Studies of CP Violation in $B \to J/\psi K^*$ Decays


(Belle Collaboration)
An angular analysis of $B$ meson decay to two vector mesons is a sensitive probe of new physics. There are three classes of parameters obtained through the angular analysis. The first is the measurement of the decay amplitudes of the three helicity states. These can be obtained by the time-integrated angular analysis of flavor specific decays. The comparison of the amplitudes between flavor separated samples probes direct CP violation. The second is the triple product correlation, which can be extracted from the measured decay amplitudes. This quantity is sensitive to $T$-odd CP violation. The third class is composed of the CP parameters ($\sin 2\phi_1$ and $\cos 2\phi_1$) that are measured through a time-dependent angular analysis. In particular, the measurement of $\cos 2\phi_1$, which appears in the time-dependent interference terms, is important both to solve the twofold ambiguity in $2\phi_1$ and to test the consistency of this determination with the more precise value from other $b \rightarrow c\bar{c}s$ decays. In this Letter, we report the measurements of these parameters for $B \rightarrow J/\psi K^*$ decays.

The data sample used in this analysis corresponds to an integrated luminosity of 253 fb$^{-1}$ recorded with the Belle detector at the KEKB $e^+e^-$ collider. Four decay modes are reconstructed: $B^0 \rightarrow J/\psi K^{*0}$; $K^{*0} \rightarrow K^+ \pi^-$ and $K^0_S \pi^+$, and $B^+ \rightarrow J/\psi K^{*+}$; $K^{*+} \rightarrow K^+ \pi^0$ and $K^0_S \pi^+$. The charge conjugate modes are included everywhere unless otherwise specified. The reconstruction is done using the criteria described in Ref. [3]. A $J/\psi$ candidate is reconstructed from two oppositely charged tracks identified as leptons. A $K^*$ candidate is selected if the absolute difference between the invariant mass of an identified $K \pi$ pair and the nominal $K^*(892)$ mass is less than 75 MeV/c$^2$. Candidate $B$ mesons are reconstructed by combining a $J/\psi$ candidate with a $K^*$ candidate and examining two quantities in the center of mass of the $Y(4S)$: the beam-constrained mass calculated using the beam energy in place of the reconstructed energy ($M_{bc}$), and the energy difference between the $B$ candidate and the beam energy ($\Delta E$). $M_{bc}$ is required to be in the range 5.27–5.29 GeV/c$^2$. For the modes with a charged (neutral) pion, the energy difference must satisfy $|\Delta E| < 30$ MeV ($-50 < \Delta E < 30$ MeV). To eliminate slow $\pi^0$ backgrounds, the angle $\theta_{K^*}$ of the kaon with respect to the opposite of the $B$ direction in the $K^*$ rest frame is required to satisfy $\cos \theta_{K^*} < 0.8$. When an event contains more than one candidate passing the above requirements, the best combination is selected based on a $\chi^2$ calculated using $M_{bc}$ and $\Delta E$. Figure 1 shows the $M_{bc}$ distributions for the candidates of two neutral $B$ decay modes assembled without the selection in $M_{bc}$. After all selections are applied, the remaining numbers of events in the signal region of the $M_{bc} - \Delta E$ plane are 8194 for $J/\psi K^{*0}(K^+ \pi^-)$, 363 for $J/\psi K^{*0}(K^0_S \pi^+)$, 2222 for $J/\psi K^{*+}(K^+ \pi^0)$, and 2168 for $J/\psi K^{*+}(K^0_S \pi^+)$. The angular distribution of the products of $B \rightarrow J/\psi K^*$ decays is described using three angles in the transversity basis [4]. The direction of motion of the $J/\psi$ in the rest...
frame of the $B$ candidate is defined to be the $x$ axis. The $y$ axis is chosen along the direction of the projection of the $K$ momentum into the plane perpendicular to the $x$ axis in the $B$ rest frame. The $z$ axis is then perpendicular to the $x$-$y$ plane according to the right-hand rule. The angle between the positive lepton ($l^+$) and the $z$ axis in the $J/\psi$ rest frame is defined as $\theta_y$. The angle between the $x$ axis and the projection of the $l^+$ momentum onto the $x$-$y$ plane is defined as $\phi_y$ in the same frame. The angle $\theta_K^*$ is defined earlier.

The distribution of the angles as a function of the decay time difference between $B$ and anti-$B$ mesons ($\Delta t$) is described as follows [5]:

$$1 \frac{d^3 \Gamma(\theta_y, \phi_y, \theta_K^*, \Delta t)}{d \cos \theta_y d \phi_y d \cos \theta_K^* d \Delta t} = \frac{9}{32 \pi} \frac{e^{-|\Delta t|/\tau_B}}{2 \tau_B} \times \sum_{i=1}^{6} g_i(\theta_y, \phi_y, \theta_K^*) a_i(\Delta t),$$

(1)

where the angular terms $g_i$ are defined as

$$g_1 = 2 \cos^2 \theta_K^* (1 - \sin^2 \theta_y \cos^2 \phi_y),$$
$$g_2 = \sin^2 \theta_K^* (1 - \sin^2 \theta_y \sin^2 \phi_y),$$
$$g_3 = \sin \theta_K^* \sin \theta_y \sin \phi_y,$$
$$g_4 = -(1/\sqrt{2}) \sin \theta_K^* \sin \theta_y \sin \phi_y,$$
$$g_5 = \sin \theta_K^* \sin \phi_y,$$
$$g_6 = (1/\sqrt{2}) \sin \theta_K^* \sin \phi_y \cos \phi_y,$$

and the amplitude terms $a_i$ as

$$a_1 = |A_0|^2 (1 + \eta \sin \phi_1 \sin \Delta m \Delta t),$$
$$a_2 = |A_\parallel|^2 (1 + \eta \sin \phi_1 \sin \Delta m \Delta t),$$
$$a_3 = |A_\perp|^2 (1 - \eta \sin \phi_1 \sin \Delta m \Delta t),$$
$$a_4 = \text{Re}(A_0^* A_\parallel) (1 + \eta \sin \phi_1 \sin \Delta m \Delta t),$$
$$a_5 = \eta \text{Im}(A_0^* A_\perp) \cos \Delta m \Delta t$$
$$- \eta \text{Re}(A_0^* A_\perp) \sin \phi_1 \sin \Delta m \Delta t,$$
$$a_6 = \eta \text{Im}(A_\parallel^* A_\perp) \cos \Delta m \Delta t$$
$$- \eta \text{Re}(A_\parallel^* A_\perp) \sin \phi_1 \sin \Delta m \Delta t.$$

Here, $A_0, A_\parallel,$ and $A_\perp$ are the complex decay amplitudes of the three helicity states in the transversity basis, and $\eta = +1 (-1)$ for $B^0$ or $B^+ (B^0$ or $B^-)$, $\Delta m$ is the $B^0 - \bar{B}^0$ mixing parameter, which is zero for charged $B$ meson decays, and $\tau_B$ is the lifetime of a $B$ meson. $\Gamma$ is the decay rate to each final state. Two $CP$ violation parameters appear in the formula, viz. $\sin \phi_1$ and $\cos \phi_1$. They can take nonzero values for the decay into $CP$ eigenstate $B^0 \to J/\psi K^{*0}(K^*_L \pi^0)$ that occurs through the same quark level diagram as that of the golden mode for $\sin \phi_1$ measurement, $B^0 \to J/\psi K_S^0$.

The determination of the decay amplitudes and $CP$ parameters are performed using an unbinned maximum likelihood method, taking into account the detection efficiency and backgrounds. The probability density function (PDF) for an event is defined as

$$P = f_{\text{sig}}(M_{bc}, \Delta E) \epsilon(\theta_y, \phi_y, \theta_K^*) \times \frac{1}{\Gamma} \frac{d^3 \Gamma(\theta_y, \phi_y, \theta_K^*, \Delta t)}{d \cos \theta_y d \phi_y d \cos \theta_K^* d \Delta t} \times e^{-|\Delta t|/\tau_B} \left[ \sum_i f_i^{\text{cf}}(M_{bc}, \Delta E) \mathcal{A}_i^{\text{cf}}(\theta_y, \phi_y, \theta_K^*) \right.$$  
$$+ f_{\text{nr}}(M_{bc}, \Delta E) \mathcal{A}_{\text{nr}}(\theta_y, \phi_y, \theta_K^*) \left. + \delta(\Delta t) f_{\text{ch}}(M_{bc}, \Delta E) \mathcal{A}_{\text{ch}}(\theta_y, \phi_y, \theta_K^*) \right),$$

(2)

where $f_{\text{sig}}, f_{\text{cf}}, f_{\text{nr}},$ and $f_{\text{ch}}$ are the respective fractions of signal, cross-feeds, nonresonant production, and combinatorial background components as functions of $\Delta E$ and $M_{bc}$, while $\mathcal{A}_i^{\text{cf}}, \mathcal{A}_{\text{nr}}$, and $\mathcal{A}_{\text{ch}}$ are the corresponding angular shape functions, and $\epsilon$ is the detection efficiency function.

Three separate background sources are considered in the fit: cross-feeds that are contaminations from other $K^*$ subdecays, nonresonant production of $K \pi$ including contaminations from higher resonance tails, and combinatorial background. The fraction and angular shape of cross-feeds are estimated from Monte Carlo (MC) calculations. The fraction of nonresonant production is estimated to be 6.8% from a fit to the invariant mass distribution of $K \pi$ pairs in an alternate data sample that is assembled without the $K$ selection criteria. The fit is performed with two Breit-Wigner functions for the $K^*(892)$ and $K^*_2(1430)$ resonances.
TABLE I. Measured decay amplitudes for $B^0$ and $B^+$ decays. The first error is statistical while the second is systematic.

<table>
<thead>
<tr>
<th></th>
<th>$B^0$</th>
<th>$B^0 + B^+$</th>
<th>$B^+$</th>
<th>$B^-$</th>
<th>$B^+ + B^-$</th>
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<tbody>
<tr>
<td>$</td>
<td>A_0</td>
<td>^2$</td>
<td>$0.571 \pm 0.015$</td>
<td>$0.578 \pm 0.016$</td>
<td>$0.574 \pm 0.012 \pm 0.009$</td>
</tr>
<tr>
<td>$</td>
<td>A_1</td>
<td>^2$</td>
<td>$0.216 \pm 0.017$</td>
<td>$0.244 \pm 0.018$</td>
<td>$0.231 \pm 0.012 \pm 0.008$</td>
</tr>
<tr>
<td>$</td>
<td>A_2</td>
<td>^2$</td>
<td>$0.213 \pm 0.017$</td>
<td>$0.178 \pm 0.017$</td>
<td>$0.195 \pm 0.012 \pm 0.008$</td>
</tr>
<tr>
<td>$\arg(A_0)$</td>
<td>$-2.934 \pm 0.134$</td>
<td>$-2.851 \pm 0.114$</td>
<td>$-2.887 \pm 0.090 \pm 0.008$</td>
<td>$-3.070 \pm 0.142$</td>
<td>$-3.129 \pm 0.172$</td>
</tr>
<tr>
<td>$\arg(A_1)$</td>
<td>$2.878 \pm 0.088$</td>
<td>$2.993 \pm 0.089$</td>
<td>$2.938 \pm 0.064 \pm 0.010$</td>
<td>$2.964 \pm 0.099$</td>
<td>$2.988 \pm 0.121$</td>
</tr>
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</table>

and a threshold function describing the nonresonant production, taking into account the contaminations of crossfeeds and combinatorial background. The phase space factor includes a $J/\psi$ recoil correction. The angular shape is obtained from samples in the mass region $1.0 < M(K\pi) < 1.3$ GeV/$c^2$.

The fractions of signal and combinatorial background are estimated from a fit to the samples in the region of $5.2 < M_{bc} < 5.29$ GeV/$c^2$ and $-0.1 < \Delta E < 0.1$ GeV. The angular shape of combinatorial background is obtained from the subsample with $5.2 < M_{bc} < 5.26$ GeV/$c^2$ used in the fit above. The fractions of both signal and backgrounds are parametrized as two-dimensional functions of $M_{bc}$ and $\Delta E$. The background angular shapes are parametrized for each of three angles separately. For each signal mode, the detection efficiency is parametrized as a three-dimensional function whose parameters are obtained by a fit to a high-statistics Monte Carlo sample. The function is almost flat except in the region $\cos\theta_{K^*} \sim 1$, where the pion is slow so that the efficiency is reduced.

The decay amplitudes are determined by fitting the time-integrated angular distribution to three measured angles. Equations (1) and (2) are integrated over $\Delta t$, where terms with $e^{iA_1\tau_{K^*}}$ and $\cos m\Delta t$ become unity while terms with $\sin m\Delta t$ become 0. The value of $\eta$ is determined from the charge of the kaon for $K^+$ decays with a $K^+$ or of the pion with a $K^0_S$. The decay mode $B^0 \rightarrow J/\psi K^0_u(K^0_u\pi^0)$ is not used, since $\eta$ cannot be determined by this prescription. In the fit, the imaginary part of $A_0$ is defined to be zero since the overall phase of the decay amplitudes is arbitrary. The values of the other five parameters, $|A_0|^2$, $|A_1|^2$, $|A_2|^2$, arg($A_0$), and arg($A_1$), are determined in the fit. There is a twofold ambiguity in arg($A_0$) and arg($A_1$) [6]. We take the choice consistent with the $s$-quark helicity conservation hypothesis, which is shown by BABAR to be the physical choice [7]. The normalization condition of the amplitudes, $|A_0|^2 + |A_1|^2 + |A_2|^2 = 1$, is taken into account by adopting the extended likelihood defined as $-\ln L = -\sum_{t=1}^{N_{\exp}} \ln G_t + N_{\exp} \ln (N_{\exp})$, where $G_t$ is the value of the PDF for each event, and $N_{\exp}$ is the number of events used for the fit. $N_{\exp}$ is defined to be $N_{\exp}(|A_0|^2 + |A_1|^2 + |A_2|^2)$ to incorporate the normalization condition. The normalization of the PDF is recalculated whenever the fit parameters change. The two charged $B$ decay modes are combined by defining a single likelihood.

The decay amplitudes determined from the fit are summarized in Table I. The obtained values are consistent between the two flavors in both neutral and charged $B$ decays, indicating no evidence for direct $CP$ violation. The flavor averaged values are consistent with our previous measurement [3] and that by BABAR using $83$ fb$^{-1}$ [7]. Small discrepancies from $\pi$ are observed in arg($A_0$) and arg($A_1$) for both $B^0$ and $B^+$ decays. The difference of these two phases is $0.458 \pm 0.110$ rad in $B^0$ decays, which is shifted from 0 by more than 4$\sigma$. This is interpreted as evidence for the existence of final state interactions.

Fig. 2 shows the projected angular distributions for $B^0 \rightarrow J/\psi K^0_u(K^+\pi^-)$ decays.

Systematic uncertainties in the fit are determined for (1) detection efficiency (MC statistics and effect of polarization), (2) background angular distribution functions, (3) background fractions, (4) slow pion efficiency, and (5) nonresonant decay polarization effect. The effect of the uncertainty in the fraction of the nonresonant production is estimated by varying the value by $\pm 5\sigma$ to take into account the possible contamination of other resonance tails. The uncertainty in its angular shape is determined by comparing with results that assume phase space decay.

The triple product correlations in $B \rightarrow J/\psi K^*\pi$ decays take the form of $\bar{p} \cdot (\hat{v}_1 \times \hat{v}_2)$ in the $B$ rest frame, where $\bar{p}$ is the momentum of $J/\psi$ or $K^*$, and $\hat{v}_1$ ($\hat{v}_2$) is the polarization vector of $J/\psi$ ($K^*$) [8]. They are odd under time reversal ($T$-odd) and their asymmetries are sensitive to direct $CP$ violation even when the strong phase difference is small. The asymmetries are defined using the decay amplitudes as

$$A_T^{(1)} = \frac{\text{Im}(A_A A_{A_1}^*)}{A_0^2 + A_1^2 + A_{A_1}}, \quad A_T^{(2)} = \frac{\text{Im}(A_{A_1} A_A^*)}{A_0^2 + A_1^2 + A_{A_1}^2}. \quad (3)$$
TABLE II. Measured asymmetries in the triple product correlations. The first error is statistical while the second is systematic.

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<tr>
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<th>$B^0$</th>
<th>$B^+$</th>
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<tbody>
<tr>
<td>$A_t^{(1)}$</td>
<td>$0.091 \pm 0.034 \pm 0.007$</td>
<td>$0.062 \pm 0.038 \pm 0.005$</td>
</tr>
<tr>
<td>$A_t^{(2)}$</td>
<td>$-0.098 \pm 0.032 \pm 0.003$</td>
<td>$-0.049 \pm 0.034 \pm 0.002$</td>
</tr>
<tr>
<td>$A_T^{(1)}$</td>
<td>$0.047 \pm 0.031 \pm 0.007$</td>
<td>$0.046 \pm 0.039 \pm 0.005$</td>
</tr>
<tr>
<td>$A_T^{(2)}$</td>
<td>$-0.089 \pm 0.029 \pm 0.003$</td>
<td>$-0.031 \pm 0.039 \pm 0.002$</td>
</tr>
<tr>
<td>$</td>
<td>A_T^{(1)} - A_T^{(2)}</td>
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</tr>
<tr>
<td>$</td>
<td>A_t^{(1)} - A_t^{(2)}</td>
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The corresponding asymmetries for anti-$B$ decays are defined as $\bar{A}_t^{(1)}$ and $\bar{A}_T^{(1)}$. The standard model predicts tiny values for these asymmetries and no difference between $\bar{B}$ and anti-$B$ mesons. Substituting the measured amplitudes in Eq. (1) to the measured angles and $\Delta t$ simultaneously in the $CP$ violation parameters $\sin2\phi_1$ and $\cos2\phi_1$ are determined by fitting the time-dependent angular distribution in Eq. (2) and to the measured angles and $\Delta t$. No difference between $A_t^{(1)}$ and $\bar{A}_t^{(1)}$ nor $A_T^{(2)}$ and $\bar{A}_T^{(2)}$ is observed, which is consistent with the absence of $T$-odd $CP$ violation.

The $CP$ violation parameters $\sin2\phi_1$ and $\cos2\phi_1$ are determined by fitting the time-dependent angular distribution in Eq. (1) to the measured angles and $\Delta t$. The procedures to measure $\Delta t$ and to determine the flavor of the decaying $B^0$ meson are described elsewhere. The flavor tagging procedure gives the flavor $f$ of the tag-side $B$ meson, where $f = +1 (-1)$ for $B^0 (\bar{B}^0)$, and the probability $w$ that this flavor determination is incorrect. The value of $\eta$ is given by $-q(1-2w)$ within the signal angular distribution in Eq. (1). The difference in $w$ between $B^0$ and $\bar{B}^0$ mesons is also considered.

Each term in Eq. (2) is convolved with the appropriate resolution functions separately for the signal, backgrounds having the $B^0$ lifetime (namely, cross-feeds and nonresonant production), and the combinatorial background with a zero-lifetime $\delta$-function shape. The resolution functions are obtained from fits to the $\Delta t$ distributions measured for various data samples. Null $CP$ asymmetry is assumed for the backgrounds. In the fit, the decay amplitudes are fixed at the values obtained for $B^0$ decays. The lifetime and mixing parameter are set to PDG values [10]. From the fit to the data, we obtain

$$\sin2\phi_1 = 0.24 \pm 0.31 \pm 0.05,$$
$$\cos2\phi_1 = 0.56 \pm 0.79 \pm 0.11.$$  

When we fix the value of $\sin2\phi_1$ to the world average value (0.726) [11], the value of $\cos2\phi_1$ becomes $0.87 \pm 0.74 \pm 0.12$. The positive sign of $\cos2\phi_1$ is consistent with the measurement by BABAR [7]; however, we cannot exclude negative values with our current statistical errors. The raw asymmetry in the measured $\Delta t$ distribution between samples with $q = +1$ and $-1$ is shown in Fig. 3 with the projected result of the fit.

Systematic uncertainties in the fit are determined in the same manner as those in the $\bar{B} \rightarrow c\bar{c}s \sin2\phi_1$ measurement [9]. In addition, the uncertainties that come from the angular analysis are estimated similarly as that in the decay amplitude measurement. The possible bias in the fit is checked by applying the same fitting procedure to the sample of $B \rightarrow J/\psi K^{*0}(K^+\pi^-)$ decays. We obtain $\sin2\phi_1 = -0.047 \pm 0.067$ and $\cos2\phi_1 = -0.111 \pm 0.161$, which are consistent with zero as expected.

In summary, a full angular analysis is performed for $B \rightarrow J/\psi K^*$ decays. The complex decay amplitudes are measured by a simultaneous fit to three transversity angles. The measured values are consistent between the two $B$ flavors in both neutral and charged $B$ meson decays, and no direct $CP$-violating effect is observed. The difference of $\arg(A_0)$ and $\arg(A_\perp)$ of $B^0$ decays is shifted from 0 by more than $4\sigma$, which is interpreted as evidence for the existence of final state interactions. The differences between the asymmetries of triple product correlations for $B$ and anti-$B$ mesons are consistent with zero in both neutral and charged $B$ mesons, and no $T$-odd $CP$-violating new physics effect is observed. The time-dependent angular analysis performed for $B^0 \rightarrow J/\psi K^{*0}(K^{*0} \rightarrow K^0_S\pi^0)$ decays gives the $CP$ violation parameters $\sin2\phi_1 = 0.24 \pm 0.31 \pm 0.05$ and $\cos2\phi_1 = 0.56 \pm 0.79 \pm 0.11$. Fixing $\sin2\phi_1$ at the world average value (0.726) gives $\cos2\phi_1 = 0.87 \pm 0.74 \pm 0.12$. The sign of $\cos2\phi_1$ is positive.
although we cannot exclude negative values with current statistical errors.

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