PAPER *Special Section on Spread Spectrum Techniques and Applications*

Online SNR Estimation for Parallel Combinatorial SS Systems in Nakagami Fading Channels

Ken-ichi TAKIZAWA[†a)], ***Student Member***, **Shigenobu SASAKI**[†], **Jie ZHOU**[†], **Shogo MURAMATSU**[†], ***and*** **Hisakazu KIKUCHI**[†], ***Regular Members***

SUMMARY In this paper, an online SNR estimator is proposed for parallel combinatorial SS (PC/SS) systems in Nakagami fading channels. The PC/SS systems are called as partial-code-parallel multicode DS/SS systems, which have the higher-speed data transmission capability comparing with conventional multicode DS/SS systems referred to as all-code-parallel systems. We propose an SNR estimator based on a statistical ratio of correlator outputs at the receiver. The SNR at the correlator output is estimated through a simple polynomial from the statistical ratio. We investigate the SNR estimation accuracy in Nakagami fading channels through computer simulations. In addition, we apply it to the convolutional coded PC/SS systems with iterative demodulation and decoding to evaluate the estimation performance from the viewpoint of error rate. Numerical results show that the PC/SS systems with the proposed SNR estimator have superior estimation performance to conventional DS/SS systems. It is also shown that the bit error rate performance using our SNR estimation method is close to the performance with perfect knowledge of channel state information in Nakagami fading channels and correlated Rayleigh fading channels.

key words: *multi-code DS/SS, online SNR estimation, iterative decoding, Nakagami fading channnles*

1. Introduction

Direct sequence spread spectrum (DS/SS) technique has been successfully implemented in mobile cellular radio systems as code division multiple access (CDMA) [1] and in wireless LAN such as IEEE 802.11 [2] standard. Demand on high-rate data transmission is increasing in various wireless communication systems. To increase the data rate in DS/SS technique, an attractive solution is multi-code DS/SS technique [3]. In this multi-code DS/SS technique, the data rate is directly proportional to the number of transmitting PN codes. So far, we have investigated partial-code-parallel multi-code DS/SS systems, so-called parallel combinatorial SS systems [4] to achieve higher-rate data transmission in DS/SS systems. The PC/SS systems convey information data by transmitting some orthogonal pseudo-noise (PN) codes with polarity chosen out of pre-assigned orthogonal PN codes. They have higher-rate data transmission capability comparing to conventional multi-code DS/SS systems. Variable-rate transmission is also available by changing the number of transmitting orthogonal PN codes [5].

Since transmitted signal encounters severe channel environment such as multipath fading, some appropriate error correction coding (ECC) techniques are usually applied in recent wireless data communication systems. Since the development of turbo codes [6], serially and parallel concatenated coding with a random interleaver and iterative decoding has been recognized as a powerful ECC technique to bring a significant improvement in error rate performance. This concept has been also applied to convolutional coded PC/SS systems, and a large improvement in error rate performance has been carried out by using iterative demodulation and decoding [7]. At the receiver, knowledge of signal-to-noise ratio (SNR) is crucial so that the iterative demodulation and decoding using the MAP or the log-MAP algorithm [8] can work successfully. To avoid insertion of additional training symbols to estimate SNR, online SNR estimation is an attractive topic.

In this paper, we propose an online SNR estimator for the PC/SS systems. The proposed SNR estimator is based on a statistical ratio of the received signals at the PC/SS receiver. For binary PSK transmission, Summers and Wilson have proposed an online SNR estimation scheme and applied it to turbo coded systems [9]. Ramesh et al. have investigated this technique in Nakagami fading channels [10]. Ho et al. have investigated the Summers' SNR estimator in serially concatenated coding [11]. No attempt to apply these estimation techniques into multicode DS/SS systems has been found so far. In the PC/SS systems, a part of correlator outputs has signal and noise components, and the other part of those have only noise component. Then the SNR estimator in [9] could not be applied directly to the SNR estimator in the PC/SS receiver.

The proposed SNR estimation method is based on that described in [9]. This SNR estimation method uses all of correlator outputs, i.e. both the correlator outputs with and without signal component. SNR is estimated by the approximation using the second-, third-, and forth-order polynomials of the statistical ratio of the correlator outputs. We evaluate the accuracy of the SNR estimation and the error rate performance in the convolutional coded PC/SS systems in Nakagami

Manuscript received April 15, 2002.
Manuscript revised June 28, 2002.
Final manuscript received August 13, 2002.

[†]The authors are with the Department of Electrical and Electronic Engineering, Niigata University, Niigata-shi, 950-2181 Japan.
a) E-mail: takizawa@telecom0.eng.niigata-u.ac.jp

channels.

The organization of this paper is as follows. In the next section, a PC/SS system model is described. The online SNR estimator for PC/SS systems in Nakagami fading channels is derived in Sect. 3. Performance evaluation and discussions are given in Sect. 4. Finally, we draw some conclusions in Sect. 5.

2. PC/SS System Model

PC/SS systems convey information data by transmitting r orthogonal PN codes with polarities out of pre-assigned M orthogonal PN codes. In this paper, BPSK is employed in order to give the polarities. The amount of the information data bits per PN period is given by

$$K = \left\lfloor \log_2 \binom{M}{r} \right\rfloor + r \tag{1}$$

where $\lfloor p \rfloor$ denotes the maximum integer that is less than p. This PC/SS system is referred to as the (M, r, K)-PC/SS system. PC/SS systems can achieve higher-rate data transmission than conventional multicode DS/SS systems by choosing the value of r successfully.

A baseband system model for the (M, r, K)-PC/SS transmitter and receiver is shown in Fig. 1. At the transmitter, a set of K-bit binary information data $\mathbf{u}$ is converted into a constant weight code (CWC) codeword $\mathbf{x}$ with length M that is given by

$$\mathbf{x} = \{x_1, \cdots, x_j, \cdots, x_M\} \tag{2}$$

The Hamming weight of a codeword $\mathbf{x}$ is r. Each element of codeword x_j denotes the on-off sign and the polarity for corresponding PN code. The transmitting signal vector $\mathbf{s}$ is represented as

$$\mathbf{s} = \sqrt{E_c}\mathbf{x} \cdot \mathbf{H} \tag{3}$$

where $\mathbf{H}$ is an $M \times l$ matrix whose individual rows corresponds to the pre-assigned orthogonal PN codes with length l. It is expressed by

$$\mathbf{H} = \begin{pmatrix} h_{1,1} & \dots & h_{1,l} \\ \vdots & \ddots & \vdots \\ h_{M,1} & \dots & h_{M,l} \end{pmatrix} \tag{4}$$

It is assumed that the product $\mathbf{H} \cdot \mathbf{H}^t$ becomes the unit matrix $\mathbf{I}$, where $\mathbf{H}^t$ denotes the transpose matrix of $\mathbf{H}$. In Eq. (3), E_c denotes the energy per transmitting PN code. It is related to the energy per information bit E_b by

$$E_c = R_c \cdot K/r \cdot E_b \tag{5}$$

where R_c is the coding rate.

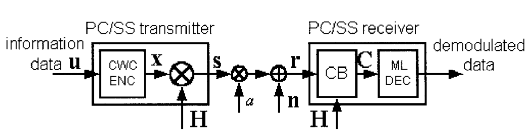


Fig. 1 (M,r,K)-PC/SS system model.

The transmitting signal is corrupted by a fading channel. In this paper, fading amplitude a is assumed to be constant during a PN period. We also assume that the probability density function (pdf) of a follows Nakagami m-distribution, which is given by

$$p(a) = \frac{2m^m a^{2m-1}}{\Gamma(m)} e^{-ma^2} \tag{6}$$

where $\Gamma(.)$ is the Gamma function. Note that the second moment of the fading amplitude $E(a^2)$ is normalized to unity. The Nakagami m-distribution spans the widest range of multipath fading distributions via the parameter m. For instance, it includes the Rayleigh distribution (m=1) as a special case. In the limit as $m \to +\infty$, the Nakagami fading channel converges to a non-fading AWGN channel. When $m \geq 1$, a one-to-one mapping between the parameter m and the Rice factor allows the Nakagami m-distribution to closely approximate the Rice distribution.

At the receiver, the received signal vector $\mathbf{r}$ corrupted by the fading is given by

$$\mathbf{r} = a \cdot \mathbf{s} + \boldsymbol{\eta} \tag{7}$$

where $\boldsymbol{\eta}$ is an l-tuple AWGN sample vector. The two-sided power spectral density of the channel noise process is $N_0/2$ W/Hz. We assume that the fading amplitude a is independent of the AWGN vector $\boldsymbol{\eta}$. In this case, the outputs from the M correlators are

$$\mathbf{C} = \mathbf{r} \cdot \mathbf{H}^t = a\sqrt{E_c}\mathbf{x} + \boldsymbol{\eta} \cdot \mathbf{H}^t \tag{8}$$

$$\mathbf{C} = \{C_1, \dots, C_j, \dots, C_M\} \tag{9}$$

In Eq. (8), replacing the noise term $\boldsymbol{\eta} \cdot \mathbf{H}^t$ by $\mathbf{n}$ yields

$$\mathbf{C} = a\sqrt{E_c}\mathbf{x} + \mathbf{n} \tag{10}$$

where $\mathbf{n}$ is an M-tuple AWGN sample vector with zero mean and variance

$$\sigma^2 = \frac{N_0}{2} \tag{11}$$

The PC/SS detector estimates the PN codes corresponding to the correlator outputs that have the first to the r th largest magnitude as the transmitted PN codes. The received information data is demodulated based on these transmitted PN code estimates.

3. Online SNR Estimation for PC/SS Systems

We want to estimate the average SNR at the correlator outputs,

$$\gamma = \left(\frac{E_c}{N_0}\right) \cdot E(a^2) = \frac{E_c}{N_0} = \frac{E_c}{2\sigma^2} \tag{12}$$

At the output of the bank of correlators described in Eq. (10), the actual transmitted codes and their polarities are unknown. Moreover, a set of transmitted PN codes usually changes symbol by symbol. In this paper, we propose a blind SNR estimation algorithm for the PC/SS systems based on a block observation of the C_j's.

At first, we define a ratio of two statistical computations on the block observation of the C_j's in the form

$$Z \equiv \frac{E\left[\sum_{j=1}^{M} {C_j}^2\right]}{E\left[\sum_{j=1}^{M} |C_j|\right]^2} \tag{13}$$

In the PC/SS systems, r PN codes are transmitted at the symbol of the interest. The set of correlator outputs in Eq. (10) contains r outputs including signal and noise and the other $M-r$ outputs including only noise. From Eq. (10), the correlator output corresponding to the transmitted PN code is given by

$$C_s = \pm a\sqrt{E_c} + n_n \tag{14}$$

where n_n is a Gaussian random variable having zero mean and variance σ^2. On the other hand, the correlator output corresponding to the PN code that have not been transmitted is given by

$$C_n = n_n \tag{15}$$

To obtain SNR estimation from the statistical ratio Z given in Eq. (13), we need to calculate the two expectations in Eq. (13). Noting that the correlator outputs are mutually independent random variables, the expectation in the numerator is rewritten as

$$E\left[\sum_{j=1}^{M} {C_j}^2\right] = r \cdot E[{C_s}^2] + (M-r) \cdot E[{C_n}^2] \tag{16}$$

In Eq. (16), the expectation of ${C_s}^2$ is expressed by

$$\begin{aligned} E\left[{C_s}^2\right] &= E\left[\left(\pm a\sqrt{E_c} + n_n\right)^2\right] \\ &= E_c \cdot E[a^2] \pm 2\sqrt{E_c} \\ &\quad \cdot E[a] \cdot E[n_n] + E[{n_n}^2] \end{aligned} \tag{17}$$

Since $E[n_n]$ is zero and $E[a^2]$ is normalized to unity, we have

$$E\left[{C_s}^2\right] = E_c + \sigma^2 \tag{18}$$

It is straightforward that

$$E\left[{C_n}^2\right] = \sigma^2 \tag{19}$$

Substituting Eqs. (18) and (19) into Eq. (16), we obtain

$$\begin{aligned} E\left[\sum_{j=1}^{M} {C_j}^2\right] &= r \cdot (E_c + \sigma^2) + (M-r) \cdot \sigma^2 \\ &= r \cdot E_c + M \cdot \sigma^2 \end{aligned} \tag{20}$$

The expectation in the denominator is given by

$$E\left[\sum_{j=1}^{M} |C_j|\right] = r \cdot E[|C_s|] + (M-r) \cdot E[|C_n|] \tag{21}$$

It is straightforward that

$$E\left[|C_n|\right] = \sigma\sqrt{\frac{2}{\pi}} \tag{22}$$

To find $E[|C_s|]$, we first calculate the conditional expectation of $|C_s|$ conditioned by the fading amplitude a, and then take its expectation over a. The derivation is summarized in Appendix A. The final results on $E[|C_s|]$ is obtained by

$$\begin{aligned} E\left[|C_s|\right] = &\ \sigma\sqrt{\frac{2}{\pi}}\left(\frac{m}{m+E_c/2\sigma^2}\right)^2 \\ &+ \sqrt{E_c}\frac{\Gamma(m+1/2)}{\sqrt{m}\Gamma(m)}\left(1 - \frac{2}{\pi}I(m)\right) \end{aligned} \tag{23}$$

where

$$\begin{aligned} I(m) = &\ \cos^{-1}\left(\sqrt{\frac{\beta}{1+\beta}}\right) \\ &+ \sum_{k=0}^{m-1} \sum_{l=0}^{2(m-k)-1} \binom{m}{k}\binom{2(m-k)-1}{l} \\ &\cdot \left(\frac{-\beta}{1+\beta}\right)^{m-k} \frac{e^{\theta_1(2l-2(m-k)+1)}-1}{2^{2(m-k)-1}(2l-2(m-k)+1)} \end{aligned} \tag{24}$$

In Eq. (24),

$$\beta = \frac{E_c}{2m\sigma^2}, \text{ and } \theta_1 = \cos^{-1}\sqrt{\frac{1+\beta}{\beta}}$$

Substituting Eqs. (22) and (23) into Eq. (21), we have

$$\begin{aligned} E\left[\sum_{j=1}^{M} |C_j|\right] = &\ r \cdot \sigma\sqrt{\frac{2}{\pi}}\left(\frac{m}{m+E_c/(2\sigma^2)}\right)^2 \\ &+ r \cdot \sqrt{E_c}\frac{\Gamma(m+1/2)}{\sqrt{m}\Gamma(m)}\left(1 - \frac{2}{\pi}I(m)\right) \\ &+ (M-r)\sigma\sqrt{\frac{2}{\pi}} \end{aligned} \tag{25}$$

Therefore, the statistical ratio Z is obtained in Eq. (26) (the bottom of this page) where $\gamma = E_c/2\sigma^2$ as defined in Eq. (12).

Since the PC/SS parameters M and r are known at the receiver, Z is considered as a function of γ and m. The Nakagami parameter m can be computed with reasonable accuracy using the method given in [12]. In this paper, we assume that the perfect knowledge of the Nakagami parameter m is available at the receiver. By this assumption, the statistical ratio Z is viewed as a function of γ.

For $m = 1$, the Nakagami m-distribution becomes the Rayleigh distribution with the pdf $p(a) = 2ae^{-a^2}$. The corresponding Z for the Rayleigh fading can be derived from Eq. (27) by substituting $m = 1$. It is obtained as

$$Z_{rayleigh} = \frac{\pi}{2} \cdot \frac{r \cdot 2\gamma + M}{\left[M + r \cdot \sqrt{\gamma}\left(\frac{\pi}{2} - \cos^{-1}\sqrt{\frac{\gamma}{1+\gamma}}\right)\right]^2} \tag{27}$$

For a given value of Z, the corresponding estimate of γ can be found by Eq. (26). However, it is not easy to determine a closed-form solution for γ from the statistics Z due to the complexity of the function in Eq. (26). This difficulty may be alleviated by utilizing a simple polynomial function to approximate the relationship between Z and γ. The second-order (*quad*), the third-order (*cubic*), and the fourth-order (*bi-quad*) polynomial are found to approximate the relation with γ. These are given by

$$\gamma_{quad} = a_2 Z^2 + a_1 Z + a_0, \tag{28}$$

$$\gamma_{cubic} = b_3 Z^3 + b_2 Z^2 + b_1 Z + b_0, \tag{29}$$

and

$$\gamma_{bi-quad} = c_4 Z^4 + c_3 Z^3 + c_2 Z^2 + c_1 Z + c_0 \tag{30}$$

The coefficients for these polynomials are determined to minimize the square of errors. For example, the coefficients for each polynomial in the (8,2,6)-PC/SS system are given by

$$\begin{cases} a_2 = 1.855604E + 03 \\ a_1 = -8.997686E + 02 \\ a_0 = 1.111973E + 02, \end{cases} \tag{31}$$

$$\begin{cases} b_3 = 9.411177E + 03 \\ b_2 = -7.344657E + 03 \\ b_1 = 1.999708E + 02 \\ b_0 = -1.825527E + 02, \end{cases} \tag{32}$$

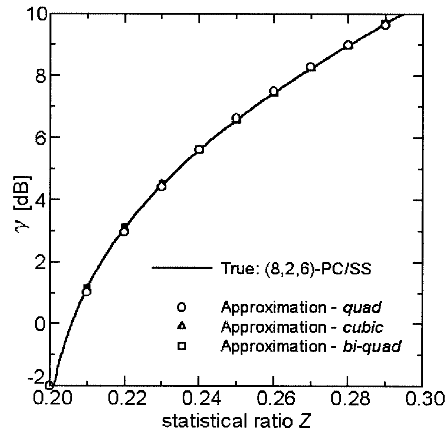


Fig. 2 The relation between SNR and the statistical ratio.

and

$$\begin{cases} c_4 = 4.137810E + 04 \\ c_3 = -4.479876E + 04 \\ c_2 = 1.868680E + 04 \\ c_1 = -3.422994E + 03 \\ c_0 = 2.306108E + 02, \end{cases} \tag{33}$$

In Fig. 2, the relation between γ and the statistics Z is plotted in the (8,2,6)-PC/SS systems. The approximations by using the coefficients given in Eqs. (31), (32) and (33) are also plotted. It is demonstrated that the polynomial approximation is sufficiently accurate. In this paper, we utilize the polynomial approximation to determine an estimate of γ. Another valid method from the implementation viewpoint is a table lookup method, which is described in [13].

The expectations in Eq. (13) could not be obtained by the finite observations of C_j. In order to obtain an estimate for Z, the expectations in Eq. (13) are replaced with the averages over a distinguished block of the received PC/SS symbols, yielding

$$\hat{Z} = \frac{\frac{1}{N_S}\sum_{i=1}^{N_S}\sum_{j=1}^{M} C_{i,j}^2}{\left[\frac{1}{N_S}\sum_{i=1}^{N_S}\sum_{j=1}^{M} |C_{i,j}|\right]^2} \tag{34}$$

where N_S is the number of the PC/SS symbols per block, and $C_{i,j}$ denotes the correlator output corresponding to the j th orthogonal code at the i th symbol. Substituting $\hat{Z}$ for the Z in Eqs. (28), (29), and (30), the SNR estimate $\tilde{\gamma}$ is obtained.

$$Z = \frac{r \cdot 2\gamma + M}{\left[r \cdot \sqrt{\frac{2}{\pi}}\left(\frac{m}{m+\gamma}\right)^m + \sqrt{\frac{2\gamma}{m}}\frac{\Gamma(m+1/2)}{\Gamma(m)}\left(1 - \frac{2I(m)}{\pi}\right) + (M - r) \cdot \sqrt{\frac{2}{\pi}}\right]^2} = f(M, r, m, \gamma) \tag{26}$$

4. Numerical Results and Discussions

4.1 SNR Estimation Accuracy versus the Block Size of Z

We evaluate the SNR estimation accuracy for the block size in the Rayleigh fading channel. The accuracy of the SNR estimates $\tilde{\gamma}$ is evaluated by the expected value $E[\tilde{\gamma}]$ and the standard deviation $SD[\tilde{\gamma}]$. A (8,2,6)-PC/SS system is considered. Every result is determined by 2000 trials. The coefficients of polynomial approximations given by Eqs. (31), (32) and (33) are used for polynomials in Eqs. (28), (29) and (30), respectively. The block size N_S is set to 10 and 100 symbols. No channel encoding is employed.

The results are listed in Table 1. The true SNR γ ranges from 4.77 dB to 12.77 dB by 2 dB, which corresponds to the SNR per information bit, say γ_b, in the range from 0 dB to 8 dB. As the block size N_S increases, the SNR estimate $\tilde{\gamma}$ becomes close to the true SNR γ on average and the standard deviation of the estimates decreases.

The estimation error in our scheme comes from two factors: one is the approximation of Eq. (26) by a polynomial, and the other is finite observation of the correlator output samples to obtain Eq. (34). Estimation error by the latter can be suppressed by increase of the block size N_S. However, estimation error by the former cannot be reduced. Table 2 shows the comparison of true SNR value and approximated SNR values obtained by Eqs. (28), (29) and (30). Z for these polynomials is obtained by Eq. (26) with true SNR value. Thus, the difference between the approximated SNR and true SNR is caused by the polynomial approximation. It is found that the *bi-quad* polynomial can provide the most accurate approximation over a wide range. In the following discussion, we use the *bi-quad* polynomials. In Table 1, it is also confirmed that the SNR estimates $\tilde{\gamma}$ approach the approximated SNR values displayed in Table 2 with increase of the block size N_S.

Table 1 Mean values and standard deviations of the SNR estimates in the (8,2,6)-PC/SS system. The Nakagami parameter m is set to unity

(a) Block size N_S is 10 symbols

True SNR γ [dB]	*quad* $E[\tilde{\gamma}]$ [dB]	*quad* $SD[\tilde{\gamma}]$ [dB]	*cubic* $E[\tilde{\gamma}]$ [dB]	*cubic* $SD[\tilde{\gamma}]$ [dB]	*bi-quad* $E[\tilde{\gamma}]$ [dB]	*bi-quad* $SD[\tilde{\gamma}]$ [dB]
4.77	4.63	1.94	4.65	4.22	4.67	3.71
6.77	6.33	2.26	6.53	2.93	6.53	2.64
8.77	8.67	2.41	8.73	2.29	8.75	2.30
10.77	10.63	2.29	10.60	2.16	10.62	2.17
12.77	12.80	2.19	12.86	2.17	12.90	2.19

(b) Block size N_S is 100 symbols

True SNR γ [dB]	*quad* $E[\tilde{\gamma}]$ [dB]	*quad* $SD[\tilde{\gamma}]$ [dB]	*cubic* $E[\tilde{\gamma}]$ [dB]	*cubic* $SD[\tilde{\gamma}]$ [dB]	*bi-quad* $E[\tilde{\gamma}]$ [dB]	*bi-quad* $SD[\tilde{\gamma}]$ [dB]
4.77	4.33	0.71	4.81	0.83	4.74	0.84
6.77	6.36	0.80	6.77	0.71	6.76	0.73
8.77	8.69	0.77	8.76	0.66	8.80	0.67
10.77	10.91	0.73	10.78	0.70	10.79	0.68
12.77	12.86	0.66	12.76	0.71	12.76	0.70

4.2 SNR Estimation Accuracy on the PC/SS Parameters and the Nakagami Parameter

Accuracy of SNR estimation may change with the PC/SS parameters, say M and r, and the Nakagami parameter m. At first, we compare SNR estimation performance for different PC/SS parameters. (8,r,K)-PC/SS systems are considered. Rayleigh fading channel (i.e. m=1) is assumed. The block size N_S is set to 100 symbols and 1000 symbols. The *bi-quad* polynomials given in Eq. (30) are used for approximation of SNR. The coefficients of the polynomials for each PC/SS system are listed in Table 3. The coefficients for the (8,2,6)-PC/SS system are identical to the set given in Eq. (33).

Table 4 displays the mean values and the standard deviations of the SNR estimates. Since these PC/SS systems have the different SNR per transmitting PN code γ, the estimation accuracy is measured by SNR per information bit γ_b. γ_b is related to γ, which is given by

$$\gamma_b = \frac{E_b}{N_0} = \frac{1/R_c \cdot r/K \cdot E_c}{2\sigma^2} = 1/R_c \cdot r/K \cdot \gamma \qquad (35)$$

The proposed SNR estimation algorithm turns out to show sufficiently accurate on average, when the number of transmitting PN codes r is smaller. In the (8,6,10)- and (8,7,10)-PC/SS systems, more symbols in the observation block is necessary to estimate the SNR with high accuracy than the PC/SS systems with less r.

Next, we examine the influence of the Nakagami parameter m on the accuracy of the SNR estimation in PC/SS systems. The (8,2,6)-PC/SS system is considered. The Nakagami parameter m ranges from 2 to 6 by 2. When $m \geq 1$, the Nakagami-m distribution closely approximates the Rice distribution with the Rice factor K_r by using the following relation

$$K_r = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}} \qquad m \geq 1 \qquad (36)$$

Table 2 Comparison of the SNR values for polynomial approximation. The Nakagami parameter m is set to unity.

SNR γ [dB]	*quad*	*cubic*	*bi-quad*
4.77	4.32	4.85	4.76
6.77	6.37	6.79	6.78
8.77	8.67	8.74	8.78
10.77	10.89	10.75	10.76
12.77	12.89	12.79	12.77

Table 3 The coefficients of the *bi-quad* polynomials for (8,r,K)-PC/SS systems.

	(8,1,4)	(8,2,6)	(8,3,8)	(8,4,10)	(8,5,10)	(8,6,10)	(8,7,10)
c_0	−2.293468E+01	4.137810E+04	5.100613E+02	5.313600E+03	7.444457E+04	−3.554460E+06	1.103422E+07
c_1	2.081392E+02	−4.479876E+04	−8.925671E+03	−9.742067E+04	−1.447717E+06	7.012536E+07	−2.396742E+08
c_2	−7.154755E+02	1.868680E+04	5.838827E+04	6.704237E+05	1.056422E+07	−5.180607E+08	1.952244E+09
c_3	1.471638E+03	−3.422994E+03	−1.704007E+05	−2.055332E+06	−3.429149E+07	1.693281E+09	−7.06749E+09
c_4	−6.701005E+02	2.306108E+02	1.896894E+05	2.373219E+06	4.178950E+07	−2.084007E+09	9.594623E+09

Table 4 Mean values and standard deviations of the SNR estimates in the (8,r,K)-PC/SS systems. The Nakagami parameter m is set to unity.

(a) N_S is 100 symbols

True SNR γ_b [dB]	r=1, K=4		r=2, K=6		r=3, K=8		r=4, K=10		r=5, K=10		r=6, K=10		r=7, K=10	
	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]
0.00	−0.01	0.73	−0.04	0.82	−0.68	1.12	0.02	1.55	0.40	2.70	5.43	8.90	11.42	5.66
2.00	1.98	0.63	2.03	0.75	1.92	1.22	2.06	1.43	2.40	2.81	6.38	9.10	10.13	4.82
4.00	4.00	0.61	4.00	0.68	4.22	0.94	4.07	1.26	4.41	2.65	3.43	8.89	11.36	5.32
6.00	5.98	0.59	6.01	0.68	6.16	0.83	6.11	1.36	6.47	2.49	0.68	8.81	14.07	6.21
8.00	7.98	0.57	8.00	0.68	7.99	0.88	8.17	1.47	8.64	2.67	8.50	8.42	15.89	6.84
10.00	10.00	0.55	9.98	0.72	9.93	1.03	10.21	1.56	10.75	2.67	12.11	7.81	16.98	7.09

(b) N_S is 1000 symbols

True SNR γ_b [dB]	r=1, K=4		r=2, K=6		r=3, K=8		r=4, K=10		r=5, K=10		r=6, K=10		r=7, K=10	
	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]	$E[\tilde{\gamma}_b]$ [dB]	$SD[\tilde{\gamma}_b]$ [dB]
0.00	0.00	0.24	−0.01	0.26	−0.02	0.34	−0.06	0.50	−0.05	1.08	0.71	3.83	2.36	1.07
2.00	2.01	0.20	2.01	0.23	2.03	0.30	2.03	0.44	2.05	0.89	1.76	2.50	3.49	1.58
4.00	4.00	0.19	4.02	0.22	4.00	0.28	4.03	0.42	4.06	0.76	3.05	3.18	4.54	2.26
6.00	6.00	0.19	5.98	0.22	5.98	0.28	6.01	0.43	6.05	0.76	5.95	1.40	7.29	2.44
8.00	8.00	0.18	8.01	0.22	8.00	0.30	8.01	0.47	8.03	0.80	8.26	1.72	9.32	2.83
10.00	10.00	0.18	10.00	0.23	10.01	0.32	10.03	0.49	10.07	0.85	10.40	1.24	10.55	3.04

According to Eq. (36), the Nakagami parameter m=2, 4 and 6 corresponds to 3.8 dB, 8.1 dB and 10.2 dB in the Rice factor K_r, respectively. The *bi-quad* polynomials are used for approximation of γ. The sets of the coefficients are given by

$$\begin{cases} c_4 = 7.966192e+4 \\ c_3 = -8.163101e+4 \\ c_2 = 3.205733e+4 \\ c_1 = -5.554898e+3 \\ c_0 = 3.555381e+2, \end{cases} \tag{37}$$

for m=2,

$$\begin{cases} c_4 = 2.763376E+04 \\ c_3 = -1.966963E+04 \\ c_2 = 5.390161E+03 \\ c_1 = -5.725974E+02 \\ c_0 = 1.297315E+01, \end{cases} \tag{38}$$

for m=4, and

$$\begin{cases} c_4 = 2.783869E+04 \\ c_3 = -1.846459E+04 \\ c_2 = 4.528591E+03 \\ c_1 = -3.689946E+02 \\ c_0 = -3.189972, \end{cases} \tag{39}$$

for m=6, respectively. N_S is set to 100 symbols.

The mean values and the standard deviations for the SNR estimates are displayed in Table 5. It is found that the SNR estimates become more accurate on average when the Nakagami parameter m gets larger. Standard deviations of the SNR estimate become smaller gradually except the low SNR range as the Nakagami parameter m increases.

Table 5 Mean values and standard deviations of the SNR estimates in the (8,2,6)-PC/SS systems. The observation block size N_S is 100 symbols.

True SNR γ [dB]	m=2		m=4		m=6	
	$E[\tilde{\gamma}]$ [dB]	$SD[\tilde{\gamma}]$ [dB]	$E[\tilde{\gamma}]$ [dB]	$SD[\tilde{\gamma}]$ [dB]	$E[\tilde{\gamma}]$ [dB]	$SD[\tilde{\gamma}]$ [dB]
4.77	4.52	0.82	4.78	0.82	4.75	0.83
6.77	6.80	0.73	6.83	0.61	6.81	0.59
8.77	8.95	0.60	8.84	0.51	8.80	0.47
10.77	10.99	0.52	10.82	0.45	10.80	0.43
12.77	12.94	0.51	12.84	0.43	12.81	0.40

4.3 Accuracy Comparison with Conventional Multicode DS/SS Systems Using the Ramesh's Estimator

Conventional multicode DS/SS systems, i.e. all-code-parallel DS/SS systems, can employ the Ramesh's SNR estimator described in [10]. In this subsection, we compare the accuracy of the SNR estimates between conventional DS/SS systems with the Ramesh's estimator and the PC/SS systems with the proposed SNR estimator over Nakagami fading channels. In this comparison, the information bit rate is set to 8 bit/symbol. Conventional multicode DS/SS system with $M = 8$, the (8,3,8)-PC/SS system and the (16,2,8)-PC/SS system are considered. The block size N_S is set to 10 and 100 symbols. The *bi-quad* polynomials are used for approximation. The set of the coefficients are listed in Table 6.

Table 6 The coefficients of the *bi-quad* polynomials for approximation.

(a) multicode DS/SS system with the Ramesh's SNR estimator

	m=1	m=2	m=4
c_0	1.575842E+05	3.152314E+04	1.780328E+04
c_1	−4.418236E+05	−9.402801E+04	−5.505882E+04
c_2	4.646410E+05	1.052111E+05	6.387049E+04
c_3	−2.172077E+05	−5.232718E+04	−3.292702E+04
c_4	3.808127E+04	9.758763E+03	6.363434E+03

(b) (8,3,8)-PC/SS system with the proposed SNR estimator

	m=1	m=2	m=4
c_0	5.100613E+02	8.709800E+02	8.541608E+02
c_1	−8.925671E+03	−1.626220E+04	−1.735304E+04
c_2	5.838827E+04	1.134714E+05	1.304641E+05
c_3	−1.704007E+05	−3.529309E+05	−4.342627E+05
c_4	1.896894E+05	4.168624E+05	5.457976E+05

(c) (16,2,8)-PC/SS system with the proposed SNR estimator

	m=1	m=2	m=4
c_0	−2.285070E+01	−3.448197E+01	−4.646357E+01
c_1	4.136072E+02	7.035925E+02	1.015922E+03
c_2	−2.831423E+03	−5.533741E+03	−8.559805E+03
c_3	1.162452E+04	2.284140E+04	3.580744E+04
c_4	−1.045940E+04	−2.500379E+04	−4.374314E+03

The mean values and the standard deviations for SNR estimates are listed in Table 7. It found that the PC/SS systems with the proposed SNR estimator provide more accurate SNR estimation than the multicode DS/SS systems with the Ramesh's estimator. This advantage becomes more remarkable when the block size N_S is small. Standard deviations of SNR estimates decrease as the Nakagami parameter m increases.

At the low SNR range, the SNR estimation in conventional multicode DS/SS system does not work at all. The reason of these results is as follows. Figure 3 displays the true SNR values versus the statistical ratio Z for the multicode DS/SS system with the Ramesh's estimator. The SNR values approximated by the corresponding *bi-quad* polynomials versus the statistical ratio Z are also plotted. At low SNR range, it is found that approximated SNR $\tilde{\gamma}$ is apart from the true SNR γ seriously. The best approximation is demonstrated when the Nakagami parameter m is unity, but the worst mismatch is shown in Table 7. If the Nak-

Table 7 The mean values and the standard deviations of the SNR estimates in the multicode DS/SS system with the Ramesh's estimator and the PC/SS systems with the proposed estimator.

(a) $m = 1$

True SNR γ_b [dB]	multicode DS/SS				(8,3,8)-PC/SS				(16,2,8)-PC/SS			
	N_S=10		N_S=100		N_S=10		N_S=100		N_S=10		N_S=100	
	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$
0.00	11.78	5.61	2.31	1.93	0.04	3.31	-0.05	1.07	-0.22	2.36	0.01	0.63
2.00	11.83	5.39	2.71	1.62	2.17	3.17	2.06	0.95	1.90	1.90	2.00	0.56
4.00	12.45	5.60	4.62	1.93	4.23	3.01	4.08	0.88	3.92	1.78	4.00	0.54
6.00	15.04	6.32	7.26	2.51	6.28	2.86	6.03	0.88	5.94	1.71	5.99	0.53
8.00	16.74	6.90	9.54	3.00	8.36	2.91	8.05	0.94	7.94	1.68	8.01	0.54
10.00	18.15	6.98	11.57	3.29	10.32	2.98	10.03	1.01	9.75	1.61	9.96	0.51

(b) $m = 2$

True SNR γ_b [dB]	multicode DS/SS				(8,3,8)-PC/SS				(16,2,8)-PC/SS			
	N_S=10		N_S=100		N_S=10		N_S=100		N_S=10		N_S=100	
	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$
0.00	6.64	3.85	1.36	1.13	0.10	5.72	0.10	1.15	-0.05	1.94	0.04	0.54
2.00	4.89	2.63	2.19	0.90	2.35	3.78	2.20	0.86	1.98	1.49	2.07	0.45
4.00	5.98	5.60	2.73	4.13	4.26	2.64	4.23	0.73	4.02	1.32	4.06	0.41
6.00	7.99	3.18	5.91	1.02	6.40	2.31	6.27	0.71	6.05	1.27	6.07	0.40
8.00	10.52	3.70	7.92	1.36	8.39	2.21	8.33	0.73	8.08	1.22	8.08	0.39
10.00	11.83	3.81	9.61	1.46	10.50	2.22	10.37	0.74	10.01	1.14	10.04	0.36

(c) $m = 4$

True SNR γ_b [dB]	multicode DS/SS				(8,3,8)-PC/SS				(16,2,8)-PC/SS			
	N_S=10		N_S=100		N_S=10		N_S=100		N_S=10		N_S=100	
	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$	$E[\tilde{\gamma}_b]$	$SD[\tilde{\gamma}_b]$
0.00	5.01	3.34	0.54	0.95	0.00	6.52	0.03	1.36	-0.07	1.72	0.03	0.49
2.00	3.02	1.75	2.06	0.71	2.14	4.78	2.09	0.91	1.99	1.34	2.03	0.39
4.00	4.57	1.65	4.05	0.43	4.20	2.50	4.11	0.65	4.07	1.15	4.01	0.34
6.00	6.82	1.95	5.85	0.62	6.19	1.93	6.09	0.58	6.02	1.03	6.02	0.32
8.00	8.93	2.21	8.03	0.78	8.20	1.73	8.12	0.54	8.06	1.00	8.03	0.31
10.00	10.62	2.24	9.79	0.79	10.17	1.58	10.14	0.50	9.93	0.91	9.97	0.28

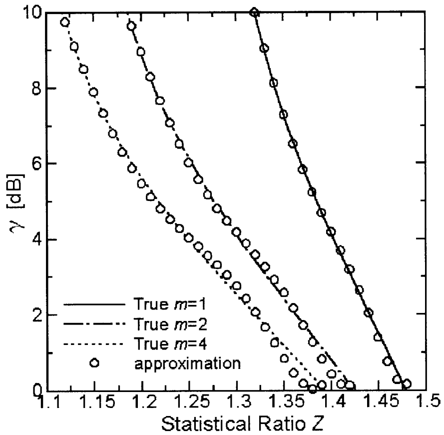


Fig. 3 SNR versus statistical ratio at the correlator output in the multicode DS/SS system with the Ramesh's estimator.

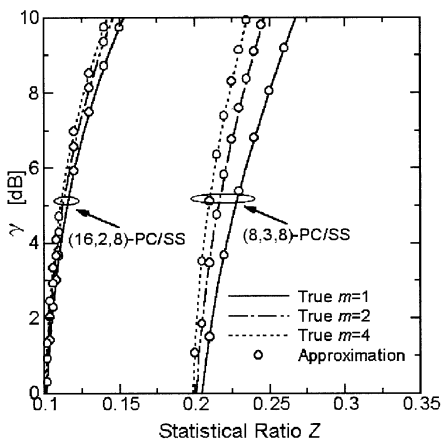


Fig. 4 SNR versus statistical ratio at the correlator output in the PC/SS systems with the proposed estimator.

agami parameter m is small, the statistical ratio Z will fluctuate widely because of large fluctuation in the correlator outputs. This leads the larger mismatch in the SNR estimation since $\tilde{\gamma}$ varies large with small change in $\hat{Z}$ where the Nakagami parameter m is small.

In contrast, the proposed SNR estimation in the PC/SS systems maintains sufficient accuracy at the low SNR region. In Fig. 4, true SNR values versus statistical ratio Z in the PC/SS systems with proposed estimator are plotted. Approximated SNR by the *bi-quad* polynomial is almost the same as true SNR. In addition, in the PC/SS systems, SNR per transmitted PN codes is larger than that in multicode DS/SS systems when energy per information bit is equal. These factors yield good SNR estimation in the PC/SS systems. It also found that the best accuracy is achieved in the (16,2,8)-PC/SS system in each Nakagami fading channel. The superiority over the (8,3,8)-PC/SS system diminishes if the Nakagami parameter m becomes larger.

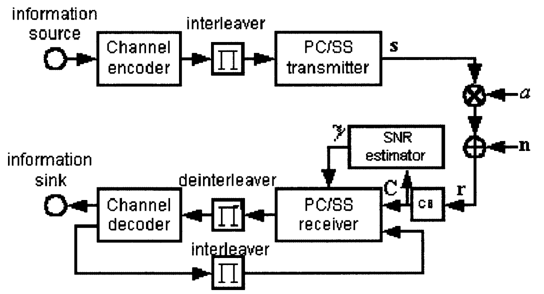


Fig. 5 System model of convolutional coded PC/SS systems with iterative demodulation and decoding.

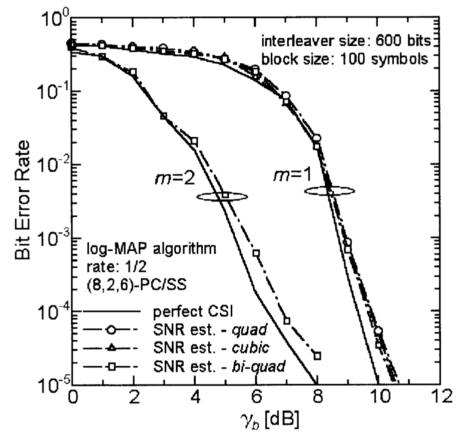


Fig. 6 Comparison of the bit error rate performance between using perfect CSI and SNR estimates on i.i.d. Nakagami fading channels.

4.4 BER Performance Comparison in Convolutional Coded PC/SS Systems with Iterative Demodulation and Decoding

The proposed online SNR estimator is applied to the iterative demodulation and decoding in convolutional coded PC/SS systems [7]. I.i.d. Nakagami fading channels and correlated Rayleigh fading channels are considered in these simulations. The system model is shown in Fig. 5. In this paper, a rate-1/2 convolutional coded (8,2,6)-PC/SS system is considered. The convolutional encoder is the nonsystematic feed-forward encoder with generator [5], [7] in an octal form. A random interleaver is employed between the convolutional encoder and the PC/SS transmitter. The log-MAP algorithm [8] is used in the PC/SS decoder and the convolutional decoder. The number of decoding iterations is set to five. When the estimator is employed, the average SNR estimate is computed from the C_j's over the symbols in one interleaver size. This average SNR is provided for the PC/SS receiver as the channel state information.

Figure 6 shows the bit error rate (BER) performance in the coded PC/SS system over i.i.d. Nakagami fading channels. The SNR per bit E_b/N_0 is equivalent to γ_b. We evaluate the error rate performance in the case of $m = 1$ and 2. The interleaver size is set to 600 bits. Thus, the block size N_S for SNR estimation is 100 symbols. In the case of $m = 1$, the set of coefficients in Eqs. (31), (32) and (33) are applied. For reference, BER performance in the coded PC/SS system with perfect channel state information (perfect CSI) on the fade amplitude and the SNR is displayed. When perfect CSI is available, the required E_b/N_0 to maintain the 10^{-4} BER is about 9.3 dB. On the other hand, the required E_b/N_0 is about 9.7 dB when the proposed SNR estimators are applied. When the *bi-quad* fit is applied in the SNR estimator, BER performance becomes the closest to that in perfect CSI case. However, the BER degradation caused by the difference of the order of polynomial is hardly observed.

In the case of $m = 2$, the set of coefficients in Eq. (37) is employed. In this case, the required E_b/N_0 to maintain the 10^{-4} BER is about 6.2 dB and 6.4 dB when the PC/SS receiver can obtain the perfect CSI and the receiver employs the proposed SNR estimator, respectively. It turns out that the proposed SNR estimator with the *quad*, the *cubic*, and the *bi-quad* fits show almost the same error rate performance. On both channels, the error rate performance in the coded PC/SS system using the SNR estimator is close to that with perfect CSI in both channels, within 0.5 dB.

The BER performances in the coded PC/SS system over correlated Rayleigh fading channels are plotted in Fig. 7. The correlated Rayleigh fading samples are simulated by using the Jakes' model [14]. The correlation in the fading process is characterized by the Bessel function of zeroth order and first kind, $J_0(2\pi f_d T_s)$, where f_d and T_s are the Doppler frequency and the symbol duration, respectively. In the computer simulations, the normalized Doppler frequency $f_d T_s$ are set to 0.1 and 0.01 to evaluate the effect of varying the correlation in the fading process. The size of interleaver between the encoder and the transmitter is set to 1200 bits. This means the block size N_S using at the SNR estimator is 200 symbols. No channel interleaver to destroy the fading correlation is employed. It is observed that the iterative demodulation and decoding scheme yields better performance as the normalized Doppler frequency $f_d T_s$ becomes larger. It is shown that the bit error rate performance with our SNR estimator is quite close to that with perfect CSI. The difference of the required E_b/N_0 to maintain the 10^{-4} BER between both systems is within 0.5 dB.

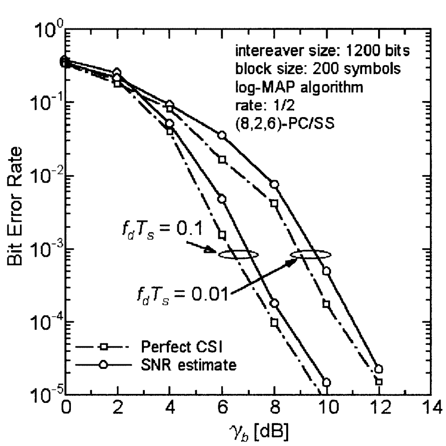


Fig. 7 Comparison of the bit error rate performance between using perfect CSI and SNR estimates on correlated Rayleigh fading channels.

5. Conclusions

In this paper, an online SNR estimator was proposed for PC/SS systems in Nakagami fading channels. This estimator estimates the SNR at the correlator outputs based on a statistical ratio of the correlator outputs. A simple polynomial was introduced to obtain the SNR estimates from this statistical ratio. Accuracy of the SNR estimation was investigated through computer simulations in various fading channels and PC/SS systems. When the Nakagami parameter m becomes large, the average SNR estimate gets closer to the true SNR. The use of more transmitting PN codes leads to the larger standard deviation of the SNR estimation.

The proposed estimator was compared with the Ramesh's SNR estimator described in [10] that would be available in conventional multicode DS/SS systems. If the data rate is equal, the proposed estimator in PC/SS systems is superior to that of the Ramesh's estimator in conventional multicode DS/SS systems, especially at lower SNR region.

We have evaluated the performance of the proposed estimator by BER evaluation of convolutional coded PC/SS systems with iterative demodulation and decoding. I.i.d. Nakagami fading channels and correlated Rayleigh fading channels were assumed. Simulation results have shown that the BER performance with the proposed SNR estimator was close to that with perfect CSI in both channels. The difference of the required SNR to maintain the 10^{-4} BER is within 0.5 dB.

Acknowledgement

This work is supported in part by the Telecommunication Advancement Foundation of Japan and by Unisia Jecs Corporation.

References

[1] R. Kohno, R. Meidan, and L.B. Milstein, "Spread-spectrum access methods for wireless communications," IEEE Commun. Mag., pp.58–67, Jan. 1995.
[2] R. van Nee, G. Awater, M. Morikura, H. Takanashi, M. Webster, and K.W. Halford, "New high-rate wireless LAN stan-dards," IEEE Commun. Mag., pp.82–88, Dec. 1999.

[3] F. Adachi, K. Ohno, A. Higashi, and Y. Okumura, "Coherent multicode DS-CDMA mobile radio access," IEICE Trans. Commun., vol.E79-B, no.9, pp.1316–1325, Sept. 1996.
[4] S. Sasaki, J. Zhu, and G. Marubayashi, "Performance of parallel combinatory spread spectrum communication systems," Proc. IEEE PIMRC'91, London, pp.204–208, Sept. 1991.
[5] N. Guo and L.B. Milstein, "On rate-variable multidimensional DS/SSMA with sequence sharing," IEEE J. Sel. Areas Commun., vol.17, no.5, pp.912–917, May 1999.
[6] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon limit error correcting coding and decoding: Turbo-codes," Proc. IEEE ICC'93, pp.1064–1070, May 1993.
[7] K. Takizawa, S. Sasaki, S. Muramatsu, and H. Kikuchi, "Iterative demodulation and decoding for parallel combinatorial SS systems," IEICE Trans. Fundamentals, vol.E84-A, no.12, pp.2991–2999, Dec. 2001.
[8] B. Vucetic and J. Yuan, Turbo codes principles and applications, Kluwer Academic Publishers, 2000.
[9] T.A. Summers and S.G. Wilson, "SNR mismatch and online estimation in turbo decoding," IEEE Trans. Commun., vol.46, no.4, pp.421–423, April 1998.
[10] A. Ramesh, A. Chokalingam, and L.B. Milstein, "SNR estimation in generalized fading channels and its application to turbo decoding," Proc. IEEE ICC'01, vol.4, pp.1094–1098, Helsinki, Finland, June 2001.
[11] M.S.C. Ho and S.S. Pietrobon, "A variance mismatch study for serial concatenated turbo codes," Proc. ISTC'00, Brest, pp.483–485, France, Sept. 2000.
[12] A. Abidi and M. Kaveh, "Performance comparison of three different estimator for the Nakagami m parameter using Monte Carlo simulations," IEEE Commun. Lett., vol.4, no.4, pp.119–121, April 2000.
[13] S.S. Pietrobon, "Implementation and performance of a turbo/MAP decoder," Int. J. of Satellite Commun., vol.16, pp.23–46, Jan./Feb. 1998.
[14] W.C. Jakes, Microwave Mobile Communications, IEEE Press, 1974.
[15] M.K. Simon and M.S. Alouini, Digital Communication over Fading Channels: A Unified Approach to Performance Approach, John Wiley & Sons, 2000.

Appendix: Derivation of $E[|C_s|]$

The detail derivation of $E[|C_s|]$ in Eq. (20) is given in [10]. We summarize the derivation for reference. Consider the random variable C_s as

$$C_s = a \cdot X_n + n_n \qquad (\text{A}\cdot 1)$$

where X_n is a binary random variable taking values $\pm\sqrt{E_c}$ with equal probability. X_n is independent of a and n_n. In (A·1), C_s depends on the random variables X_n, a and n_n. To derive the expected value of $|C_s|$, we first average $|C_s|$ over random variable X_n, then average over n_n conditioned on a, and finally average over a. First, averaging $|C_s|$ over X_n, we get

$$E\left[|C_s| \;\middle|\; n_n, a\right] = \frac{1}{2}E\left[|-a\sqrt{E_c}+n_n|\right] + \frac{1}{2}E\left[|a\sqrt{E_c}+n_n|\right] \qquad (\text{A}\cdot 2)$$

Denoting $A = |-a\sqrt{E_c}+n_n|$ and $B = |a\sqrt{E_c}+n_n|$ in the above equation, and averaging over n_n, conditioned on a, we have

$$E[A|a] = \int_{-\infty}^{\infty} A \cdot p(n_n)dn_n = \sigma\sqrt{\frac{2}{\pi}}e^{-\frac{a^2E_c}{2\sigma^2}} + a\sqrt{E_c} - 2a\sqrt{E_c}Q\left(\frac{a\sqrt{E_c}}{\sigma}\right) \qquad (\text{A}\cdot 3)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_{u=x}^{\infty} e^{-\frac{u^2}{2}}du \qquad (\text{A}\cdot 4)$$

Similarly,

$$E[B|a] = \sigma\sqrt{\frac{2}{\pi}}e^{-\frac{a^2E_c}{2\sigma^2}} - a\sqrt{E_c} + 2a\sqrt{E_c}Q\left(\frac{-a\sqrt{E_c}}{\sigma}\right) \qquad (\text{A}\cdot 5)$$

By noting that $Q(-x) = 1 - Q(x)$ and substituting Eqs. (A·3) and (A·5) into Eq. (A·2), we get

$$E[|C_s| \mid a] = \sigma\sqrt{\frac{2}{\pi}}e^{-\frac{a^2E_c}{2\sigma^2}} + a\sqrt{E_c} - 2a\sqrt{E_c}Q\left(\frac{a\sqrt{E_c}}{\sigma}\right) \qquad (\text{A}\cdot 6)$$

Now, we take the expectation over a to get $E[|C_s|]$. The expectation of $e^{\frac{-a^2E_c}{2\sigma^2}}$ in the first term in Eq. (A·6) is obtained as

$$E\left[e^{-\frac{a^2E_c}{2\sigma^2}}\right] = \frac{m^m}{\left(m+\frac{E_c}{2\sigma^2}\right)^2} \qquad (\text{A}\cdot 7)$$

The expectation of the second term in Eq. (A·6) is obtained as

$$E\left[a\sqrt{E_c}\right] = \sqrt{E_c}\frac{\Gamma(m+1/2)}{\sqrt{m}\Gamma(m)} \qquad (\text{A}\cdot 8)$$

To derive $E\left[aQ\left(\frac{a\sqrt{E_c}}{\sigma}\right)\right]$ for the third term in Eq. (A·6), we use the alternative for $Q(x)$ given by [15]

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{x^2}{2\sin^2\phi}}d\phi \;;\, x \geq 0 \qquad (\text{A}\cdot 9)$$

This gives

$$E\left[aQ\left(\frac{a\sqrt{E_c}}{\sigma}\right)\right] = \frac{\Gamma(m+1/2)}{\pi\Gamma(m)\sqrt{m}}\int_0^{\pi/2}\frac{\sin^{2m+1}\phi}{(\sin^2\phi+\beta)^{m+\frac{1}{2}}}d\phi \qquad (\text{A}\cdot 10)$$

where $\beta = \frac{E_c}{2m\sigma^2}$. Let us define

$$I(m) = \int_0^{\pi/2}\frac{\sin^{2m+1}\phi d\phi}{(\sin^2\phi+\beta)^{m+\frac{1}{2}}} \qquad (\text{A}\cdot 11)$$

Substituting $\sin^2\phi + \beta = t^2$, Eq. (A·11) becomes

$$I(m) = \int_{\sqrt{\beta}}^{\sqrt{1+\beta}} \frac{(t^2-\beta)^m}{t^{2m}\sqrt{1+\beta-t^2}} dt$$
$$= \sum_{k=0}^{m-1} \binom{m}{k} (-\beta)^{m-k} \int_{\sqrt{\beta}}^{\sqrt{1+\beta}} \frac{dt}{t^{2(m-k)}\sqrt{1+\beta-t^2}}$$
$$+ \cos^{-1}\left(\sqrt{\frac{\beta}{1+\beta}}\right) \qquad \text{(A· 12)}$$

In order to further simplify Eq. (A·12), let us define $I_1(m)$ as

$$I_1(m) = \int_{\sqrt{\beta}}^{\sqrt{1+\beta}} \frac{1}{t^{2p}\sqrt{1+\beta-t^2}} dt \ ; p = m-k, \ p > 0 \qquad \text{(A· 13)}$$

Substituting $t = 1/u$ in Eq. (A·13), we get

$$I_1(m) = \frac{1}{\sqrt{1+\beta}} \int_{u=1/\sqrt{1+\beta}}^{1/\sqrt{\beta}} \frac{u^{2p-1}}{t^{2p}\sqrt{u^2 - \frac{1}{1+\beta}}} du \qquad \text{(A· 14)}$$

Substituting $u = \frac{\cosh\theta}{\sqrt{1+\beta}}$ in Eq. (A·14), Eq. (A·14) becomes

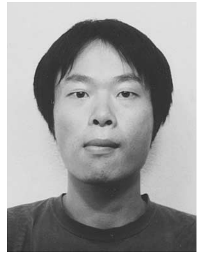
$$I_1(m) = \int_{\theta=0}^{\theta_1} \cosh^{2p-1}\theta d\theta \ ; \theta_1 = \cosh^{-1}\sqrt{\frac{1+\beta}{\beta}}$$
$$= \frac{1}{(1+\beta)^p} \frac{1}{2^{2p-1}} \sum_{l=0}^{2p-1} \binom{2p-1}{l} \frac{e^{\theta_1(2l-2p+1)}-1}{2l-2p+1} \qquad \text{(A· 15)}$$

Plugging the above expression for $I_1(m)$ in Eq. (A·12), we obtain $I(m)$ as

$$I(m) = \cos^{-1}\left(\sqrt{\frac{\beta}{1+\beta}}\right)$$
$$+ \sum_{k=0}^{m-1} \sum_{l=0}^{2(m-k)-1} \binom{m}{k} \binom{2(m-k)-1}{l}$$
$$\cdot \left(\frac{-\beta}{1+\beta}\right)^{m-k} \frac{e^{\theta_1(2l-2(m-k)+1)}-1}{2^{2(m-k)-1}(2l-2(m-k)+1)} \qquad \text{(A· 16)}$$

Finally, combining Eqs. (A·16), (A·10), (A·8), (A·7) and (A·6), we have the expression for $E[|C_s|]$ as

$$E[|C_s|] = \sigma\sqrt{\frac{2}{\pi}} \left(\frac{m}{m+E_c/2\sigma^2}\right)^2$$
$$+ \sqrt{E_c}\frac{\Gamma(m+1/2)}{\sqrt{m}\Gamma(m)} \left(1 - \frac{2}{\pi}I(m)\right) \qquad \text{(A· 17)}$$

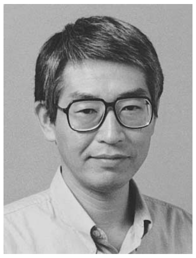


Ken-ichi Takizawa was born in Niigata, Japan, in 1975. He received B.E. and M.E. degrees from Niigata University, Niigata, in 1998 and 2000, respectively. He is currently a Ph.D. candidate at Graduate School of Science and Technology, Niigata University. His research interests include digital wireless communications and channel coding. He is a student member of IEEE.

Shigenobu Sasaki received B.E., M.E. and Ph.D. degrees from Nagaoka University of Technology, Nagaoka, Japan, in 1987, 1989 and 1993, respectively. Since 1992, he has been with Niigata University, where he is an Associate Professor in the Department of Electrical and Electronic Engineering. From 1999 to 2000, he was a visiting scholar at the Department of Electrical and Computer Engineering, University of California, San Diego. His research interests are in the area of digital communications with special emphasis on spread spectrum communication systems and wireless communications. He is a member of IEEE and SITA.

Jie Zhou was born in Sichuan, China, on February 25, 1964. He received B.E. and M.E. degrees from Nanjing University of Posts and Telecommunications, China in 1985 and 1990, respectively. Received Dr.Eng. degree in Dept. of Computer Science, Gunma University, Japan in March 2001. After which he joined Chongqing University of Posts and Telecommunications since 1990, where he became an engineer in 1992, an associate professor in 1998. Since April 2001, he has been with Dept. of Electrical and Electronic Engineering, Niigata University, Japan where he is currently a Research Associate. Performed several communication projects about measuring instruments, the industrial automatic control system and presented some papers both abroad and at home. His research interests lie in the areas of DATA transmission, wireless ATM and radiowave propagation in mobile communications.

Shogo Muramatsu was born in Tokyo, Japan in 1970. He received B.E., M.E. and Dr.Eng. degrees in electrical engineering from Tokyo Metropolitan University in 1993, 1995 and 1998, respectively. In 1997, He joined Tokyo Metropolitan University. In 1999, he joined Niigata University, where he is currently an Associate Professor at the Department of Electrical and Electronic Engineering. His research interests are in digital signal processing, multirate systems, image processing and VLSI architecture. He is a Member of IEEE.



Hisakazu Kikuchi was born in Niigata, Japan, in 1952. He received B.E. and M.E. degrees from Niigata University, Niigata, in 1974 and 1976, respectively, and Dr.Eng. degree in electrical and electronic engineering from Tokyo Institute of Technology, Tokyo, in 1988. From 1976 to 1979 he worked at Information Systems Laboratories, Fujitsu Ltd. Since 1979 he has been with Niigata University, where he is a Professor in electrical engineering. During a year of 1992 to 1993, he was a visiting scientist at the Electrical Engineering Department, University of California, Los Angeles. His research interests include digital signal processing, image processing, wavelets, and mobile telecommunications. He is a member of IEEE, Institute of Image Information and Television Engineers, and Japan SIAM.