

# Blind Array Calibration Technique Using ICA

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## 1. Introduction

High-resolution direction of arrival (DOA) estimation algorithms, (e.g. MUSIC algorithms [1]), require array calibration for their mode vectors. One of the calibration techniques is the method which employs a calibration matrix for a covariance matrix of the array. The calibration matrix is estimated before the operation with a set of reference sources having known DOAs in general. Since the method requires known reference signals, it is difficult to apply the method for arrays in actual operation.

We propose a array calibration technique with observed signals in the actual operation. Since the observed signals are the mixed signals of several sources, we employ independent component analysis (ICA) [2] [3] to resolve each signal component. The ICA is a method which can resolve mixed signals into individual signal components when the signals are independent and maximally nongaussian. In the proposed calibration technique, the calibration matrix is estimated with these independent signal components. EM (expectation maximization)-type algorithm is employed for the estimation since DOA of the resolved signal components is unknown.

In this report, we show the performance of the proposed calibration technique with computer simulations of a 4-element dipole array.

## 2. Calibration Data Set Extraction Using ICA

### 2.1 Data Model

The observed signal model and the array configuration are depicted in Fig.1, where  $L$  is the number of array elements, and  $K(< L)$  is the number of impinging signals. Here, we assume that  $M$  trials are available for the calibration data set extraction by the ICA. The DOAs of signals in each trial are assumed to be different. The  $m$ -th ( $m = 1, 2, \dots, M$ ) observed signal vector  $\mathbf{x}^{(m)}(t)$  is given by,

$$\begin{aligned}\mathbf{x}^{(m)}(t) &= \tilde{\mathbf{A}}^{(m)}\mathbf{s}^{(m)}(t) + \mathbf{n}(t) \\ &= \mathbf{C}\mathbf{A}^{(m)}\mathbf{s}^{(m)}(t) + \mathbf{n}(t),\end{aligned}\quad (1)$$

where  $\tilde{\mathbf{A}}^{(m)}$  is an  $L \times K$  mixing matrix,  $\mathbf{C}$  is the  $L \times L$  calibration matrix,  $\mathbf{A}^{(m)}$  is the  $L \times K$  mode matrix,  $\mathbf{s}^{(m)}(t)$  is the  $L \times 1$  source signal vector, and  $\mathbf{n}(t)$  is additive white Gaussian noise vector. In this model the signal components of  $\mathbf{s}^{(m)}(t)$  are assumed to be nongaussian and mutually independent from each other.

### 2.2 Signal Separation Based on ICA

The signals can be separated from the observed signal using ICA since the signals are mutually independent from each other.

By using ICA, the independent component vector  $\mathbf{y}^{(m)}(t)$  is given by

$$\mathbf{y}^{(m)}(t) = \mathbf{W}^{(m)}\mathbf{x}^{(m)}(t) = \mathbf{W}^{(m)}\tilde{\mathbf{A}}^{(m)}\mathbf{s}^{(m)}(t), \quad (2)$$

where  $\mathbf{W}^{(m)}$  is the  $K \times L$  demixing matrix. In this formulation, the noise vector is ignored for simplicity. Since the independent component vector  $\mathbf{y}^{(m)}(t)$  is equal to the signal vector  $\mathbf{s}^{(m)}(t)$ ,  $\tilde{\mathbf{A}}^{(m)}$  can be given by

$$\tilde{\mathbf{A}}^{(m)} = (\mathbf{W}^{(m)H}\mathbf{W}^{(m)})^{-1}\mathbf{W}^{(m)H}, \quad (3)$$

where  $^H$  denotes the conjugate transpose.

The columns of the  $\tilde{A}^{(m)}$  are the mode vectors of the corresponding signal that can be used as reference data sets for the array calibration in the next section. When we apply the above procedure to  $M$  trials and each trial assumes to have  $K$  signals, then we obtain  $M \times K$  reference data having unknown DOAs for calibration.

### 3. Iterative Estimation of Calibration Matrix

Reference data for the calibration are resolved by using ICA in the Sect.2. In the calibration matrix estimation, we select  $N$  array calibration data from these reference data, and construct a mode matrix  $\tilde{A}$  of the signals, which is given by

$$\tilde{A} = \begin{bmatrix} \tilde{a}_1 & \tilde{a}_2 & \dots & \tilde{a}_N \end{bmatrix}. \quad (4)$$

The mode vectors for coherent multipath signals are rejected by this selection. If DOAs of the signals in  $A$  is known, the calibration matrix  $C$  is given by [4]

$$C = \tilde{E}_s G A^H (A A^H)^{-1} \quad (5)$$

$$\tilde{E}_s = \begin{bmatrix} \tilde{e}_1 & \tilde{e}_2 & \dots & \tilde{e}_N \end{bmatrix} \quad (6)$$

$$\tilde{A} = \tilde{E}_s G \quad (7)$$

$$P = I - A^H (A A^H)^{-1} A, \quad (8)$$

where  $\tilde{e}_i$  is the principal eigenvector of  $\tilde{a}_i^H \tilde{a}_i$ .  $G$  is the diagonal matrix which is made from eigenvector corresponding the smallest eigenvalue of  $(\tilde{E}_s^H \tilde{E}_s) \odot P$  where  $\odot$  denotes the Hadamard product of matrix.

Since the mode vectors of  $A$  are unknown in this case, the calibration matrix is estimated iteratively. The proposed iterative procedure is summarized below:

#### Procedure of iterative estimation of calibration matrix

**Step.1.** Estimate DOA using  $\tilde{A}$ .

**Step.2.** We set the initial mode matrix  $\hat{A}^{(0)}$ , and the initial calibration matrix is computed from  $\hat{C}^{(0)} = \tilde{E}_s \hat{G}^{(0)} \hat{A}^{(0)H} (\hat{A}^{(0)} \hat{A}^{(0)H})^{-1}$ .

**Step.3.** We set the initial value of  $l$  to 0.

**Step.4.** Re-estimate DOA using  $\hat{C}^{(l)}$  and  $\tilde{A}$ . Then, the mode matrix  $\hat{A}^{(l+1)}$  is computed.

**Step.5.** Compute  $\hat{C}^{(l+1)}$ .

**Step.6.** if  $\|\hat{C}^{(l+1)} - \hat{C}^{(l)}\|_F$  is converged,  $\hat{C}^{(l+1)}$  is the estimated calibration matrix, otherwise set  $l = l + 1$  and return to Step.4.

Where  $\|\cdot\|_F$  is the Forbenius norm of the matrix.

### 4. Simulation Example

In this section, we evaluate performance of the proposed method by computer simulations. The model parameters in the simulations are given in Table 1.

The observed data are calculated by the moment method. The data contains mutual coupling effect among the elements. In this simulation, 4 ( $M = 4$ ) trial are employed. Each trial has the signals coming from  $(-60^\circ, 10^\circ)$ ,  $(-30^\circ, 15^\circ)$ ,  $(-15^\circ, 30^\circ)$ , and  $(-10^\circ, 60^\circ)$ , respectively. The signals having DOAs of  $(-60^\circ, -30^\circ, -15^\circ, -10^\circ, 10^\circ, 15^\circ, 30^\circ, 60^\circ)$  are selected in the calibration. The calibration results are

Table 1: Parameters of the Simulation Model

4-element Uniform Linear Array	
Element	0.5 wave length dipole
Element Spacing	0.5 wave length
Number of Reference Data	8(=2+2+2+2)
Number of Snapshots	5000 samples/ 4 trials
SNR	20dB

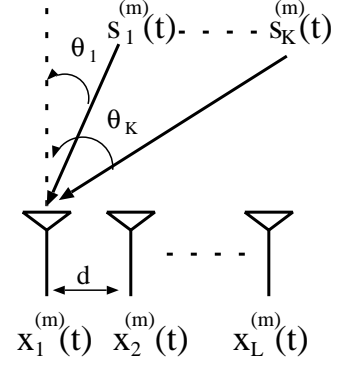


Figure 1: Data model

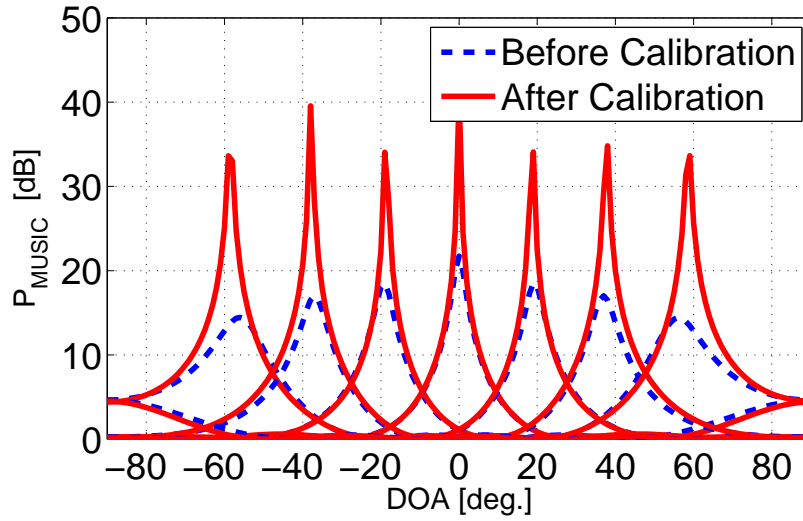


Figure 2: MUSIC spectrum

shown in Fig.2, where we show the results of DOA estimation by MUSIC. The peaks of the MUSIC spectrum with proposed calibration technique are sharper than the peaks without calibration.

Fig.3 shows the RMSE of DOA estimation. The maximum RMSE without calibration is larger than 4 degrees. On the other hand, the RMSE with proposed calibration becomes less than 2 degrees. As can be seen in this figure, DOA error still remains after the calibration. This is due to the ambiguity of the weight matrix  $\mathbf{G}$  in (5).

When the weight matrix is assumed to be known, the RMSE of DOA estimation becomes almost zero as shown in Fig.4. In this case, the RMSE is less than 0.3 degrees. The assumption that the  $\mathbf{G}$  is known a priori is unrealistic assumption for blind array calibration. Then, the other constraints will be required for practical arrays. However, the simulation results show that the proposed calibration improves DOA estimation accuracy to some extent without this assumption, and it can be further improved with proper assumptions.

## 5. Conclusions

We proposed a new blind array calibration technique using ICA and EM algorithm. This method extracts reference data for calibration from observed signals by using ICA, and estimate the calibration matrix by EM algorithm. Computer simulation results show that the proposed technique decrease DOA estimation error. Modification with proper assumptions is the further study.

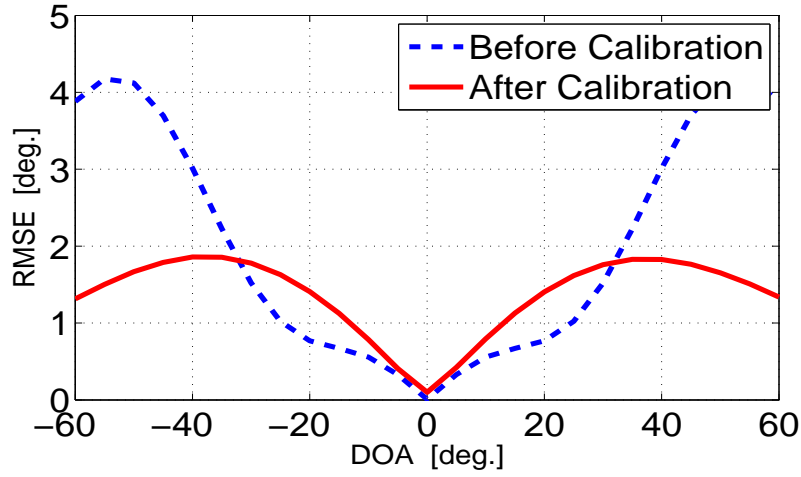


Figure 3: RMSE of DOA Estimation

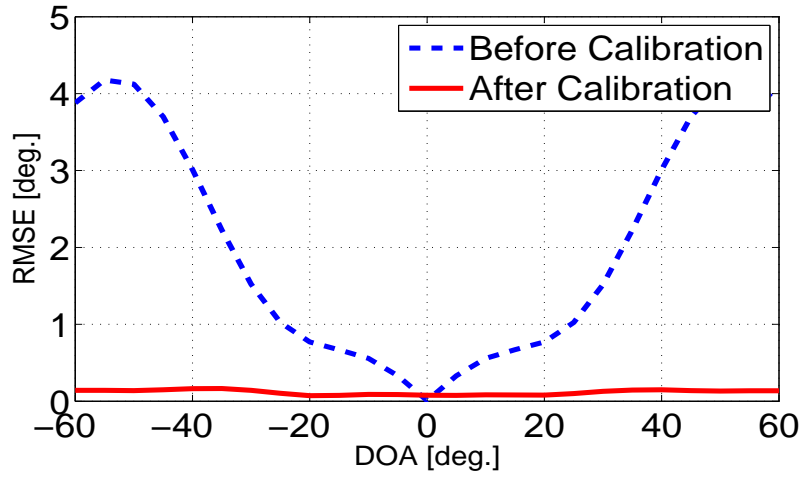


Figure 4: RMSE of DOA Estimation with known  $G$  matrix

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