed a measurement technique using a 90-degree hybrid coupler and a sliding short. It can be applied to the radiation efficiency measurement of the monopole antenna based on Wheeler cap method, and the uncertainty is
Table 1: Comparison of extended uncertainty $U_c$ at 1.75GHz for a monopole antenna with a length of 40mm

<table>
<thead>
<tr>
<th>Extended uncertainty, $U_c$ [%] ($k = 2$)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_c(\eta_{90HD-4pt-mc})$</td>
<td>3.52</td>
</tr>
<tr>
<td>$U_c(\eta_{90HD-8pt-mc})$</td>
<td>3.24</td>
</tr>
<tr>
<td>$U_c(\eta_{90HD-16pt-mc})$</td>
<td>3.07</td>
</tr>
<tr>
<td>$U_c(\eta_{VNA-mc})$</td>
<td>3.58</td>
</tr>
</tbody>
</table>

evaluated by the Monte Carlo simulation. Also, we examine the uncertainty by changing the number of selecting the length of the sliding short. To save the measurement time, this number should be reduced, however, the uncertainty is smaller as the number is larger. In the future, this trade-off should be solved.

References


