FFLO state and anomalous flux quantization in the one-dimensional attractive Hubbard models with imbalanced spin populations

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We examine anomalous flux quantization of the one-dimensional attractive Hubbard models with imbalanced spin populations by using the exact diagonalization method. In the single chain Hubbard model with sufficiently large attractive interaction and imbalanced spin populations, the period of the flux quanta is determined by the difference between the system size N_L and electron number N_e as $h/(N_L - N_e)e$, in contrast to the superconducting flux quanta of $h/2e$ based on usual Cooper pairs and/or the period $h/4e$ derived by the mean-field theory. We find that similar anomalous flux quantization appears in the zigzag and ladder Hubbard models with imbalanced spin populations, when the band structure near the Fermi levels is regarded to be almost equivalent to that of the single chain Hubbard model.

KEYWORDS: FFLO state, flux quantization, spin imbalance, attractive Hubbard model, exact diagonalization

1. Introduction

Recently, Yoshida and Yanase [1] showed that the so-called Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state [2, 3] indicates an anomalous flux quantization of period $h/4e$, which is a half of the superconducting flux quantum $h/2e$. They analyzed the one-dimensional (1D) attractive Hubbard chain with 200 sites by using the mean-field approximation based on the Bogoliubov-de Gennes equation in the weak-coupling region [1]. Here the FFLO state is characterized by the formation of Cooper pairs with finite center-of-mass momentum caused by the imbalance of the Fermi surfaces of two-component fermions and exhibits inhomogeneous superconducting phases with a spatially oscillating order parameter [4–13].

It is known that the strong quantum fluctuation effect is crucial for low-dimensional systems such as the 1D mesoscopic chain [14–19] and the strong coupling theory is more suitable for the experimental situation of the FFLO state realized in ultracold atomic gases [20–23]. However, most of the theoretical works for the FFLO state have been performed within the mean-field approximation whose application is limited in the weak-coupling region. To clarify the strong coupling effects, we think that the nonperturbative and reliable approach beyond the mean-field approximation is required.

In the previous work [24], we investigated the 1D Hubbard chain with the attractive interaction in the presence of the spin imbalance by using the exact diagonalization (ED) method for finite-size systems. To analyze the anomalous flux quantization, we numerically calculated the periodicity of the ground-state energy $E(\Phi)$ with respect to the magnetic flux Φ without any approximation. When the absolute value of the attractive interaction $|U|$ is sufficiently larger than the hopping energy of electrons, the period of the flux quanta was found to be determined by the difference between the system size N_L and electron number N_e as $h/(N_L - N_e)e$ [24]. The result is a contrast to the superconducting flux quanta of $h/2e$ based on usual Cooper pairs and/or the period $h/4e$ claimed by the analysis of the mean-field theory for the FFLO state [1]. It is noted that this anomalous behavior is different with the the anomalous flux quantization discussed in the repulsive ($U > 0$) Hubbard chain in the strong correlation regime with $U \gg n|t|$, where $E(\Phi)$ shows oscillation with a period of h/N_e [25, 26].

In the present study, we investigate the anomalous flux quantization of the zigzag and ladder Hubbard models with the attractive interaction, in addition to the single chain model by using ED method. Although our calculation is restricted to small systems, we think that the essential features of the anomalous flux quantization in the FFLO state can be well described even in finite-size systems [7–12, 14, 15].

2. Model and Formulation

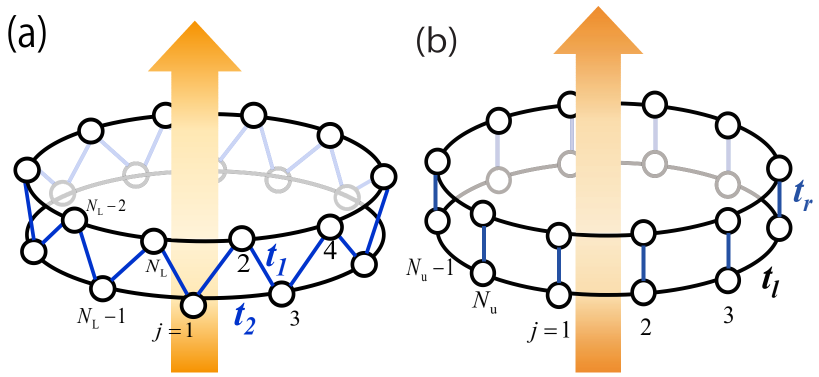

Fig. 1. (Color online)Schematic diagrams of (a) the zigzag Hubbard model and (b) the ladder Hubbard model under the magnetic flux Φ.

We consider the zigzag 1D Hubbard chain given by the following Hamiltonian:

$$H = -t_1 \sum_{i,\sigma} (e^{i2\pi\Phi/N_L} c^\dagger_{i,\sigma} c_{i+1,\sigma} + h.c.) - t_2 \sum_{i,\sigma} (e^{i4\pi\Phi/N_L} c^\dagger_{i,\sigma} c_{i+2,\sigma} + h.c.) - |U| \sum_i n_{i,\uparrow} n_{i,\downarrow}, \quad (1)$$

where $c^\dagger_{i,\sigma}$ stands for the creation operator for an electron with spin σ $(=\uparrow,\downarrow)$ at site i and $n_{i,\sigma} = c^\dagger_{i,\sigma} c_{i,\sigma}$. Here, t_1 and t_2 represent the hopping integral between nearest-neighbor sites and next-nearest-neighbor (NNN) sites, respectively. Φ corresponds to the magnetic flux through the chain measured in units of h/e, and N_L is the system size. The interaction parameter $|U|$ stands for the strength of the attractive interaction on the site.

We also treat the Hubbard ladder model as follows:

$$H = -t_l \sum_{i,m,\sigma} (e^{i2\pi\Phi/N_u} c^\dagger_{i,m,\sigma} c_{i+1,m,\sigma} + h.c.) - t_r \sum_{i,\sigma} (c^\dagger_{i,1,\sigma} c_{i,2,\sigma} + h.c.) - |U| \sum_{i,m} n_{i,m,\uparrow} n_{i,m,\downarrow} \quad (2)$$

where $c^\dagger_{i,m,\sigma}$ stands for the creation operator for an electron with spin σ at site (i, m). Here, $m(= 1, 2)$ denotes the index of legs and i is the position on rungs. Here, t_l and t_r represent the hopping integral between nearest-neighbor rungs and intra rung, respectively. N_u indicates the number of the rungs and corresponds to half of the number of the total sites. Schematic diagrams of these models are given Fig. 1.

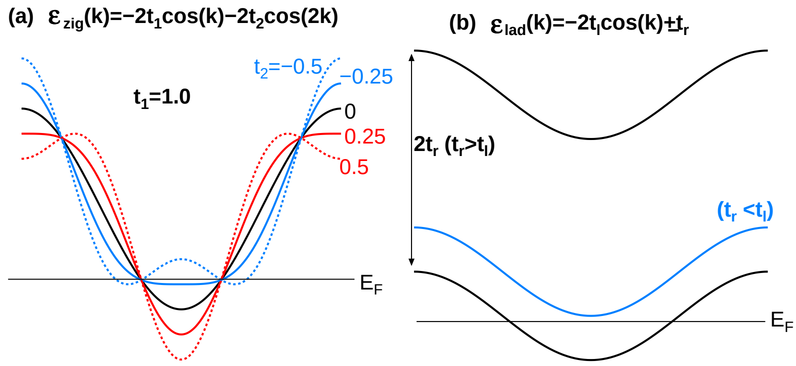

Fig. 2. (Color online) Dispersion relation $\epsilon(k)$ of (a) the zigzag chain model with several values of t_2 and (b) the ladder model at a typical value of t_r, where Fermi energy E_F is set to correspond to $n = 0.5$.

We numerically diagonalize the zigzag-chain Hamiltonian [eq. (1)] of up to 8 sites, and the ladder model Hamiltonian [eq. (2)] of up to 8 rungs, using the standard Lanczos algorithm. If flux quantization appears, the lowest energy level is expected to alternate and oscillate with increasing Φ [14, 15]. To carry out a systematic calculation, we use the periodic boundary condition when $N_\uparrow$ and $N_\downarrow$ are odd numbers and the antiperiodic boundary condition when they are even, where $N_\uparrow$ and $N_\downarrow$ are the total numbers of up- and down-spin electrons, respectively [16]. The filling n of electrons is given by $n = N_e/N_L$ for the zigzag chain, and $n = N_e/N_r$ for the ladder chain, where $N_e(= N_\uparrow + N_\downarrow)$ is the total number of electrons, and the spin imbalance is defined by $p = \frac{N_\uparrow - N_\downarrow}{N_e}$.

In the noninteracting case ($U = 0$), these Hamiltonians are easily diagonalized and yield dispersion relations representing for the zigzag chain model $\epsilon_{\text{zig}}(k) = -2t_1\cos(k) - 2t_2\cos(2k)$, and for the ladder model $\epsilon_{\text{lad}}(k) = -2t_r\cos(k) \pm t_l$, where k is the wavenumber and these dispersion relations are depicted for several parameters in Fig. 2. Hereafter, we set $t_1 = 1$ and $t_l = 1$.

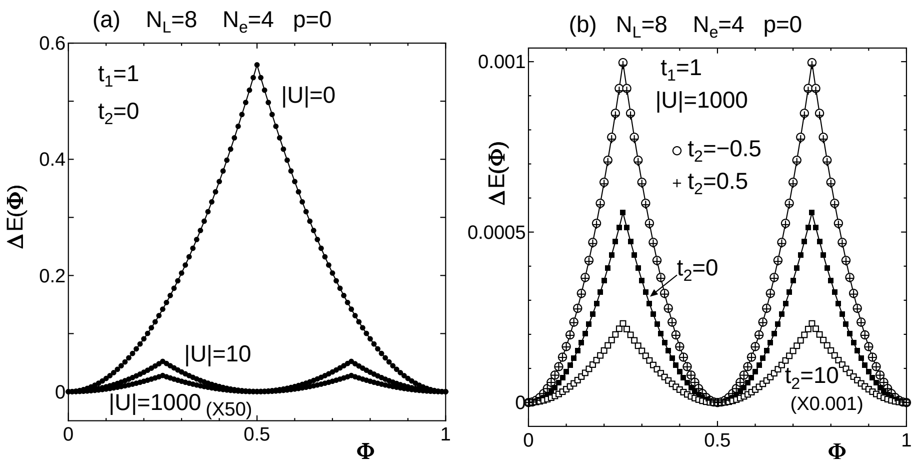

Fig. 3. Difference in the ground-state energy $\Delta E = E(\Phi) - E(0)$ as a function of the magnetic flux Φ with $p = 0$ and n=0.5 (4 electrons/8 sites) for (a) $|U| = 0, 10, 1000$ at $t_2 = 0$, and (b) $t_2 = -0.5, 0, 0.5, 10$ at $|U| = 1000$, respectively. Here, the open circle denotes the result of $t_2 = -0.5$ and the cross is that of $t_2 = 0.5$.

3. Numerical Results

3.1 Zigzag model

At first, we examine the flux quantization of the zigzag Hubbard model as shown in fig. 1(a). Figure 3(a) shows the difference in the ground-state energy $\Delta E(\Phi) = E(\Phi) - E(0)$ as a function of Φ at quarter filling n=0.5 (4 electrons/8 sites) for $|U| = 0$, 10 and 1000 in the absence of spin imbalance, i.e., $p = 0$. When $t_2 = 0$, the model reduces the usual Hubbard chain and we expect the usual flux quantization of Cooper pairs with zero momentum, i.e. the period $h/2e$ [14, 15]. We can see that the energy levels cross at $\Phi \sim 0.25$ and 0.75 for all values of the attractive interaction ($|U| = 10, 1000$), where the usual superconducting flux quantization of period $h/2e$ ($= 0.5$ in the present unit of $h/e = 1$) is observed. The energy scale of the flux quantization of period $h/2e$ seems to be proportional to $\sim 1/|U|$, because the energy scale of the pair hopping is considered to be given as $\sim t_1^2/|U|$.

Figure 3(b) shows the result of $t_2 = -0.5, 0, 0.5$ and 10 at $|U| = 1000$ for $p = 0$. We find that the flux quantization is almost independent of t_2. Especially, the difference between the result of $t_2 = -0.5$ and 0.5 is very small and it is hard to distinguish both cases in this figure. When $t_2 = 10$, the system should be considered as a double chain connected by electron hopping t_1. It also clearly indicates the flux quantization of $h/2e$.

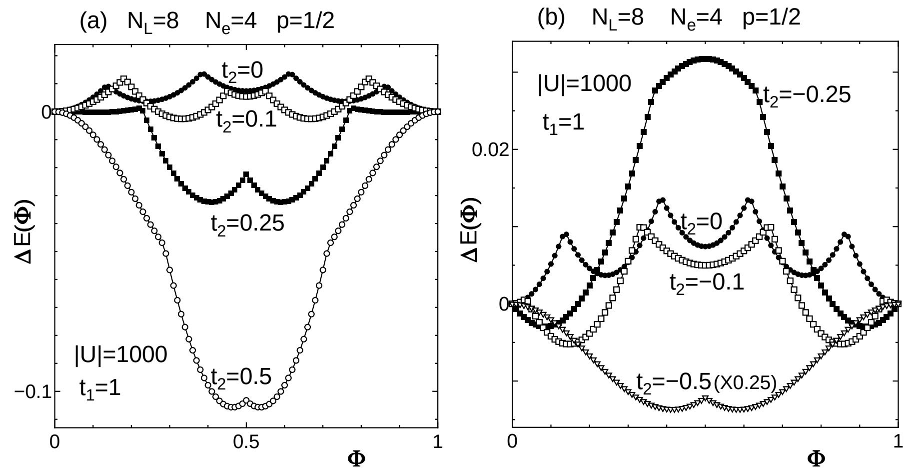

Fig. 4. Difference in the ground-state energy $\Delta E = E(\Phi) - E(0)$ as a function of the magnetic flux Φ with $p = 0.5$ at n=0.5 (4 electrons/8 sites) for (a) t_2=0, 0.1, 0.25, 0.5 and (b) t_2=0, -0.1, -0.25, -0.5.

Figure 4(a) gives the $\Delta E(\Phi)$ for t_2=0, 0.1, 0.25, 0.5 in the presence of spin-imbalance with $p = 0.5$. Our previous work [24] has already pointed out that the period of the flux quantization is given by $h/(N_L - N_e)e$ for $t_2 = 0$ with sufficiently large negative interaction $|U|$. The result is reconfirmed in Fig4(a), which indicates the period to be $h/4e$ with $N_L - N_e = 4$. It indicates that the anomalous flux quantization of period $h/4e$ survives, although its shape is fairly deformed for t_2=0.25, in contrast with the usual flux quantization of period $h/2e$. When $t_2 = 0.5$, the NNN hopping prevents the appearance of the anomalous flux quantization and it is difficult to find the sign of the anomalous flux quantization in the Φ dependence of $E(\Phi)$.

Figure 4(b) shows $\Delta E(\Phi)$ in the case of t_2=0, -0.1, -0.25, -0.5. In this case, deformation of $\Delta E(\Phi)$ as a function of Φ becomes significant for $t_2 \gtrsim 0.25$ and the anomalous flux quantization vanishes even for $t_2 = 0.25$. It indicates that the influence of the NNN hopping upon the anomalous flux quantization is larger than the case t_2 being positive. It may be caused by the band structure that the change at E_F for $t_2 < 0$ is larger that for $t_2 > 0$ as shown in Fig.2. The result also suggests that the anomalous flux quantization is sensitive to the band structure.

3.2 Ladder model

Next, we address the case of Ladder model depicted in Fig.1(b), where each of the upper and lower bands is completely equivalent to the band of the single chain as indicated in the dispersion relation of $\epsilon_{\rm lad}(k)$ in the noninteracting model. Here, we note that the number of unit cells N_u is corresponding to the N_L in the chain model.

Figure 5 gives the $\Delta E(\Phi)$ as a function of Φ of the ladder model at $p = 1/2$ with the result of the single chain model. It clearly indicates that the period of the anomalous flux quantization appears as $h/(N_u - N_e)e$ for $t_r/t_l = 10$ at $|U| = 500$, where this period agrees with that of the single chain with the same parameters [24].

On the other hand, the anomalous periodicity completely disappears for $t_r/t_l = 0.1$ and 1. In the case of $t_r/t_l = 0.1$, the band gap between the upper and lower bands of the ladder model, which is yielded by $2t_r - 2t_l$, vanishes and both bands fairly close each other. Then, four Fermi points appear at E_F as a typical 1D two-band system and the electronic state is considered to be different from that of the single band model. However, in the case of $t_r/t_l = 1.0$, the band structure at E_F seems to be almost equivalent to that of the single chain model with two Fermi points. It is strange and interesting that the anomalous flux quantization vanishes at $t_r/t_l = 1.0$. We have also confirmed that the anomalous flux quantization is not clearly observed even if $t_r/t_l = 5.0$(not shown). The result suggests that the large band gap is crucial to the periodicity of the anomalous flux quantization of $h/(N_u - N_e)e$. Further, the anomalous flux quantization is not determined by only the electronic state at E_F, but relates to that of the whole band structure.

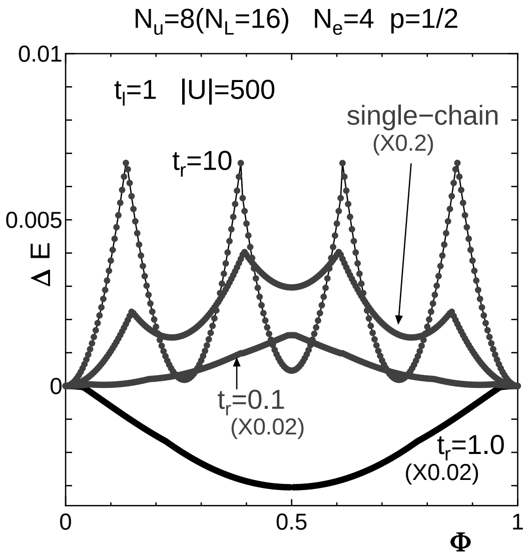

Fig. 5. (Color online) Difference in the ground-state energy $\Delta E = E(\Phi) - E(0)$ as a function of the magnetic flux Φ with $p = 1/2$ at n=0.5 (4 electrons/8 units) and $|U| = 500$ for $t_r = 0.1$, 1.0 and 10. Here, the result of the single chain is also added to compare with the ladder model at the same parameters.

4. Summary

We sturdy the anomalous flux quantization of the zigzag Hubbard and the ladder Hubbard models with large attractive interaction and imbalanced spin populations by using the exact diagonalization method. We found that the period of the flux quanta are determined as $h/(N_L - N_e)e$ for zigzag model, and $h/(N_u - N_e)e$ for ladder model. These behavior are corresponding to the result of the anomalous flux quantization obtained in the single chain Hubbard model. However, the condition of the anomalous flux quantization is restricted to the case of the electronic state is regarded to be close to that of the single chain model.

At this stage, we could not explain the mechanism of the anomalous flux quantization of the zigzag model and/or the ladder model except the case of the single chain model [24]. Further, it is not clear how our results relate to the result of the mean-field analysis [1], and whether the anomalous flux quantization appears in the other models such as ladder model with three or more legs and the two-dimensional model [13]. These problems may be very interesting and we will address them in future.

Acknowledgments

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