PAPER *Special Section on Surveys of Researches in CAS Fields in the Last Two Decades, II*

Development in Graph- and/or Network-Theoretic Research of Cellular Mobile Communication Channel Assignment Problems

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SUMMARY The demand for mobile communication services is rapidly increasing, because the mobile communication service is synonymy of an ideal communication style realizing communication in anytime, anywhere and with anyone. The development of economic and social activities is a primary factor of the increasing demand for mobile communication services. The demand stimulates the development of technology in mobile communication including personal communication services. Thus mobile communication has been one of the most active research in communications in the last several years. There exist various problems to which graph & network theory is applicable in mobile communication services [1]. (for example, channel assignment algorithm in cellular system, protocol in mobile communication networks and traffic control in mobile communication). A model of a cellular system has been formulated using a graph and it is known that the channel assignment problem is equivalent to the coloring problem of graph theory. Recently, two types of coloring problems on graphs or networks related to the channel assignment problem were proposed. Mainly, we introduce these coloring problems and show some results on these problems in this paper.

key words: *graph and network, coloring problem, mobile communication, channel assignment problem*

1. Introduction

In cellular mobile communication systems (we simply call a cellular mobile communication system a cellular system, hereafter), on which are based many of today's mobile communication schemes, efficient use of channels is a very important issue. In a cellular system, the service area is divided into many small cells and each channel is re-used in some cells, simultaneously. Channel assignment methods affect the system capacity and they are classified into Fixed Channel Assignment (FCA) and Dynamic Channel Assignment (DCA) [2]-[4]. In FCA, channels are permanently allocated to each cell. Basically, all channels are available in every cell in DCA. In general, DCA gives better performance than FCA. However, FCA is better than DCA under over load. The size of a cell becomes smaller in future cellular systems. As it becomes smaller, the variance of traffic fluctuations in each cell becomes larger. DCA is more flexible than FCA, so DCA can cope with the traffic fluctuations. Generally, to obtain the solution of the channel assignment problem is NP-hard [3]. So, approximate algorithms must be used in order to obtain the solution in a practical time when DCA is applied to the cellular system. Therefore, it is one of the most important problems to find the algorithms in a cellular system.

The usual model of a cellular system has been formulated using a graph [3], [5]. There exist the merits of this formulation using a graph as follows. In usual, a service area in a cellular system is divided into regular hexagon cells. A lot of studies on channel assignment in a cellular system deal with this regular location of cells. However, a cellular system may have different size cells and non-uniform interferences from cells to cells. The formulation using a graph deals with the cellular system with different size cells and non-uniform interference consistently. Moreover, we can apply results in graph & network theory to the assignment problem.

Recently, two types of coloring problems on graphs or networks related to the channel assignment problem were proposed. In this paper, we introduce these coloring problems and show some results on these problems. The channel scheme on one coloring problem is introduced by offsetting each channel by $1/k$ of its channel bandwidth. Another coloring problem takes the degree of cochannel interference into consideration. For general terminology in graph theory, we refer the reader to [6], [7].

2. Cell structure

There exist different structures of service areas (see Figs. 1(a), (b), ⋯, (f)). The service areas in Figs. 1 (b), (c), (d) are called a band shaped service area, ring shaped service area and tree shaped service area, respectively. The service area in Fig. 1(e) has three dimensional cells. This is a model of an in-building mobile communication. In Fig. 1(f), a cell is divided into three cells, called a higher cell, a middle cell and

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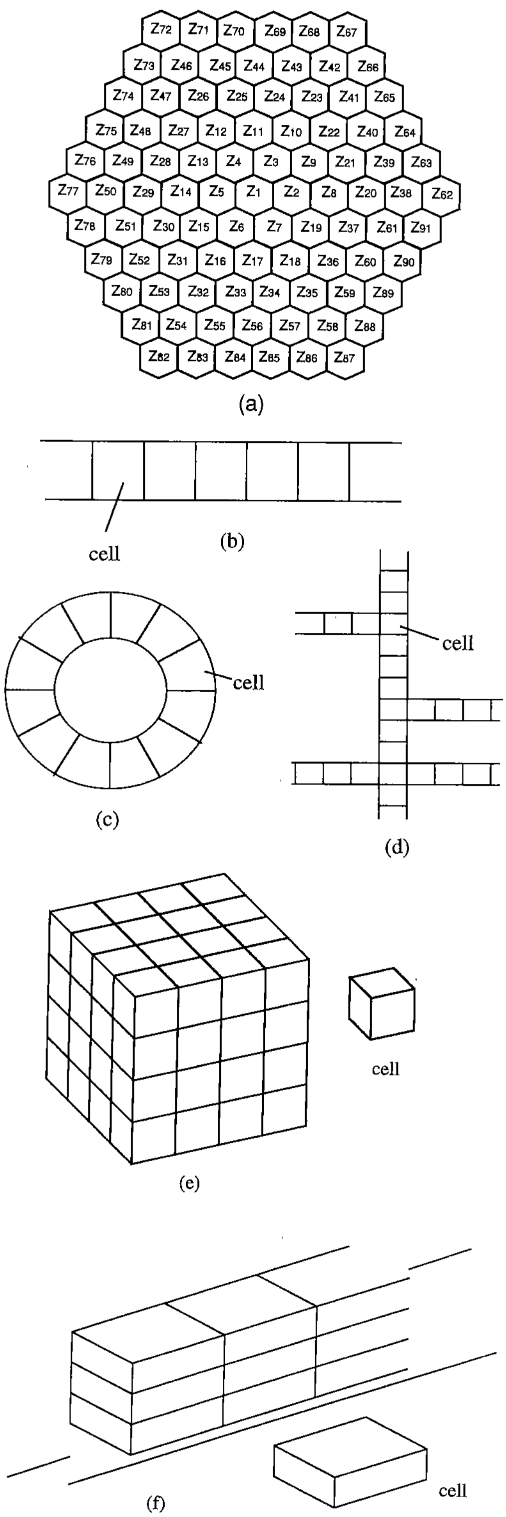

Fig. 1 Various structures of cells.

a lower cell. Large-sized cars such as buses use higher cells and small-sized cars such as private cars use middle or lower cells. Assuming radio propagation is on only roads in urban area, the service area is shown in Fig. 1(d). The service areas in Figs. 1(d), (e) and (f) are models of micro (or pico) cellular systems.

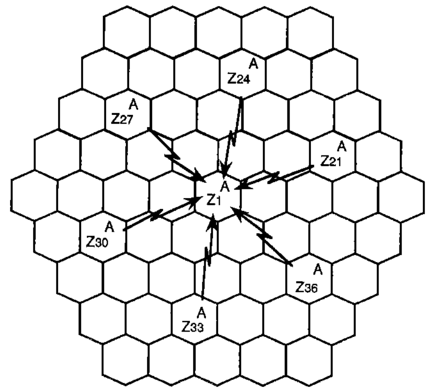

Fig. 2 Interference to z_1.

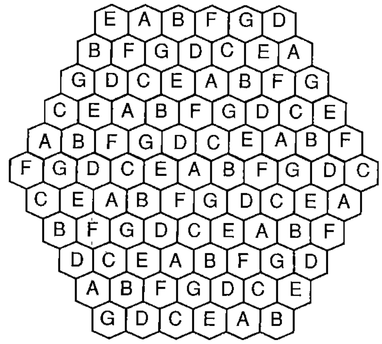

Fig. 3 A channel allocation.

If a channel frequency is used in a cell, it can not be used in some other cells. These other cells are called buffer cells of the cell. If the buffer cells of a given cell consist of all closer than k cells away from it, the buffering system is called the k-belt buffering. For example, we assume two-belt buffering system in Fig. 1 (a). cell z_{15} is a buffer cell of cell z_1. A channel A can be use in z_1, z_{21}, z_{24}, z_{27}, z_{30}, z_{33} and z_{36} at the same time (see Fig. 2). In the case of FCA and two-belt buffering, a channel allocation using channels A, B, C, D, E, F and G is shown in Fig. 3.

3. Usual Formulation Using a Graph

We construct an undirected graph G for a cellular system. G is a graph in which a vertex v_i corresponds to a cell z_i in the cellular system and an edge $e=(v_i, v_j)$ represents that the cochannel interference between cells z_i and z_j can not be neglected. The graph G is called the interference graph of the cellular system. An assignment of colors (element of some set) to vertices of a graph G, one color to each vertex, so that adjacent vertices are assigned different colors is called a coloring of G. For every coloring of G, the minimum number of colors is called the chromatic number of G denoted by $\chi(G)$. In the interference graph, assigning channels to cells is equal to assigning colors to vertices. So, in the formulation of a cellular system using a

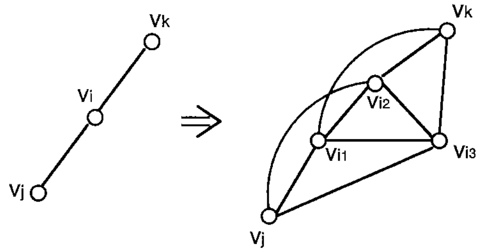

Fig. 4 A replacement of a vertex.

graph, the number of channels for any channel assignment for a cellular system, is not less than the chromatic number $\chi(G)$ of the interference graph G.

If some users are in a cell, we have to assign a channel to each user. In this case, we reconstruct or modify the interference graph as follows. We consider assigning t channels $(t \geqq 1)$ to a cell z_i. We replace the vertex v_i, with a complete graph with t vertices. An example of the replacement in the case of $t=3$ is shown in Fig. 4. Therefore, we consider only assigning a channel to each vertex of the interference graph hereafter.

4. Channel Offset Scheme

Usually, a spectrum which is divided into channels is shown in Fig. 5. Recently, a channel offset scheme has been introduced [8], [9]. A $1/k$ channel offset scheme is introduced by offsetting each channel by $1/k$ of its channel unit bandwidth. The 1/3 channel offset scheme is shown in Fig. 6. We call the channel scheme in Fig. 5 the offsetless scheme. The number of channels for a given bandwidth in a channel offset scheme is larger than in the scheme (offsetless scheme) in Fig. 5. For simplicity, let us represent the axis of frequency spectrum by the number line as shown in Fig. 7, where intervals $[0, 1)$, $[1, 2)$, $[1.5, 2.5)$, $\cdots$ represent channels. The simple expression of the spectrum in Figs. 5 and 6 are shown in Figs. 7(a) and 7(b), respectively. The channel $[x, y)$, where $y-x=1$ which means the channel bandwidth, is represented by x, that is, a channel x means channel $[x, y)$. The degree $d(f_i, f_j)$ of interchannel interference between two channels f_i and f_j is determined, depending on f_i, f_j, $1-|f_i-f_j|$, and so on. If $d(d_i, f_j)$ is large, then it means that the degree of interchannel interference is large.

The interference $g(z_i, z_j)$ is the upper bound of the interference between two cells z_i and z_j guaranteeing good voice quality and we assume $0 \leqq g(z_i, z_j) \leqq 1$, where $g(z_i, z_j)$ is generally determined by the measurement of interference. That is, $g(z_i, z_j)$ depends on the location of service area, the size of cells, and so on.

In this section, we assume that a service area, the interference $g(z_i, z_j)$ for every pair of cells in the service area and the degree $d(f_i, f_j)$ of interchannel interference for every pair of channels are given. Here,

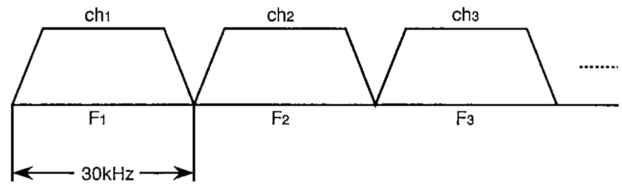

Fig. 5 A channel scheme.

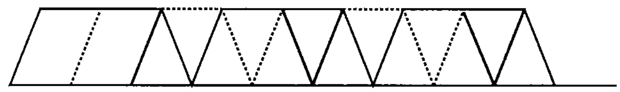

Fig. 6 1/3 channel offset scheme.

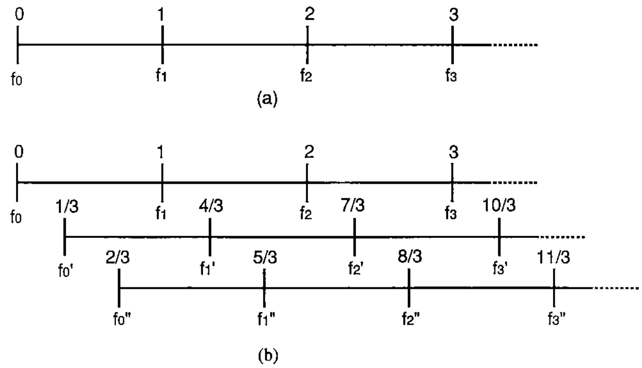

Fig. 7 Simple representations of channel offset schemes.

we define $d(f_i, f_j)$ as follows†.

$$d(f_i, f_j) = \begin{cases} 1-|f_i-f_j| & \text{if } |f_i-f_j| \leqq 1, \\ 0 & \text{otherwise.} \end{cases}$$

We assume that the bandwidth of a channel is one. So, if $|f_i-f_j| \geqq 1$ for two channels f_i and f_j, then we can ignore the interference between f_i and f_j. Let c be a mapping from a cell to a channel. For each cells z_i, z_j, if $d(c(z_i), c(z_j)) \leqq g(z_i, z_j)$, then c is called a feasible assignment of channels. Let us define total bandwidth B, as follows:

$$B = \max_{z_i, z_j} |c(z_i) - c(z_j) + 1|.$$

Example 1: Consider a service area in Fig. 1(a). Let the cellular system be three-belt buffering system, where

$$g(z_i, z_j) = \begin{cases} 0, & \text{if } z_i \text{ and } z_j \text{ are contiguous} \\ 0.5, & \text{if } z_j \text{ is located at 1 or 2 cells away from } z_i \\ 1, & \text{otherwise.} \end{cases}$$

The 1/2 channel offset scheme called interleave is

† In [9], $d(f_i, f_j) = |f_i - f_j|$. For consistency in this paper, we change the definition of $d(f_i, f_j)$ as stated above.

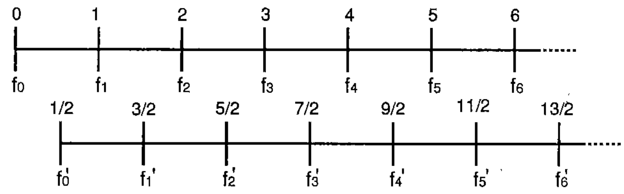

Fig. 8 1/2 channel offset scheme.

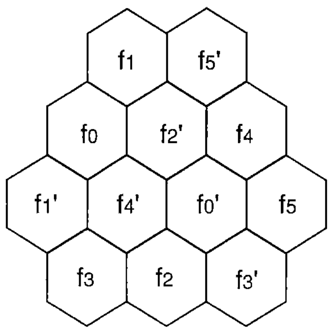

Fig. 9 A cluster.

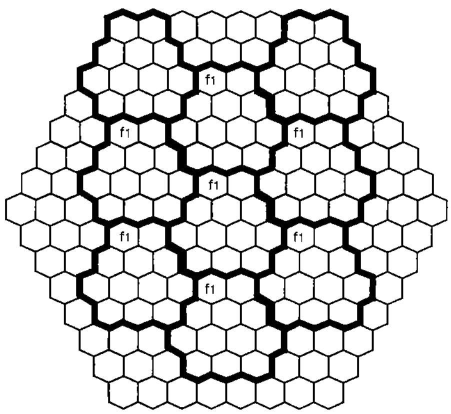

Fig. 10 A feasible assignment of channels using a cluster.

shown in Fig. 8. Figure 9 illustrates a cluster of 12 cells for a feasible assignment of channels in a service area in Fig. 1(a) using the 1/2 channel offset scheme. Using this cluster, we can assign channels to all cells (see Fig. 10). The total bandwidth is 6.5. If we use the offsetless scheme, the total bandwidth is 12.

In a cellular system, if the channel offset scheme is changed, the total bandwidth is change in general. However, there exists the optimal channel offset scheme if each $g(z_i, z_j)$ is a rational number [9]. Let a_0 be a positive integer. If $a_0 \cdot g(z_i, z_j)$ is a nonnegative integer for each cell pair, then the $1/a_0$ channel offset scheme is optimum. Actually, the 1/2 channel offset scheme is optimal in Example 1. $B=6.5$ is the best (minimum) bandwidth using 1/2 channel offset system. And there does not exist the total bandwidth less than 6.5 using any other scheme.

Next, we formulate the channel assignment problem as an assignment problem of colors to vertices of a network. $N=(V(N), E(N); g)$ is a network with vertex set $V(N)$ and edge set $E(N)$. Each vertex v_i corresponds to cell z_i and $(v_i, v_j) \in E(N)$ if and only if $g(z_i, z_j)<1$. With each edge (v_i, v_j), the nonnegative real number $g(z_i, z_j)$ called the weight of the edge are associated. The network N is called the interference network of the cellular system. If each edge weight is zero in a network N and we use offsetless scheme, namely $k=1$, then the minimum total bandwidth of N is equal to the chromatic number of the underlying graph of N. Therefore, this assignment problem is a generalized coloring problem of graphs.

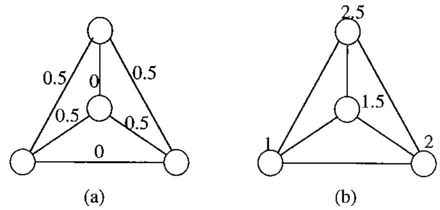

Fig. 11 An interference network and a coloring of the network.

Example 2: In Fig. 11(a), let N be an interference network. A channel assignment using 1/2 channel offset scheme is shown Fig. 11(b). The total bandwidth is $2.5-1+1=2.5$. It is obvious that this value is optimal.

5. Coloring Taking the Degree of Interference into Consideration

In this section, we introduce another new coloring problem on graphs or networks related to the channel assignment problem [10]-[13]. This coloring takes the degree of cochannel interference into consideration.

In previous section, if we assume the two-belt buffering system in Fig. 1(a). A channel A can be use in z_1, z_{21}, z_{24}, z_{27}, z_{30}, z_{33} and z_{36} at the same time. The interference to z_1 may be the sum of the interference from these cells except for z_1. A two-belt buffering system means that the buffering is sufficient for effective immunity to such interference in the worst case. However, if z_{21} and z_{24} only use channel A, a cell that is a buffer cell of z_1 may be able to use channel A. For example, z_1, z_{21}, z_{25} and z_{15} may be able to use channel A at the same time.

We explain this coloring using a simple example. Figure 12 illustrates an interference graph. In the graph, we need three channels A, B and C. Figure 13 (a) illustrates a network called an interference network, where each edge weight represents the degree of cochannel interference. For example, if channel A is assigned to v_1, v_2 and v_3, the cochannel interference is $1+0.5=1.5$. Now, we assume that if the cochannel interference to each cell is not greater than 1, the assignment guarantees good voice quality. This means that the shreshold is one. In this case, we need only

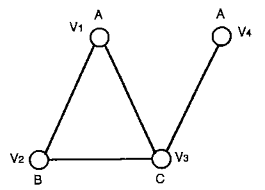

Fig. 12 A interference graph.

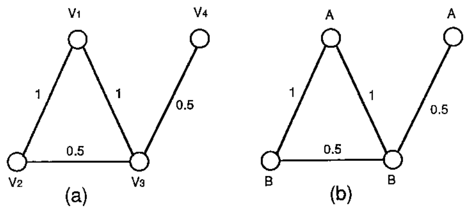

Fig. 13 An interference network and a coloring of the network.

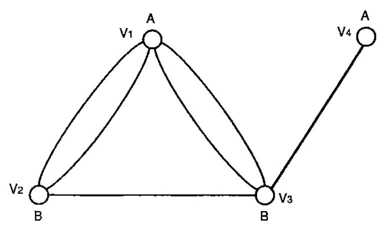

Fig. 14 A interference (multiple) graph.

two channels (see Fig. 13(b)). Let each edge weight be 1 and the threshold be 0. Then, the coloring problem is the same as the coloring problem of graphs. Therefore, the new coloring problem is a generalization of the usual coloring problem.

Assuming that the ratio of all edge weights can be represented using natural numbers, we can reconstruct the interference network, that admits multiple edges, such that all edges have same positive weight. In this case, the new coloring problem is rewritten into a coloring problem of (multiple) graphs. For example, the network in Fig. 13(a) is translated into the graph in Fig. 14. The minimum number of colors for graph G is denoted by $\chi_h(G)$, where h is the threshold. We show an upper and a lower bounds of $\chi_h(G)$, as follows [13].

$$\frac{\chi(G)}{h+1} \leqq \chi_h(G) \leqq \left\{\frac{r+1}{h+1}\right\}, \tag{1}$$

where r is the maximum number of edges that are incident to v for each vertex v and $\{x\}$ represents the minimum integer that is not less than x.,

Example 3: In Fig. 15(a), let $h=2$. Since $r=5$, the upper bound is $\{6/3\}=2$. An assignment using two colors is shown in Fig. 15(b).

In K_4 that is a complete graph with 4 vertices, since $r=3$ and $\chi(K_4)=4$, the equalities hold in (1) if

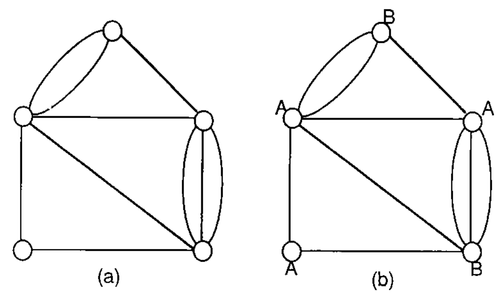

Fig. 15 An interference graph and a coloring of the graph.

$h=1$. Therefore the upper and lower bounds are sharp. And if $h=0$, the upper bound is $r+1$. This is a well-known result in graph theory [6].

6. Mixture of Previous Coloring Concepts

To efficiently use spectrum further, we combine the concepts of colorings in Sects. 4 and 5. Roughly speaking, this coloring is a coloring taking the degree of interchannel interference, including cochannel interference, into consideration using fractions as colors. The degree of interference between two channels f_i and f_j is denoted by $d^*(f_i, f_j)$. Here, we define $d^*(f_i, f_j)$ as follows.

$$d^*(f_i, f_j) = \begin{cases} 1-|f_i-f_j| & \text{if } |f_i-f_j| \leqq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Let G be a multiple graph and h be a fixed nonnegative integer. We consider a mapping c from a nonnegative rational number to a vertex. Let $N(v) = \{u \in V(G) | u \text{ is adjacent to } v\}$ and $m(u, v)$ be the number of edges that join u and v. If for any v,

$$\sum_{u \in N(v)} m(u, v)\, d^*(c(u), c(v)) \leqq h,$$

then c called a feasible coloring with h. For a feasible coloring c with h, let us define the total bandwidth B

$$B = \max_{u,v \in V(G)} |c(u) - c(v) + 1|.$$

The minimum number of total bandwidths for feasible colorings with h is denoted by $\chi_h^*(G)$. It is easily shown that $\chi_0^*(G) = \chi(G)$ and $\chi_h^*(G) \leqq \chi_h(G)$.

Example 4: A simple example is shown in Fig. 16. The coloring in Fig. 16 is a feasible coloring with 1. For example.

$$\begin{aligned} &\sum_{u \in N(v_0)} m(u, v_0)\, d^*(c(u), c(v_0)) \\ &= 2 \cdot d^*(c(v_1), c(v_0)) + 1 \cdot d^*(c(v_3), c(v_0)) \\ &\quad + 1 \cdot d^*(c(v_4), c(v_0)) \\ &= 2 \cdot \left(1 - \left|\frac{4}{5} - 0\right|\right) + 1 \cdot \left(1 - \left|\frac{3}{5} - 0\right|\right) + 1 \cdot 0 \\ &= 0.8 < 1. \end{aligned}$$

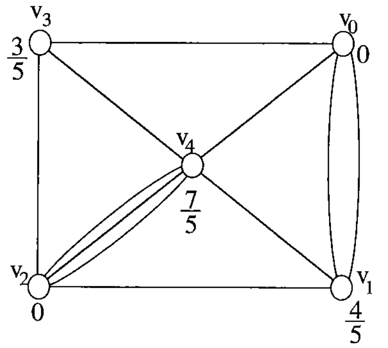

Fig. 16 A feasible coloring with 1 of a graph.

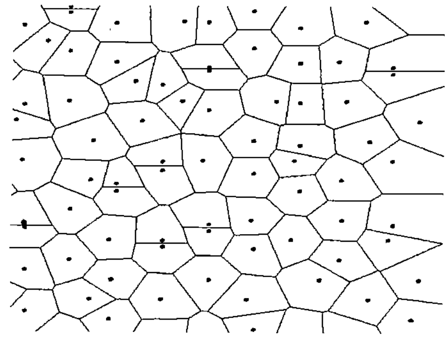

Fig. 17 Voronoi diagram.

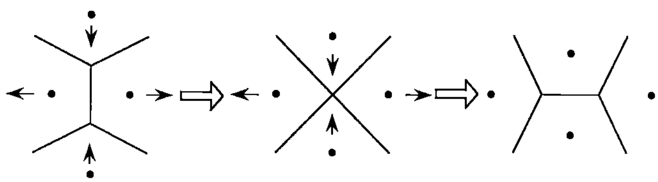

Fig. 18 Topological change of Voronoi diagram.

And the total bandwidth is $(7/5)-0+1=12/5$.

There exists the following lower bound of this coloring [14].

$$\frac{\chi(G)}{2h} \leqq \chi_h^*(G). \quad (h \neq 0) \tag{2}$$

In K_4, $\chi(K_4)=4$ and we easily show that $\chi_1^*(K_4)=2$. In this case, the equality in (2) holds.

7. Cell Structure and Computational Geometry

Computational geometry is concerned with the design and analysis of algorithms for solving geometric problems. The field of computational geometry includes geographic information processing, VLSI design, pattern recognition and robotics. The Voronoi Diagram [15] is one of the fundamental concepts in computational geometry with many applications. For two points p and q, let $d(p, q)$ denote the Euclidean distance between p and q. For a finite set $P=\{p_1, \cdots, p_n\}$ of points in the plane, region $V(p_i)$ is defined by

$$V(p_i)=\{p \mid d(p, p_i)<d(p, p_j) \text{ for any } j(\neq i)\}.$$

$V(p_i)$ is called the Voronoi region of p_i. The Voronoi regions $V(p_1), \cdots, V(p_n)$ make a partition of the plane, and this partition is called the Voronoi diagram for P (see Fig. 17). It is well-known that a Voronoi diagram on n points in the plane can be obtained in $O(n \log n)$ time and $O(n)$ space. Recently, the Voronoi diagram for moving objects has been investigated in connection with motion planing in robotics and geometric optimization problem in computational geometry [16]. A topological change of Voronoi diagram is shown in Fig. 18.

Now if we regard each element of P as a cell site and assume that radio signal strength drops in proportion to distance, then we can regard Voronoi regions as cells. In micro (or pico) cellular systems, by miniaturization of devices, mobile units may be able to have the function of exchanges, and connect other mobile units and cell sites. In this case, not only mobile units but also cell sites move in service area. Therefore, cell structure changes dynamically. If we know how to change cell structure, it is available for assignment of channels and channel switching. The computational geometry is applicable to cell structure of mobile communications.

8. Other Problems in Channel Assignment Problems

There exist various graph & network theoretical approaches to channel assignment problems. For example, in [17], some lower bounds of the number of channels are given. In [18], computational complexity with respect to channel assignment problem is discussed. In general, to obtain the solutions of the channel assignment problems is intractable. So approximate algorithms must be used in order to obtain the solution in a practical time. One example is the application of neural networks [19]-[21].

9. Conclusion

This paper reviewed application of graph & network theory to mobile communication, especially, to channel assignment problems in cellular systems. We introduced new coloring problems on graphs or networks related to the channel assignment problems, compared these problems with the usual coloring problem and showed some results with respect to these problems. We considered the relation between cell structure and the Voronoi diagram.

The demand for mobile communication services is rapidly increasing. We should work out various problems in mobile communication services in near future. Therefore, it is important to apply graph & network theory to these problems.

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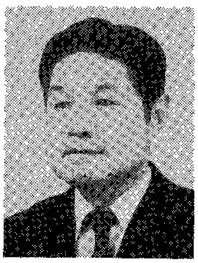

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