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Performance of Parallel Combinatory SS Communication Systems in Rayleigh Fading Channel

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SUMMARY The performance of parallel combinatory spread spectrum (PC/SS) communication systems in the frequency-nonselective, slowly Rayleigh fading channel is studied. Performance is evaluated by symbol error rate using numerical computation. To overcome the performance degradation caused by fading, we also studied the effects of selection diversity and Reed-Solomon coding applied to the PC/SS system. As a result, a remarkable improvement in error rate performance is achieved with Reed-Solomon coding and diversity technique. The coding rate for the maximum coding gain is almost a half of that in the additive white gaussian noise channel.
key words: *spread spectrum technology, Rayleigh fading, selection diversity, Reed-Solomon codes*

1. Introduction

Recently, spread spectrum (SS) technique receives a growing interest as an effective way to provide high channel capacity for outdoor and indoor cellular mobile communication systems [1].

In Japan, 2.4 GHz ISM band was opened to SS systems at the end of 1992, many studies concerned on SS wireless information networks have been started.

Generally speaking, radio channel has a limited bandwidth. Thus, if we wish to achieve high-speed data transmission by SS technique, a part of the advantage of using SS technique could be lost because of insufficient spectrum spreading. To maintain performance merit offered by spectrum spreading, we need to develop a new SS technique with high data transmission rate in a limited bandwidth.

For this purpose, parallel combinatory spread spectrum (PC/SS) communication systems have been proposed [2], [3]. In the PC/SS system, several pseudo-noise (PN) sequences are simultaneously transmitted out of a pre-assigned spreading sequence set. The transmitting PN sequences are selected from the sequence set corresponding to the state of a set of data bits. Up to now, some performance analysis was reported for the additive white gaussian noise (AWGN) environment [2]-[4].

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This paper addresses the symbol error rate performance of the PC/SS communication system in the frequency-nonselective, slowly Rayleigh fading channel. To reduce the performance degradation caused by fading, it is well known that the diversity technique and the error-control coding are very effective. So, the performance improvement by using Reed-Solomon coding and selection diversity technique is also studied.

In the next section, a system model of the PC/SS system and a channel model are defined. In Sect. 3, we analyze the symbol error rate performance of the PC/SS system in the Rayleigh fading channel. Performance of the PC/SS system with the selection diversity and Reed-Solomon coding is also studied. Numerical results and discussion are described in Sect. 4, and we conclude our results in Sect. 5.

2. System and Channel Model

2.1 PC/SS Transmitter Model

In the transmitter, a set of M orthogonal sequences with chip duration T_c is assigned. That is,

$$\boldsymbol{a}=(\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \cdots, \boldsymbol{a}_M) \tag{1}$$

$$\boldsymbol{a}_i=(a_{i,0}, a_{i,1}, a_{i,2}, \cdots, a_{i,(N-1)}) \quad a_{i,j}\in\{-1, +1\}$$

where $a_{i,j}$ is the jth element of ith spreading sequence. N represents the length of assigned sequence. The input sequence of data with duration T_d is described as

$$\boldsymbol{d}_{in}=(d_1, d_2, \cdots, d_k) \quad d_i\in\{0, 1\} \tag{2}$$

The data in (2) is converted to the data of k parallel channels with duration $T(=kT_d=NT_c)$. In the mapping circuit, r transmitting PN sequences are chosen from the assigned M orthogonal PN sequences which are assigned for a particular user.

The mapping method carried out as follows. First, we split $\boldsymbol{d}_{in}$ into two parts:

$$\boldsymbol{d}_s=(d_1, d_2, \cdots, d_r) \tag{3}$$

$$\boldsymbol{d}_c=(d_{r+1}, d_{r+2}, \cdots, d_k) \tag{4}$$

The sequence $(d_1, d_2, \cdots, d_r)$ represents the $+$ or $-$ signature of each transmitting PN sequence. A state of $\boldsymbol{d}_c$ specifies a combination of r transmitting PN sequences among the assigned sequences.

To select a set of PN sequences to be transmitted, $\boldsymbol{d}_c$ is coded into a constant weight code of length M and weight r. This is referred to as (M, r) constant weight code, and is written by

$$\boldsymbol{c}=(c_1, c_2, c_3, \cdots, c_M) \tag{5}$$
$$c_i \in \{0, 1\}$$

A set of r PN sequences for simultaneous transmission is determined such that by choosing ith PN sequence when $c_i=1$. The set of transmitting PN sequences is expressed as

$$\boldsymbol{v}=(\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_r). \tag{6}$$
$$\boldsymbol{v}_i=(v_{i,0}, v_{i,1}, v_{i,2}, \cdots, v_{i,(N-1)}).$$

The signature of a transmitted PN sequence is obtained from (3). A signature sequence for a set of transmitting sequence is written as

$$\boldsymbol{b}=(b_1, b_2, b_3, \cdots, b_r). \tag{7}$$
$$b_i=(-1)^{d_i} \quad i=1, 2, \cdots, r$$

Next, the above state word of transmitting sequence is multiplied with a PN sequence set to form a transmitting signal as follows.

$$s_c(t)=\sum_{i=1}^{r} b_i \sum_{j=0}^{N-1} v_{i,j} P_{T_c}(t-jT_c) \tag{8}$$

$P_{T_c}(\cdot)$ represents the chip waveform. If the chip waveform is the rectangular pulse, it is given by

$$P_{T_c}(t)=\begin{cases} 1 & 0 \leqq t < T_c \\ 0 & t<0, t \geqq T_c \end{cases} \tag{9}$$

$s_c(t)$ is a multi-level signal with $(r+1)$ levels. Multiplying it with a carrier, the transmitter output is obtained as

$$s(t)=s_c(t) \cos \omega_c t \tag{10}$$

where ω_c is the carrier frequency. For simplicity, the carrier phase is assumed to be zero. The system configuration is described in Ref. [3].

2.2 Channel Model

In Ref. [5], the multipath fading channel is represented by the following simple channel model. The low-pass equivalent impulse response of the fading channel is given by

$$h(t)=\sum_{l=1}^{L} \beta_l e^{j\theta_l} \delta(t-\tau_l) \tag{11}$$

where β, τ, θ are path gain, path delay and path phase, respectively. L is the number of resolvable paths. In this paper, slowly, frequency-nonselective fading is assumed, so that $L=1$ as in (11). Thus, the receiver input is of the form

$$r(t)=\beta e^{j\theta} s(t-\tau)+\eta(t), \tag{12}$$

where $\eta(t)$ is the additive white gaussian noise. We assume that β has the Rayleigh distribution and θ has the uniform distribution.

2.3 Receiver Model

At the receiver, M matched filters are used to despread the received signal. A matched filter is matched to the assigned spreading sequence which is the same with the counterpart in a transmitter. The matched filter output is described as

$$\boldsymbol{y}=(y_1, y_2, y_3, \cdots, y_M) \tag{13}$$

$$y_i=\int_0^T r(t)\, a_i(t) \cos \omega_c t dt \tag{14}$$

$$a_i(t)=\sum_{j=0}^{N-1} a_{i,j} P_{T_c}(t-jT_c)$$

In the decision circuit, the r-out-of-M combination of transmitted PN sequences is estimated from the matched filter outputs. According to the descending order of the squared magnitude of y_i, r elements of y_i^2 are decoded to '1', and the others are decoded to '0'. The estimate is transformed into a constant weight code

$$\boldsymbol{c}'=(c_1', c_2', c_3', \cdots, c_M') \tag{15}$$

Passing $\boldsymbol{c}'$ through the (M, r) constant weight decoder, the data are decoded into

$$\boldsymbol{Z}_c=(Z_{r+1}, Z_{r+2}, Z_{r+3}, \cdots, Z_M). \tag{16}$$

From the matched filter outputs in (13) and constant weight code $\boldsymbol{c}'$, we get the signature-dependent data

$$\boldsymbol{Z}_s=(Z_1, Z_2, Z_3, \cdots, Z_r) \tag{17}$$

$$\begin{cases} z_i=1 & \text{if} \quad y_i \geqq 0 \\ z_i=0 & \text{otherwise} \quad i=1, 2, \cdots, r \end{cases} \tag{18}$$

Finaly, the receiver output is obtained through parallel to serial conversion.

In this paper, we assume the perfect synchronization of carrier and PN sequences between a transmitter and a receiver.

3. Performance Analysis

The performance of the PC/SS system described above is evaluated in terms of symbol error rate (SER) characteristics. Assuming complete coherent detection in the receiver, the SER of the PC/SS system in an additive white gaussian noise channel is obtained as [3]:

$$P_e=1-\left\{\int_0^\infty \chi^2\left(u;1,\frac{2E_s}{rN_0}\right)\right.$$

$$\cdot\left[\int_0^u \chi^2(v;1)\,dv\right]^{M-r} du\Bigg\}^r \tag{19}$$

where $\chi^2(u;m,\lambda)$ and $\chi^2(u;m)$ are the probability density functions (*pdf*) of u which have noncentral and central chi-square distribution with m degrees of freedom and noncentral parameter λ, respectively. E_s denotes the energy per symbol, and N_0 is the one-sided spectral density of the white gaussian noise.

In a fading environment, the received power varies over an extremely wide range. If M and r are constant in (19), SER is written as a function of γ defined by E_s/N_0. Generally, the SER performance in the fading channel is given by

$$P_e'=\int_0^\infty P_e(\gamma)p(\gamma)\,d\gamma \tag{20}$$

when $p(\gamma)$ is the *pdf* of the input symbol energy-to-noise ratio in the receiver. The Rayleigh fading *pdf* of γ is written by

$$p(\gamma)=\frac{1}{\Gamma}\exp\left(-\frac{\gamma}{\Gamma}\right), \tag{21}$$

where Γ is the average symbol energy-to-noise ratio. Substituting (19) and (21) into (20), the SER is obtained as

$$\begin{aligned}P_e'&=\frac{1}{\Gamma}\int_0^\infty P_e(\gamma)\exp\left(-\frac{\gamma}{\Gamma}\right)d\gamma\\&=\frac{1}{\Gamma}\int_0^\infty \exp\left(-\frac{\gamma}{\Gamma}\right)\cdot\left(1-\left\{\int_0^\infty \chi^2\left(u;1,\frac{2\gamma}{r}\right)\right.\right.\\&\qquad\left.\left.\cdot\left[\int_0^u \chi^2(v;1)\,dv\right]^{M-r} du\right\}^r\right)d\gamma.\end{aligned} \tag{22}$$

In the fading environment, the SER performance degradation is so large, since there is large channel attenuation. To improve the performance degradation, the diversity techniques and error control coding are very effective.

A. Performance of the PC/SS system with selection diversity

It is possible to reduce the influence of large channel attenuation by using diversity techniques. One of these methods is to choose the strongest signal from a set of signals carrying the same information. It is called selection diversity, and its implementation is easy.

The received signal of each branch is assumed to be equally distributed and mutually independent. The Rayleigh fading *pdf* of $\gamma=E_s/N_0$ with selection diversity is expressed as

$$p(\gamma)=\frac{J}{\Gamma}\exp\left(-\frac{\gamma}{\Gamma}\right)\left\{1-\exp\left(-\frac{\gamma}{\Gamma}\right)\right\}^{J-1} \tag{23}$$

where J is the number of branches and Γ is the average SNR of each branch. Substituting (23) into $p(\gamma)$ of (20), we get the SER as follows:

$$\begin{aligned}P_e'&=\frac{J}{\Gamma}\int_0^\infty \exp\left(-\frac{\gamma}{\Gamma}\right)\left\{1-\exp\left(-\frac{\gamma}{\Gamma}\right)\right\}^{J-1}\\&\quad\cdot\left(1-\left\{\int_0^\infty \chi^2\left(u;1,\frac{2\gamma}{r}\right)\right.\right.\\&\quad\left.\left.\cdot\left[\int_0^u \chi^2(v;1)\,dv\right]^{M-r} du\right\}^r\right)d\gamma.\end{aligned} \tag{24}$$

B. Performance of the PC/SS system with Reed-Solomon coding

It is advantageous to combine the error control coding with the PC/SS technique, to improve the error rate performance. Since the PC/SS system transmits k bits of data during a single PN period, the 2^k-ary Reed-Solomon (RS) coding scheme brings about a considerable improvement of error rate performance in AWGN channel (See Ref. [4]). Hence, we study the effect of RS coding in the PC/SS system in fading channels.

Applying bounded distance decoding, we find the expression of the SER P_e'':

$$\begin{aligned}P_e''&=\frac{1}{n}\sum_{j=2h+1}^{n} j\cdot A_j\sum_{i=h+1}^{n}\sum_{d=0}^{h}\phi(i,j,d)P_c^{n-i}P_{e1}^i\\&\quad+\frac{1}{n}\sum_{i=h+1}^{n}\left({}_nC_i q^i-\sum_{j=2h+1}^{n}\sum_{d=0}^{h}\phi(i,j,d)A_j\right)\\&\quad\cdot i\cdot P_c^{n-i}P_{e1}^i\end{aligned} \tag{25}$$

where

$$\phi(i,j,d)=\begin{cases}{}_jC_x\cdot{}_{n-j}C_{d-x}\\ \quad\text{if } x=(j-i+d)/2 \text{ is integer and}\\ \quad\min[2n-i-j,\,i+j]\geqq d\leqq|j-i|\\ 0\quad\text{otherwise}\end{cases} \tag{26}$$

P_c and P_{e1} are written as

$$P_c=1-P_e \tag{27}$$

$$P_{e1}=P_e/(2^k-1) \tag{28}$$

respectively. h denotes the error correction capability of RS codes. Hence, the coding rate is given by *(n-2h)/n*. A_j is the number of codes with weight j.

4. Numerical Results and Discussion

Numerical results of the SER performance are shown in this section. First we try to find the optimum coding rate of RS coded PC/SS systems. In a coded PC/SS system, the SNR per symbol is controlled by coding rate. There is a trade-off relation between the SNR per symbol and the error correction capability. Thus, there is the optimum coding rate in an RS coded PC/SS system. Figure 1 shows the plot of SER vs. coding rate of the RS coded PC/SS system. The optimum coding rate varies with the average signal-to noise ratio per symbol. In Fig. 1, SER reaches the lowest when the coding rate stays in the range of 1/4 through 3/8. In

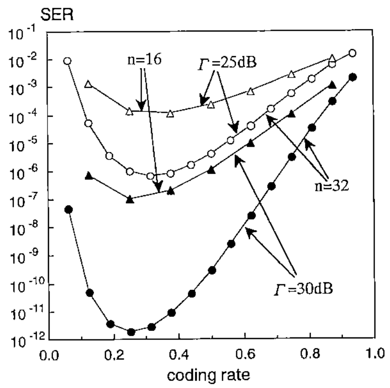

Fig. 1 SER vs. coding rate of the PC/SS system in Rayleigh fading channel ($M=16$, $r=2$, $k=8$).

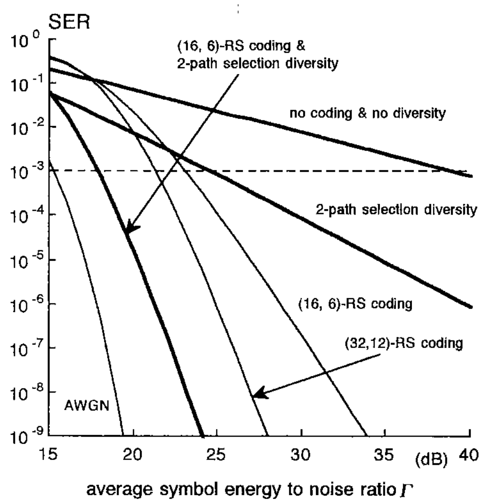

Fig. 2 SER of PC/SS system with 2-path selection diversity ($M=16$, $r=2$, $k=8$).

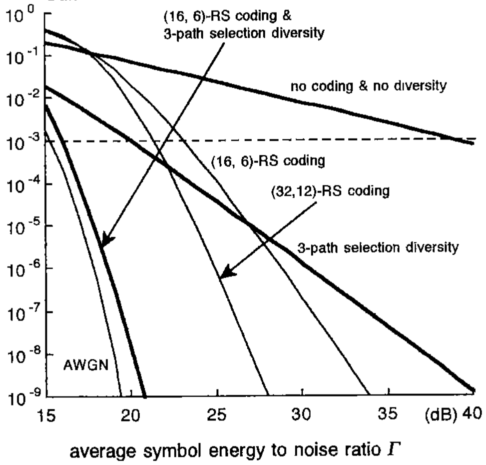

Fig. 3 SER of PC/SS system with 3-path selection diversity ($M=16$, $r=2$, $k=8$).

the AWGN channel, the optimum coding rate is almost 3/4 [4]. It is found that the optimum coding rate in this channel is almost half of that in the AWGN channel. The reason why dips appear in the plot is a direct consequence that fading channel demands higher error correction capability to reduce the error due to the attenuation of received power.

Figure 2 and Fig. 3 illustrate the symbol error rate vs. average SNR Γ of the PC/SS system in the Rayleigh fading channel. In the AWGN channel, error rate performance of the PC/SS system is better than that of conventional direct-sequence spread spectrum (DS/SS) systems if the signal-to noise ratio per bit is higher than 4 dB [3]. However, in fading channel, owing to the attenuation of received power, SER performance can be worse than conventional DS/SS systems: as found in Ref. [3], a critically lower symbol SNR environment can increase the error rate in the PC/SS system, because such an environment causes a severe degradation in the PC/SS system rather than the conventional DS/SS system [3]. However, by combining 2-path and 3-path selection diversity, required Γ decreases by 14 dB and 19 dB at 10^{-3} SER, respectively. This implies that more than 1/20 reduction is possible in terms of transmitting power by the diversity option. In other words, the attenuation of received power caused by fading is significantly improved. This improvement should come from the change of the probability distribution of received SNR, see Eqs. (21) and (23), involved with applying diversity for the Rayleigh fading channel. This change works better for the error rate performance in the PC/SS systems than that in the DS/SS systems [3].

Based on the result in Fig. 1, the SER performance in the cases of applying 3/8-rate RS coding is also illustrated in these plots. When (16, 6)-RS coding is applied to the no-diversity PC/SS system, required Γ decreases by 16 dB at 10^{-3} SER. When (32, 12)-RS code is applied, further reduction of 2 dB is possible at 10^{-3} SER compared with the case of (16, 6)-RS code. The combination of (16, 6)-RS coding and 2-path selection diversity yields about 3 dB reduction in required Γ compared to the case of applying (32, 12)-RS coding only. It is worth noting that the smaller the desired SER, the more reduction in the average SNR can be expected. Of course, from the viewpoint of system implementation, large system is needed by the diversity option. It is yet advantageous to apply just longer-code RS coding.

5. Conclusion

The symbol error rate performance of parallel combinatory SS communication systems over a Rayleigh fading channel is obtained. To address the basic performance in fading environment, we analyzed slowly, frequency-nonselective fading channels. In fading channel, the error rate performance of the PC/SS system is not always better than conventional DS/SS system because the PC/SS system is influenced by the attenuation of received power. However, selection diversity techniques provide considerable reduction of required average SNR in the PC/SS systems. It is also found that the RS coding improves the error rate performance against fading. The optimum coding rate of RS coding in the Rayleigh fading environment is almost a half of that for AWGN channel.

In the PC/SS system, acquisition and tracking problems of PN sequences are left for further research because a set of transmitting sequences constantly changes with the data symbol.

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