PAPER

A New Image Coding Technique with Low Entropy Using a Flexible Zerotree

Sanghyun JOO†, Hisakazu KIKUCHI†, Shigenobu SASAKI†, *and* Jaeho SHIN††, *Members*

SUMMARY A zerotree image-coding scheme is introduced that effectively exploits the inter-scale self-similarities found in the octave decomposition by a wavelet transform. A zerotree is useful for efficiently coding wavelet coefficients; its efficiency was proved by Shapiro's EZW. In the EZW coder, wavelet coefficients are symbolized, then entropy-coded for further compression. In this paper, we analyze the symbols produced by the EZW coder and discuss the entropy for a symbol. We modify the procedure used for symbol-stream generation to produce lower entropy. First, we modify the fixed relation between a parent and children used in the EZW coder to raise the probability that a significant parent has significant children. The modified relation is flexibly modified again based on the observation that a significant coefficient is more likely to have significant coefficients in its neighborhood. The three relations are compared in terms of the number of symbols they produce.

key words: *image compression, wavelet transform, zerotree coding, entropy coding*

1. Introduction

To efficiently transmit digital images through communication channels with limited capacities, the data is compressed. The image data has to be compressed in both lossy and lossless ways. In most cases, the original image is transformed so as to remove some of the correlation among pixels; then only the few coefficients with concentrated energies need to be processed [1]–[4].

The image transformation is based on the Fourier transform (FT), which uses periodic harmonics as basis functions. However, most natural signals and nonstationary signals need to be analyzed simultaneously in both time and frequency, and the FT does not give an analysis in the time domain. Although the short term Fourier transform (STFT) overcomes this limitation by using a window function, the same-sized window must be used for all locations in the time-frequency plane.

A wavelet transform can be considered to be a generalized STFT; it analyzes the signals while dilating and translating a prototype wavelet. That is, the wavelet adjusts the window size to fit the signal variances [5]–[10]. The goal of the transformation is to concentrate the energies to a few coefficients, and a wavelet packet is the way to represent an arbitrary signal with the best basis function. However, finding the best basis is still quite complex and takes much time, although efforts have been made to simplify the task [11], [12]. In contrast, an octave-band wavelet gives a comparatively good energy compaction when we consider its simplicity and reduced processing time. Moreover, when the signal is highly correlated, the decomposition by the wavelet packet is very similar to that by the octave-band wavelet.

Among the wavelet-based codings [13]–[23], the dependencies among the scales are well exploited in embedded zerotree coding of wavelet coefficients (EZW) [13], set partitioning in hierarchical trees (SPIHT) [14], and space-frequency quantization (SFQ) [15]. That is, one coefficient in a given scale is related to four coefficients at the same spatial location at the next finer scale in terms of a parent-child relation. This relation is applied to all coefficients except the ones in DC scale.

The EZW coder by Shapiro was the first to apply an embedded zerotree using a wavelet. The algorithm of this coder is based on three concepts: 1) prediction of the absence of significant coefficients across scales by exploiting the self-similarity inherent in images, 2) successive approximation for decoded coefficients, and 3) adaptive arithmetic coding of the produced symbols. Next came the SPIHT coder that has improved performance and faster processing. One of the advantages of the algorithm used in the EZW and SPIHT coders is that the encoder and decoder can be stopped at any point, and the decoder can reconstruct an approximated image from the information received so far. This capability is useful when using constrained communication channels. The more recent, SFQ coder outperforms the EZW and SPIHT coders. There are two versions: one uses the octave-band wavelet and the other uses the wavelet packet. They improve coding performance by pruning branches from the trees using a rate-distortion and scalar-quantizing the coefficients at the remaining nodes.

The three coding schemes described above have two common procedures: symbol-generation (model transformation) and entropy-coding of the symbol stream. The symbol stream is first produced for the purpose of representation; then the symbols are entropy-coded. In this paper, we introduce a new zerotree scheme that lead

Manuscript received December 1, 1997.

Manuscript revised July 1, 1998.

†The authors are with the Department of Electrical Engineering, Niigata University, Niigata-shi, 950–2181 Japan.

††The author is with the Department of Electronic Engineering, Dongguk University, 3-26 Phildong, Joonggu, Seoul 100–715, Korea.

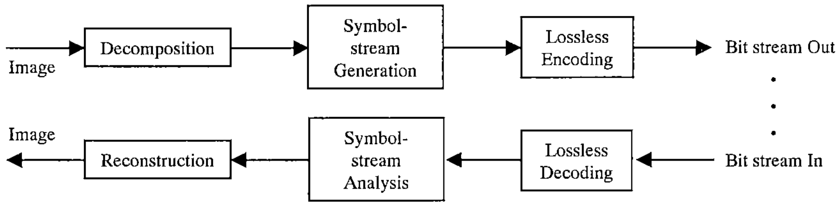


Fig. 1 Encoding and decoding with EZW algorithm.

to lower entropy and thus more compression [24]. Because the entropy is determined by the probabilities of the produced symbols, we have modified the symbol-generation process by using flexible treeing. The tree is flexibly designed so as to lead low entropy. In the EZW scheme, a node on a tree branches out into four nodes; this relation is referred to as a fixed relation in the sense that the relation does not change. In contrast, our proposed relation can be considered a "flexible zerotree" in the sense that a node on a tree branches into basic four nodes, then flexibly extends its branches to neighboring nodes. This flexible-tree approach enables branches to be extended efficiently.

This paper is organized as follows: the EZW scheme is reviewed briefly in Sect. 2. In Sect. 3, we introduce two new parent-child relations with low entropy. They are applied to two images and the numbers of produced symbols are compared with those from the fixed relation in Sect. 4. The results are discussed based on a curve drawn from an equation for the entropy-coded size. Also, performances are given for PSNR versus bit rate. We conclude with a summary in Sect. 5.

2. Embedded Zerotree Wavelet Coding

Shapiro [13] developed an efficient algorithm that transmits wavelet coefficients. In this algorithm, zerotrees are combined with bit-plane coding so that they can efficiently represent many insignificant coefficients. The wavelet coefficients have some dependencies across subbands, and these dependencies are well exploited in a quadtree structure. The compression has three steps (Fig. 1): (1) wavelet decomposition, (2) symbol-stream generation, (3) entropy coding. To help describe this tree-based coding, we define the following terms.

- Parent: a coefficient in a band that has four coefficients at the same spatial location in the next finer scale with a similar orientation.
- Child: one of the four coefficients in the definition of parent.
- Ancestors: for a given child, the set of coefficients at all coarser scales with similar orientations corresponding to the same spatial locations.
- Descendants: for a given parent, the set of coefficients at all finer scales with similar orientations corresponding to the same spatial locations

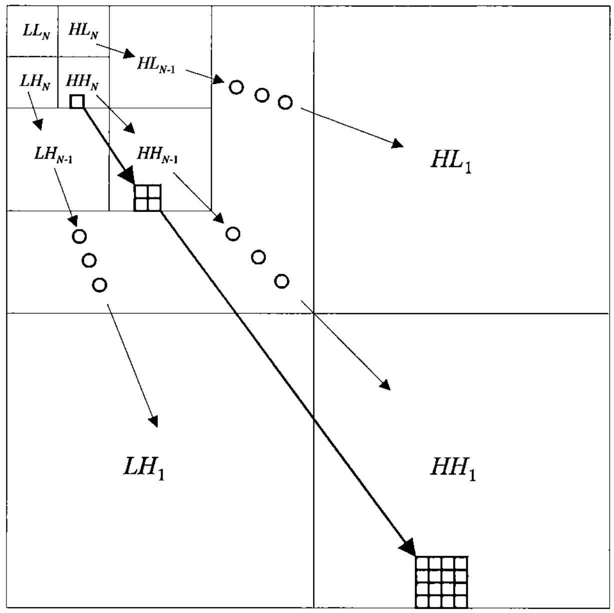


Fig. 2 Parent-child dependencies of subbands.

- Root: a coefficient that exists in all subbands, except LH_1, HL_1, HH_1, and LL_N in Fig. 2, is called a "root" with respect to its descendants.

In tree-based coding, all coefficients are scanned in such a way that no child is scanned before its parent. Therefore, bands LL_N, HL_N, LH_N, and HH_N in Fig. 2 are scanned first. Scanning then moves on to N-1 scale scanning (HL_{N-1}, LH_{N-1}, HH_{N-1}) and continues until reaching the starting scale (HL_1, LH_1, HH_1) as shown in Fig. 3. This scanning pattern arranges the coefficients in the order of their importance, allowing for embedding.

Two types of processing are performed: a dominant pass and a subordinate pass. A dominant pass finds the coefficients to a given threshold, and a subordinate pass refines these significant coefficients. The first step is to find the maximum coefficient among all coefficients, then set the initial threshold to be a maximum power of two smaller than this maximum coefficient. The threshold for each subsequent dominant pass is set to half that of the previous threshold. Four symbols are used for signaling a dominant pass to the decoder. A ZTR symbol is used for a zerotree root that is insignificant and has no significant descendants. An isolated zero

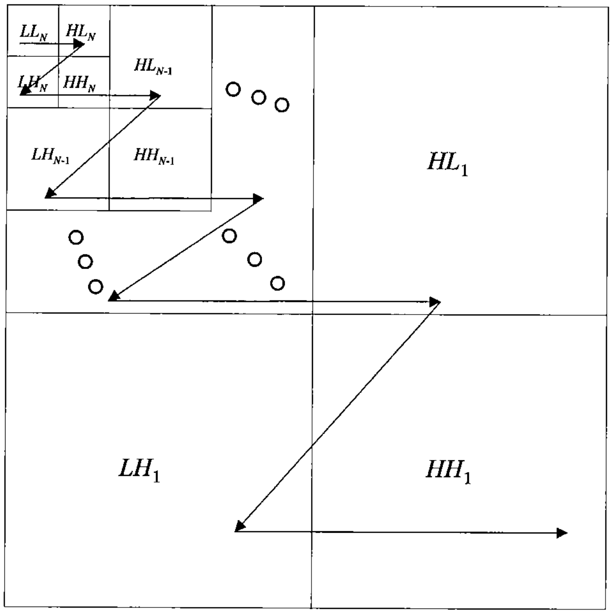


Fig. 3 Scanning order of subbands for encoding a significance map.

(IZ) symbol is used when a coefficient is insignificant but has significant descendants. Two symbols are used for a significant coefficient, POS or NEG corresponding to its sign. The ZTR and IZ symbols are used to indicate the locations of the significant coefficients, POS and NEG, as efficiently as possible.

After each dominant pass, a subordinate pass is performed to refine the coefficients found to be significant in the previous dominant passes. This pass makes lower or higher decisions of uncertainty for the given threshold to minimize the quantization errors. That is, a coefficient in the upper half of the uncertainty interval is coded with the symbol HIGH, while a coefficient in the lower half is coded with the symbol LOW. By reading the subordinate symbols corresponding to the significant coefficients and knowing the threshold, the decoder can determine the interval and approximately reconstruct the significant coefficients. Therefore, from the decoder's viewpoint, the rough estimates of the significant coefficients become more refined and accurate as more subordinate passes are made.

Four symbols are needed for the dominant passes, except for the highest bands with no IZ symbols, and two symbols are needed for subordinate passes. All output symbols are transmitted as a stream and entropy-coded using an adaptive arithmetic coder [25]. The header includes the initial threshold, the original image size, the decomposed scales, and the used filters; it is inserted prior to output stream.

3. Proposed Zerotree Coding

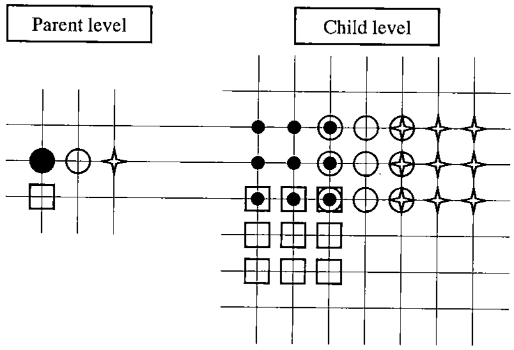
As explained in the previous section, a dominant pass in the EZW coder identifies the significant coefficients with respect to a given threshold and their signs. In the pass, four symbols—ZTR, POS, NEG, and IZ—are used to indicate the locations and signs. Assume that a symbol stream was produced from the coder. The stream contains symbols produced from both dominant and subordinate passes. The quality of the reconstructed image depends on the number of POS and NEG symbols in the dominant passes and the number of HIGH and LOW symbols in the subordinate passes. Although IZ and ZTR symbols are in the symbol stream, they do not affect image quality. They only indicate where the significant coefficients exist. Therefore, to get a better performance, we need to use the numbers of IZ and ZTR symbols as few as possible. Also, the occurrence probabilities of the symbols should be exploited, because the produced symbols are entropy-coded.

Given a set of two symbols S_1, S_2 and an accurate assessment of the probability distribution P of the symbols, Shannon [26] proved that the smallest possible number of bits needed to encode a symbol is the entropy of P, denoted by

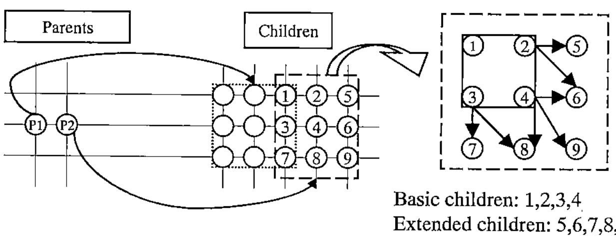
$$E(P) = -(P_1 \log_2 P_1 + P_2 \log_2 P_2) \quad (1)$$

where P_i is the probability that symbol S_i occurs. To encode the symbols that occur by using as few bits as possible, the probability of the less-probable symbols needs to be lowered. That is, the entropy needs to be lowered. Is it possible to control the probability of occurrence? The answer is "yes" for special cases like the EZW coder. This is because one parent in a scale is related to four children at the same spatial locations in the next scale. We refer to this relation as a fixed relation. To lower the probability of the less-probable symbols, the IZ symbols in the EZW coder, we introduce a new relation: each parent has nine children, as shown in Fig. 4 (a). This modified relation enables a parent to have more children than a fixed relation does. This leads to more children belonging to significant parents, so that fewer IZ symbols are produced. Note that more ZTR symbols are required due to the enlarged relation. As you can see in the figure, the children of neighboring parents are not independent of each other. Even though they are overlapped and shared among parents, the children do not need to be scanned twice. As a result, the number of ZTR symbols should not increase very much.

Nevertheless, the modified relation does increase the number of ZTR symbols because each parent has more children, so that some IZ symbols are replaced by ZTR symbols. This increase is exploited by an observation that a significant coefficient is more likely to have significant coefficients in its neighborhood than an insignificant coefficient. From the observation, the modified relation can be made flexible, as shown in Fig. 4 (b), so that each parent has four or more children. The relation is referred to as a flexible relation and the coder using this relation is named as FZW that stands for

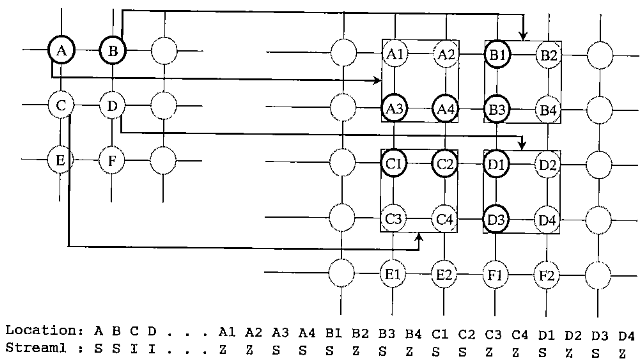


(a) Modified relation: each parent has nine children; each shape represents a parent and its children.

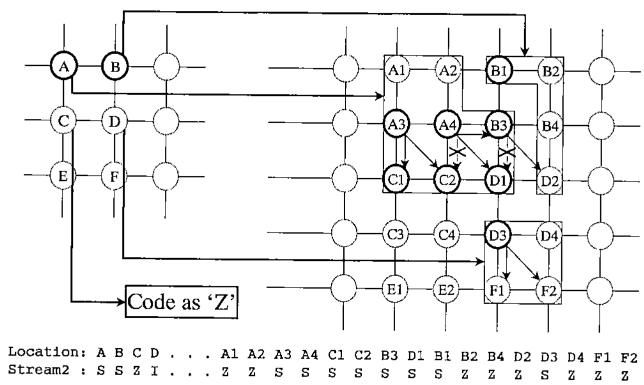


(b) Flexible relation: each parent has four basic children plus extended children, the number of which depends on the significance of the basic children.

Fig. 4 Modified and flexible parent-children relations.



(a) Searching in the EZW coder



(b) Searching in the FZW coder

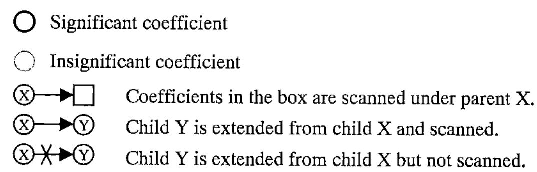


Fig. 5 Searching for significant coefficients in (a) EZW and (b) FZW coders.

"flexible relation based embedded zerotree coding of wavelet coefficients." Each parent always has four basic children (1, 2, 3, and 4), and the number of extended children depends on the significance of children 2, 3, and 4. For example, when child 2 is significant, the parent has basic children 1, 2, 3, and 4 and extended children 5 and 6 (see Fig. 4 (b)). In this case, the parent has only six children—1, 2, 3, 4, 5, and 6. The number of IZ symbols in the flexible relation is more than in the modified relation because the relation is retrenched. However, the increments of IZ symbols can be compensated enough by decrements of ZTR symbols.

Searching methods for significant coefficients in the EZW and FZW coders are illustrated in Fig. 5. There are ten significant coefficients—two, A and B, are in the parent scale and eight, A3, A4, B1, B3, C1, C2, D1, and D3, are in the child scale. The goal is to identify all significant coefficients while ensuring that children are not scanned before their parents. To simplify the symbol representations, POS and NEG are marked with an 'S' meaning a significant coefficient. Also, IZ and ZTR are marked with an 'I' and 'Z,' respectively.

With the fixed relation in the EZW coder, Fig. 5 (a), parents A, B, C and D, each have two significant children. Therefore, C and D should be marked with an 'I' to scan their significant children—C1, C2 and D1, D3. In this case, two 'I's and eight 'Z's are needed to identify the ten 'S's in the stream.

With the flexible relation in the FZW coder, Fig. 5 (b), parent A is significant and thus marked with an 'S'; its basic children are A1, A2, A3, and A4, of which A3 and A4 are significant. Thus, C1 and C2 from A3 and B3 and D1 from A4 are added as extended children. Note that there is no need to re-scan a coefficient once it has been scanned, such as with C2. In this case, parent A is related to eight children—A1, A2, A3, A4, C1, C2, B3, and D1. Their relations are clearly shown in Fig. 6 (a). The next parent, B, is also significant. Its basic children are B1, B2, B3, and B4. Because B3 was already scanned under parent A, it is not re-scanned. In this case, the extended children are only D2 from B3 because D1 was also previously scanned. Although parent C has significant basic children—C1 and C2, it can be marked with a 'Z' because their significance was already identified under parent A. We thus save one 'I' symbol compared to the fixed relation. Parent D is a little different from C because there is significant basic child D3 not yet scanned. Parent D is thus responsible for getting D3 scanned. Therefore, D is marked with an

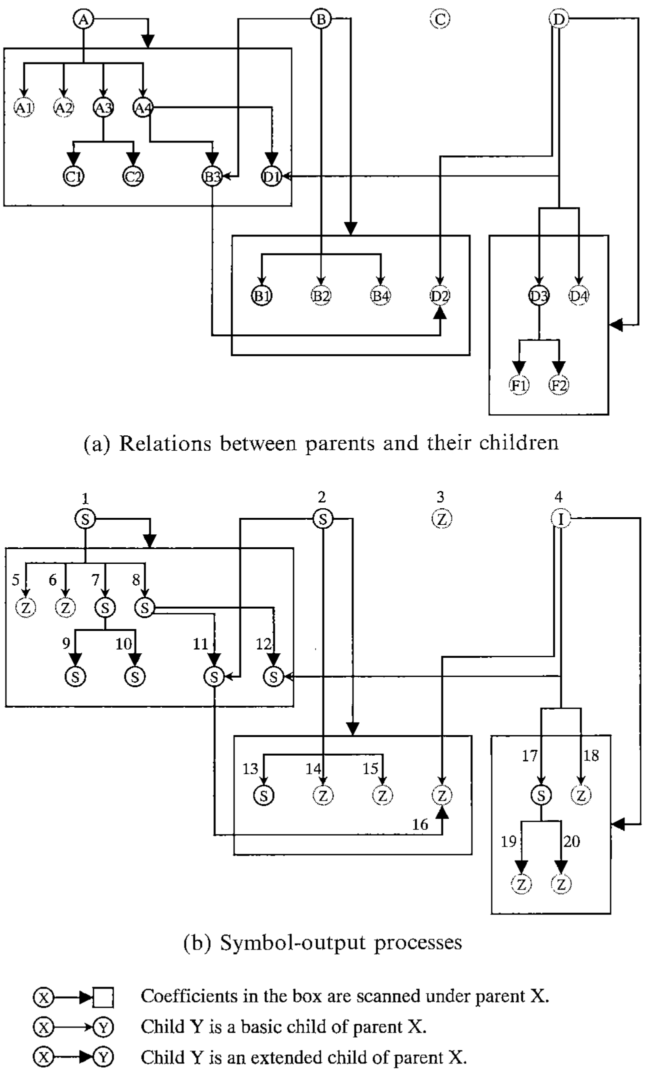


Fig. 6 Parent-child relations and symbol-output processes for the searching in Fig. 5 (b).

'I.' Once D3 is scanned, its two extended children, F1 and F2 should also be scanned in accordance with the extension rule, although they are not significant. The symbol output processes are mapped with numbers in Fig. 6 (b).

The symbol streams produced from the EZW and FZW coders are given in Fig. 5. Stream 2 contains one less 'I' symbol and one more 'Z' symbol than stream 1. Consequently, stream 2 has lower entropy. Although this example shows an increase of only one 'Z' symbol, in practice, an 'I' symbol decrease usually leads to more than one additional 'Z' symbol.

4. Experimental Results and Discussions

We simulated their use on two test images—Lena and Barbara (512×512 in grey scale). We compared the performance of the FZW coder with that of the EZW coder. The images were downloaded from the RPI site: ftp://ipl.rpi.edu/pub/image/still/usc.

All simulations were done with 6-scale octave wavelet by using a 9/7-tap filter bank [27] and a reflection extension at the image border. The filter performed better than the QMF 9-tap filter [28] that Shapiro used. However, using the QMF filter, we could not match his published performance [13], in either the flexible or fixed relations. To reproduce an image from transmitted symbols, the stream includes 8 bytes header information—4 bytes for the horizontal and vertical dimensions of the image, 1 byte for the filters, 1 byte for the wavelet scales, and 2 bytes for the maximum coefficient. With the header information, the initial threshold is set at a maximum integer power of two that does not exceed the maximum coefficient.

Along with this threshold, a dominant pass is inserted into the stream, followed by a subordinate pass. Then, the threshold is halved. These two passes are alternated until the coding is terminated. All symbols produced from the passes are entropy-coded using the Jones' coder [29]. In this entropy coding, the frequencies of the symbol-alphabet are adaptively updated according to the occurrence rates of the symbols. The initial occurrence probabilities for the symbols are evenly allocated, and they are updated according to the produced symbols. To improve the entropy coding, the probabilities are refreshed whenever the maximum frequency reaches 256.

Figure 7 displays the coding performance using the FZW coder as well as the results in [13]. The first two circles were obtained at very low bit rates by stopping the coding at 256 and 512 bytes, respectively. The other circles were computed at the bit rates of the multiples of 1024 bytes.

A comparison of the symbols produced among the fixed, modified, and flexible relations is shown in Table 1. To get the same image, the codings were terminated at the point where a threshold became 16 for Lena and Barbara images. That is, all coefficients larger than or equal to the threshold 32 were identified as significant by the dominant passes; where the corresponding subordinate passes were also transmitted. The numbers of symbols for the subordinate passes were not given in the table because they were the same. As shown in the table, the flexible relation produced symbol streams with fewer IZ symbols and more ZTR symbols than the fixed relation, while it produced more IZ symbols and fewer ZTR symbols than the modified relation. When the produced symbols were entropy-coded, the flexible relation showed the best performance among the three relations. It was proved in terms of compressed sizes and compression ratios.

The PSNRs of 32.51 and 30.39 dB for Lena and Barbara, respectively, were obtained by applying the same terminating condition. The differences in the compressed sizes were due solely to the number of IZ and

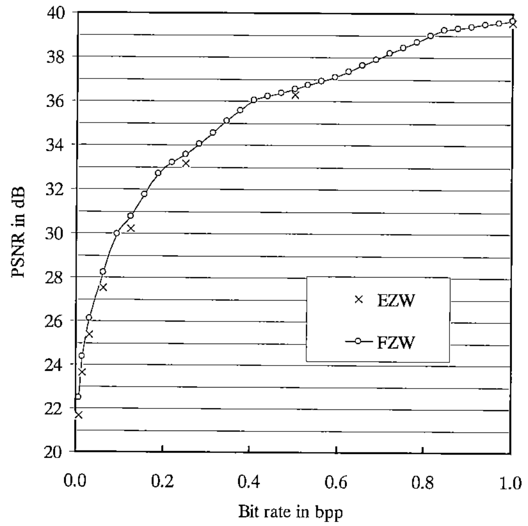
Table 1 Symbols produced using different relations for the same image quality.

(a) Lena (512×512, 8-bit grey image, original size = 262144 Bytes)

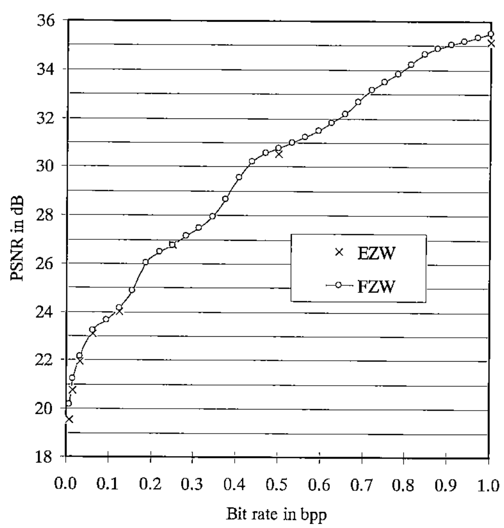
Relations	Compressed size in Bytes	PSNR in dB	Compression ratio	♯ of POS	♯ of NEG	♯ of ZTR	♯ of IZ
Fixed	6511	32.51	40.26 : 1	3876	3743	43249	1566
Modified	6388	32.51	41.04 : 1	3876	3743	65707	912
Flexible	6288	32.51	41.69 : 1	3876	3743	48102	1168

(b) Barbara (512×512, 8-bit grey image, original size = 262144 Bytes)

Relations	Compressed size in Bytes	PSNR in dB	Compression ratio	♯ of POS	♯ of NEG	♯ of ZTR	♯ of IZ
Fixed	15878	30.39	16.51 : 1	9741	9586	72003	7603
Modified	15022	30.39	17.45 : 1	9741	9586	103203	4804
Flexible	14511	30.39	18.07 : 1	9741	9586	80526	5063



(a) Lena



(b) Barbara

Fig. 7 Performance curves for test images.

ZTR symbols. To analyze the numbers, we set the numbers of IZ and ZTR symbols to variables, n_{IZ} and

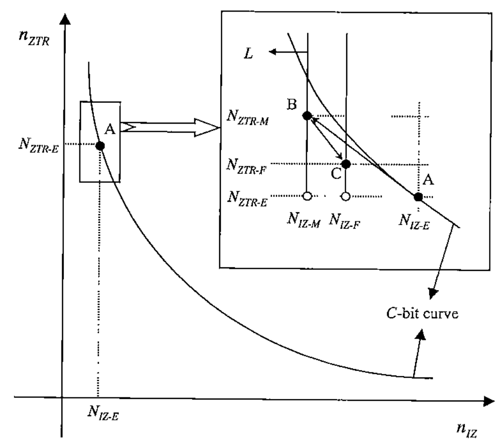


Fig. 8 Relationship between the number of ZTR symbols and the number of IZ symbols. Note that n_{IZ} and n_{ZTR} are located at N_{IZ-E} and N_{ZTR-E} in the EZW coder.

n_{ZTR}; where the same image is reproduced by the same N_{POS} and N_{NEG} using each relation. Then, we know the total number of used symbols and the occurrence probabilities of the four symbols. Therefore, the minimum entropy-coded size can be computed by multiplying the entropy for a symbol by the total number of symbols. Let the compressed size by the EZW coder be C bits. Then, an identity equation with two variables can be made for the given C bits, as shown in Eq. (2).

$$C = -\left(N_{POS}\log_2\frac{N_{POS}}{t} + N_{NEG}\log_2\frac{N_{NEG}}{t} + n_{IZ}\log_2\frac{n_{IZ}}{t} + n_{ZTR}\log_2\frac{n_{ZTR}}{t}\right) \quad (2)$$

where, t is the total number of symbols. Using Eq. (2), we can relate n_{IZ} to n_{ZTR}, as shown in Fig. 8. The same image quality using C bits can be obtained at every point on the curve. When the EZW coder is used, in practice, the number of ZTR symbols is much greater than the number of IZ symbols; that is, $N_{ZTR-E} >> N_{IZ-E}$. Therefore, the point by the EZW coder is on the far-left side of the graph, for example, point A in the figure. As we move to the left on the curve, the entropy is lowered and the total number of symbols is increased. The curve can therefore be seen as a collection of points representing the same image using the same C bits. When the combination of n_{IZ} and n_{ZTR} is below the curve, it means that the entropy coding is achieved with less cost than C bits. In contrast, if the combination is above the curve, more than C bits are needed. Therefore, the curve can be used as a criteria for good compression.

Using the C-bit curve, we found the point where the numbers of IZ and ZTR symbols, N_{IZ-M} and N_{ZTR-M}, in the modified relation are located. Recall that the modified relation was designed to produce fewer IZ symbols than the fixed relation. Therefore, N_{IZ-M} may be located on line L. Moreover, the children of neighboring parents are not independent of each other. They are overlapped and shared among parents: that is, a child does not need to be scanned again once it has been scanned. As the result, N_{ZTR-M} was found to be below the curve. That is, the numbers, N_{IZ-M} and N_{ZTR-M}, were found to be at point B.

Recall that the flexible relation was designed to produce fewer ZTR symbols than in the modified relation. The flexible relation was exploited by the observation that a significant coefficient is more likely to have significant coefficients in its neighborhood than an insignificant coefficient. In the flexible relation, some of children in the modified relation are retrenched. Therefore, the flexible relation is at point C (N_{IZ-F}, N_{ZTR-F}). From points A, B, and C, we derived inequalities, (1) and (2). From these inequalities, our simulation results showed that inequality (3) is satisfied.

(1) Number of IZ symbols:
$N_{IZ-E} > N_{IZ-F} > N_{IZ-M}$

(2) Number of ZTR symbols:
$N_{ZTR-M} > N_{ZTR-F} > N_{ZTR-E}$

(3) Compressed size in Bytes:
Fixed > Modified > Flexible

5. Conclusions

We have proposed a new relation in which a parent is related to its children in a flexible manner. We started by modifying the fixed relation. This modified relation produces a symbol stream that has lower entropy because each parent has nine children. The children of neighboring parents are not independent of each other; they are overlapped and shared by parents. This enlarged relation means that fewer IZ symbols and more ZTR symbols are produced than in the fixed relation.

The increases of the ZTR symbols were exploited by the observation that a significant coefficient is apt to have significant coefficients in its neighborhood than an insignificant coefficient. Based on this, we derived a flexible relation and compared the numbers of symbols produced by the fixed, modified and flexible relations. Although the symbol streams produced the same image, their compressed sizes were quite different. For rate-controlled results, the FZW coder outperformed the EZW coder [13] by 0.2–1.0 dB in terms of PSNR.

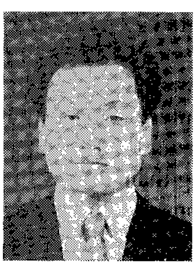
Acknowledgement

This work was in part supported by the Grant-in-Aid for Scientific Research, No. 07650419, from the Ministry of Education, Science and Culture of Japan and the research grant by Telecommunications Advancement Organization of Japan.

References

[1] T. Ebrahimi, E. Reusens, and W. Li, "New trends in very low bitrate video coding," Proc. IEEE, vol.83, no.6, pp.877–891, June 1995.
[2] G. Strang and T. Nguyen, "Wavelets and filter banks," Wellesley-Cambridge Press, 1996.
[3] J.W. Woods, "Subband Image Coding," Kluwer Academic Publishers, Boston, MA, 1991.
[4] M. Vetterli and J. Kovacevic, "Wavelets and Subband Coding," Prentice Hall, 1995.
[5] I. Daubechies, "Ten Lectures on Wavelets," SIAM, Philadelphia, PA, 1992.
[6] I. Daubechies, "Orthonormal bases of compactly supported wavelets," Commun. Pure and Appl. Math., vol.41, pp.909–996, Nov. 1988.
[7] I. Daubechies, "The wavelet transform, time-frequency localization and signal analysis," IEEE Trans. Inf. Theory, vol.36, no.5, pp.961–1005, Sept. 1990.
[8] S. Mallat, "A theory for multi-resolution signal decomposition: The wavelet representation," IEEE Trans. Pattern Anal. & Mach. Intell., vol.11, no.7, pp.674–693, July 1989.
[9] S. Mallat, "Multi-frequency channel decompositions of images and wavelet models," IEEE Trans. Acoust., Speech, Signal Processing, vol.37, no.12, pp.2091–2110, Dec. 1989.
[10] E.J. Stollnits, T.D. Derose, and D.H. Salesin, "Wavelets for Computer Graphics," Morgan Kaufmann Publishers, 1996.
[11] R. Coifman and M.V. Wickerhauser, "Entropy-based algorithms for best basis selection," IEEE Trans. Inf. Theory, IT-38, pp.713–718, March 1992.
[12] K. Ramchandran and M. Vetterli, "Best wavelet packet bases in a rate-distortion sense," IEEE Trans. Image Processing, vol.2, no.2, pp.160–176, April 1993.
[13] J.M. Shapiro, "Embedded image coding using zerotrees of wavelet coefficients," IEEE Trans. Signal Processing, vol.41, no.12, pp.3445–3462, Dec. 1993.
[14] A. Said and W.A. Pearlman, "A new fast and efficient image codec based on set partitioning in hierarchical trees," IEEE Trans. Circuits and Systems for Video Technology, vol.6, no.3, pp.243–250, June 1996.
[15] Z. Xiong, K. Ramchandran, and M.T. Orchard, "Space-frequency quantization for wavelet image coding," IEEE Trans. Image Processing, vol.6, no.5, pp.677–693, May 1997.
[16] Z. Xiong, K. Ramchandran, M.T. Orchard, and K. Asai, "Wavelet packets-based image coding using joint space-frequency quantization," Proc. ICIP '94, vol.3, pp.324–328, Austin, Texas, Nov. 1994.

[17] Z. Xiong, K. Ramchandran, and M.T. Orchard, "Wavelet packets image coding using space-frequency quantization," IEEE Trans. Image Processing, Jan. 1996.
[18] Z. Xiong, O. Guleryuz, and M.T. Orchard, "A DCT-based embedded Image coder," IEEE Signal Processing Letters, pp.289–290, Nov. 1996.
[19] D. Taubman and A. Zakhor, "Multirate 3-D subband coding of video," IEEE Trans. Image Processing, vol.3, no.5, pp.572–588, Sept. 1994.
[20] M.J. Tsai, J. Villasenor, and F. Chen, "Stack-run 6," IEEE Trans. Circuits and Systems for Video Technology, vol.6, pp.519–521, Oct. 1996.
[21] T. Strutz and E. Mueller, "Image data compression with pdf-adaptive reconstruction of wavelet coefficients," Proc. SPIE, vol.2569, pp.747–758, July 1995.
[22] C. Chrysafis and A. Ortega, "Efficient context-based entropy coding for lossy wavelet image compression," Data Compression Conference, Snowbird, UT, March 1997.
[23] S.M. LoPresto, K. Ramchandran, and M.T. Orchard, "Image coding based on mixture modeling of wavelet coefficients and a fast estimation-quantization framework," Data Compression Conference, pp.221–230, UT, March 1997.
[24] S.H. Joo, H. Kikuchi, S. Sasaki, and J. Shin, "A flexible zerotree coding with low entropy," Proc. ICASSP'98, vol.5, pp.2685–2688, Seattle, May 1998.
[25] H. Witten, R. Neal, and J.G. Cleary, "Arithmetic coding for data compression," Comm. ACM, vol.30, no.6, pp.520–540, June 1987.
[26] C.E. Shannon, "The Mathematical Theory of Communication," reprinted in University of Illinois Press, Urbana, 1949.
[27] M. Antonini, M. Barlaud, P. Mathieu, and I. Daubechies, "Image coding using wavelet transform," IEEE Trans. Image Processing, vol.1, pp.205–220, April 1992.
[28] E.H. Adelson, E. Simoncelli, and R. Hingorani, "Orthogonal pyramid transforms for image coding," Proc. SPIE, vol.845, Cambridge, MA, pp.50–58, Oct. 1987.
[29] C.B. Jones, "An efficient coding system for long source sequences," IEEE Trans. Inf. Theory, vol.IT-27, no.3, pp.280–291, May 1981.



Sanghyun Joo He received B.E. and M.E. degrees from Dongguk University, Korea, in 1989 and 1994, respectively. From 1994 to 1996 he worked at Korea Academy of Industrial Technology (KAITech). He is now a doctoral student at Niigata University. His research interests include digital image processing, source coding, channel coding and wavelets. He is a member of IEEE.

Hisakazu Kikuchi He was born in Niigata in 1952. He received B.E. and M.E. degrees from Niigata University, Niigata, in 1974 and 1976, respectively, and Dr.Eng. degree in electrical and electronic engineering from Tokyo Institute of Technology, Tokyo, in 1988. From 1976 to 1979 he worked at Systems Laboratories, Fujitsu Ltd. Since 1979 he has been with Niigata University, where he is Professor in electrical engineering. During a year of 1992 to 1993, he was a visiting scientist at the Electrical Engineering Department, University of California, Los Angeles. His research interests include digital signal processing, image processing, wavelets, and mobile telecommunications. He is a member of IEEE, Inst. of Image Information and Television Engineers, and Japan SIAM.

Shigenobu Sasaki He was born in Nara, Japan on April 18, 1964. He received B.E., M.E. and Dr.Eng. degrees from Nagaoka University of Technology, Nagaoka, Japan, in 1987, 1989 and 1993, respectively. Since 1992, he has been with the Department of Electrical and Electronics Engineering, Faculty of Engineering, Niigata University, Niigata, Japan, where he is now Assistant Professor. His research interests are in communication systems and signal processing, especially spread spectrum communication systems and mobile communications. Dr. Sasaki is a member of IEEE and Society of Information Theory and its Applications (SITA), Japan.

Jaeho Shin He received the B.E., M.E. and Ph.D. degrees in electronic engineering from Seoul National University, Seoul, Korea, in 1979, 1982 and 1987 respectively. He is a Professor in the Department of Electronic Engineering, Dongguk University in Seoul, Korea. His research interests include digital signal processing, digital system design, information security. He is a member of IEEE, IEEK, KICS, and KISS.