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On Exact Model-Based Scattering Decomposition of Polarimetric SAR Data

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1. Introduction

Model-based scattering decomposition [1-4] is an effective and popular tool for analyzing polarimetric SAR (PolSAR) data due to its clear physical explanation, convenient implementation, and easy visual interpretation. This technique aims to express the measured PolSAR data as the combination of different scattering mechanisms. However, the original three-component model proposed by Freeman and Durden [1] assumes azimuthal reflection symmetry and consequently does not use the complete information of the covariance or coherency matrix. Recent advances focused on increasing component numbers (e.g., the four-component model [2]) or reducing the number of knowns (e.g., by matrix rotation [3]), with the purpose to account for more elements in the matrix data. Nevertheless, it is to the best knowledge of the authors that finding a physically meaningful matrix expansion (i.e., model-based) that exactly (but not approximately) matches the measured data remains an unresolved task.

This paper is dedicated for addressing such a problem. We demonstrate that a coherency matrix can be exactly decomposed into three components respectively representing surface scattering, double-bounce scattering, and volume scattering. In particular, we show that solving for the expansion coefficients as well as the expansion matrices in fact amounts to a generalized eigenvalue problem. This equivalence guarantees that 1) positive powers are always maintained during the decomposition, and 2) the three-component expansion perfectly matches the measured data.

2. Three-Component Scattering Model

In this paper, we consider decomposition of the coherency matrix (CM) of the PolSAR data format, which is the second order statistic obtained by ensemble averaging the Pauli vectors [3]. In general, under the condition of scattering reciprocity the CM is a 3×3 complex positive-semidefinite and conjugate symmetric matrix. The rank of the CM, however, depends on the number of Pauli vectors involved in the ensemble averaging, or in SAR terminologies, on the equivalent number of looks (ENL) [5]. In this paper, we assume that the ENL of the image is sufficiently large so that the CM is always regarded as full-ranked (i.e., rank 3) and consequently positive-definite.

The purpose of model-based scattering decomposition is thus to express the CM as the weighted sum of different scattering mechanisms. In particular, we study the three-component scattering scheme where the measure CM can be expanded to three components as follows [1, 4]:

$$\boldsymbol{T} = f_S\boldsymbol{T}_S + f_D\boldsymbol{T}_D + f_V\boldsymbol{T}_V, \tag{1}$$

where $\boldsymbol{T}$ is the measured CM; $\boldsymbol{T}_S$, $\boldsymbol{T}_D$, and $\boldsymbol{T}_V$ denote the CMs for the surface scattering, double-bounce scattering, and volume scattering, respectively; f_S, f_D, and f_V are the corresponding expansion coefficients which by physical meaning should be all positive numbers because they represent component powers. The original Freeman decomposition models the three scattering components with the following CMs [1, 4]:

$$\boldsymbol{T}_S = \begin{bmatrix} 1 & \alpha & 0 \\ \alpha^* & |\alpha|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \boldsymbol{T}_D = \begin{bmatrix} |\beta|^2 & \beta & 0 \\ \beta^* & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \boldsymbol{T}_V = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

In principle, the model parameters (α and β) as well as the expansion coefficients (f_S, f_D, and f_V) can be obtained by solving (1) according to (2). Nevertheless, this only results in an inexact solution to (1) because obviously the elements T_{13} and T_{23} in $\boldsymbol{T}$ cannot be accounted for by any of the three scattering models of (2).

In order to overcome such a problem, i.e., to obtain an exact decomposition of the measured CM, we consider a more generalized model in which no specific parametric forms is specified for $\boldsymbol{T}_S$ and $\boldsymbol{T}_D$ as in (2). The only constraint for $\boldsymbol{T}_S$ and $\boldsymbol{T}_D$ is that they are single-rank matrices. Note that this constraint in fact contains (2) as a special case.

3. Exact Scattering Decomposition: A Generalized Eigenvalue Solution

In this section, we show how to solve the expansion of (1) under the constraint that both $\boldsymbol{T}_S$ and $\boldsymbol{T}_D$ are single-rank matrices. First note that (1) can be rewritten as:

$$\boldsymbol{T} - f_V\boldsymbol{T}_V = f_S\boldsymbol{T}_S + f_D\boldsymbol{T}_D. \tag{3}$$

Then it is easy to know $\boldsymbol{T} - f_V\boldsymbol{T}_V$ is at most a rank 2 matrix because both $\boldsymbol{T}_S$ and $\boldsymbol{T}_D$ are rank 1. Consequently, the determinant of $\boldsymbol{T} - f_V\boldsymbol{T}_V$ is bound to vanish, that is:

$$\left|\boldsymbol{T} - f_V\boldsymbol{T}_V\right| = 0. \tag{4}$$

Note that the above equation is a cubic equation about f_V which can be easily solved. However, it can be also seen that (4) actually corresponds to the following generalized eigenvalue problem:

$$\boldsymbol{T}\boldsymbol{x} = f_V\boldsymbol{T}_V\boldsymbol{x}, \tag{5}$$

where $\boldsymbol{x}$ is the generalized eigenvector and f_V is the generalized eigenvalue. Since both $\boldsymbol{T}$ and $\boldsymbol{T}_V$ are positive-definite and conjugate symmetric, it can be proved that all the eigenvalues of (5) are positive (cf. Appendix). Then suppose $\lambda_{\min}$ is the minimum eigenvalue and by letting $f_V = \lambda_{\min}$, it can be proved that $\boldsymbol{T} - f_V\boldsymbol{T}_V$ is a positive-semidefinite matrix (cf. Appendix). As a result, according to (3) the two other expansion coefficients f_S, f_D will be the two positive eigenvalues of $\boldsymbol{T} - f_V\boldsymbol{T}_V$ with $\boldsymbol{T}_S$ and $\boldsymbol{T}_D$ corresponding to the respective eigenvectors, i.e.:

$$\boldsymbol{T} - f_V\boldsymbol{T}_V = \lambda_1\boldsymbol{u}_1\boldsymbol{u}_1^H + \lambda_2\boldsymbol{u}_2\boldsymbol{u}_2^H = f_S\boldsymbol{T}_S + f_D\boldsymbol{T}_D. \tag{6}$$

Without loss of generality we assume $\lambda_1 \geq \lambda_2$. Assignment of λ_1 and λ_2 to f_S and f_D can be based on the elements in $\boldsymbol{T} - f_V\boldsymbol{T}_V$. If $T_{11} - 2f_V > T_{22} - f_V$, surface scattering is considered dominant and we have $f_S = \lambda_1, f_D = \lambda_2, \boldsymbol{T}_S = \boldsymbol{u}_1\boldsymbol{u}_1^H, \boldsymbol{T}_D = \boldsymbol{u}_2\boldsymbol{u}_2^H$; otherwise, the double-bounce scattering is dominant and we have $f_S = \lambda_2, f_D = \lambda_1, \boldsymbol{T}_S = \boldsymbol{u}_2\boldsymbol{u}_2^H, \boldsymbol{T}_D = \boldsymbol{u}_1\boldsymbol{u}_1^H$. It can be verified that this derivation yields an exact solution to (1) with always positive expansion coefficients.

To conclude this section, the entire procedure for exact decomposition of the PolSAR CM data is summarized in Algorithm 1.

Algorithm 1	**Exact Decomposition of CM**
INPUT:	$\boldsymbol{T}$
1:	Solve the generalized eigenvalue problem: $\boldsymbol{T}\boldsymbol{x} = \lambda\boldsymbol{T}_V\boldsymbol{x} \Rightarrow f_V = \lambda_{\min}$
2:	Solve the eigenvalue problem of $\boldsymbol{T} - f_V\boldsymbol{T}_V$: λ_1, λ_2 $(\lambda_1 \geq \lambda_2 > 0), \boldsymbol{u}_1, \boldsymbol{u}_2$
4:	**IF** $T_{11} - 2f_V > T_{22} - f_V$
5:	$f_S = \lambda_1, f_D = \lambda_2, \boldsymbol{T}_S = \boldsymbol{u}_1\boldsymbol{u}_1^H, \boldsymbol{T}_D = \boldsymbol{u}_2\boldsymbol{u}_2^H$
6:	**ELSE**
7:	$f_S = \lambda_2, f_D = \lambda_1, \boldsymbol{T}_S = \boldsymbol{u}_2\boldsymbol{u}_2^H, \boldsymbol{T}_D = \boldsymbol{u}_1\boldsymbol{u}_1^H$
8:	**ENDIF**
9:	$P_S = f_S$, $P_D = f_D$, $P_V = 4f_V$.
OUPPUT:	$P_S, P_D, P_V, \boldsymbol{T}_S, \boldsymbol{T}_D$

4. Experiment

In this section, the PolSAR data acquired by JAXA's ALOS-PALSAR over the Kyoto city of Japan is selected for method validation. The original data format is the single-look complex (SLC) scattering matrix with a resolution of 30m in the range direction and 5m in the azimuth direction. In order to obtain the second order CM data, a spatial multilooking has been performed by ensemble averaging 12×2 (azimuth×range) pixels. The final image thus possesses an approximately square spacing of 60m in both the range and azimuth directions. Fig. 1 shows the decomposition result by the proposed method. For comparison, two more results by the original Freeman decomposition [1] and the Freeman decomposition with orientation angle compensation [4] have been also shown alongside in Fig. 1. It can be seen that the proposed method has the best discrimination in the urban area with the strongest double-bounce scattering power (red) therein. We also note that positive powers are guaranteed over the entire image. However, the original Freeman decomposition appears to over-estimate the volume scattering power by which parts of the urban areas are wrongly determined as volume scattering dominant (green). It also produces negative power for 131814 in total of 958460 pixels of the image. On the other hand, the Freeman decomposition with orientation angel compensation shows performance improvement with an increased double-bounce scattering power in urban areas and reduced number of negative powers (38103 in total of 958460 pixels) but negative power yet remains.

In addition, it needs to be emphasized again that the decomposition result by the proposed method is exact in that the solution gives a perfect match to the measured CM data. The Freeman decomposition either with or without orientation angle compensation only gives a partial fit to the data by ignoring certain elements.

5. Conclusion

In this paper, we have proposed an exact method for decomposing the PolSAR CM data. We showed that the matrix expansion problem can be conveniently accomplished by solving a generalized eigenvalue problem. The method does not rely on azimuthal reflection symmetry and it gives an accurate expansion of the measured CM. Consequently, no information is lost in the decomposition result based on which further quantitative analysis can be reliably conducted.

Appendix

In this appendix, we briefly prove that the eigenvalues of the generalized eigenvalue problem of (5) are all positive. First note that by eigen-decomposition $\boldsymbol{T} = \boldsymbol{V\Sigma V}^H = \boldsymbol{MM}^H$ where $\boldsymbol{M} = \boldsymbol{V\Sigma}^{1/2}$. Then (5) can be equivalently written as the following standard eigenvalue problem:

$$\boldsymbol{M}^H \boldsymbol{T}_V^{-1} \boldsymbol{M}\boldsymbol{y} = f_V \boldsymbol{y}, \tag{A1}$$

where $\boldsymbol{y} = \boldsymbol{M}^H \boldsymbol{x}$. Recall that $\boldsymbol{T}_V = \mathrm{diag}(2,1,1)$ is a positive-definite matrix. Then it can be easily proved that $\boldsymbol{M}^H \boldsymbol{T}_V^{-1} \boldsymbol{M}$ is also positive-definite. Hence all its eigenvalues are positive.

Next we prove that if λ_{min} is the minimum eigenvalue of (A1), then $\boldsymbol{T} - \lambda_{\min}\boldsymbol{T}_V$ is positive-semidefinite. By eigen-decomposition of $\boldsymbol{M}^H \boldsymbol{T}_V^{-1} \boldsymbol{M}$ we have $\boldsymbol{M}^H \boldsymbol{T}_V^{-1} \boldsymbol{M} = \boldsymbol{U\Lambda U}^H$. Thus $\boldsymbol{T}_V = \boldsymbol{MU\Lambda}^{-1}\boldsymbol{U}^H\boldsymbol{M}^H$. Consequently we have:

$$\boldsymbol{T} - \lambda_{\min}\boldsymbol{T}_V = \boldsymbol{MU}\left(\boldsymbol{I} - \lambda_{\min}\boldsymbol{\Lambda}^{-1}\right)\boldsymbol{U}^H\boldsymbol{M}^H. \tag{A2}$$

Remember that $\lambda_{\min}$ is the minimum eigenvalue in $\boldsymbol{\Lambda}$. Hence it can easily proved that $\boldsymbol{T} - \lambda_{\min}\boldsymbol{T}_V$ is a positive-semidefinite matrix of rank 2.

Acknowledgments

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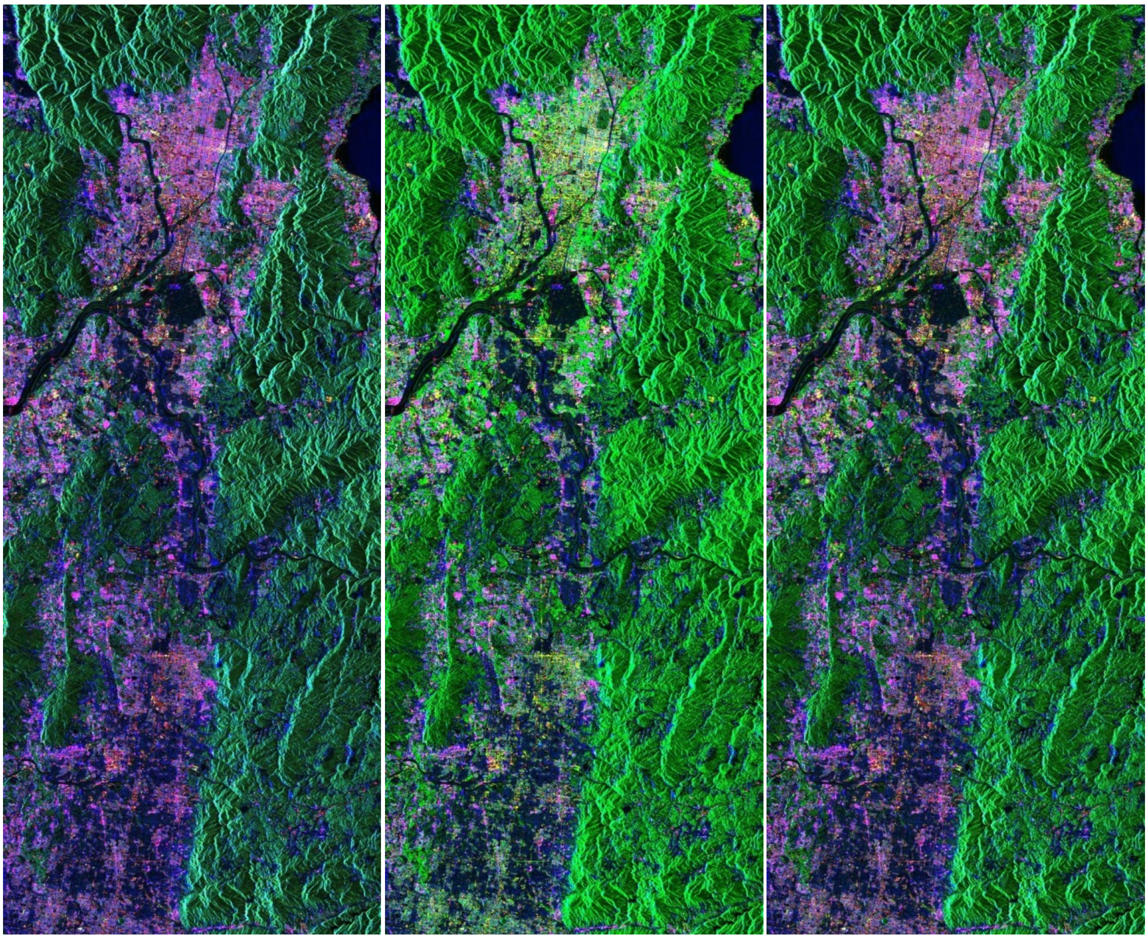

Figure 1: Decomposition Results by Proposed Method (Left), Original Freeman Decomposition (Middle), and Freeman Decomposition with Orientation Angle Compensation (Right).