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Error Analysis for Ultra-Wideband DS- and Hybrid DS/TH-CDMA with Arbitrary Chip-Duty*

Mohammad Azizur RAHMAN[†a)], ***Student Member***, **Shigenobu SASAKI**[†], ***and*** **Hisakazu KIKUCHI**[†], ***Members***

SUMMARY In this paper, ultra-wideband (UWB) multiple access systems are introduced by using direct-sequence (DS) and hybrid direct-sequence time-hopping (DS/TH) code division multiple access (CDMA) that use arbitrary chip-duty of the spreading sequences. The bit error probabilities are presented. First of all, the variances of the multiple access interference are developed by investigating the collision properties of the signals. Afterward, various approximations are applied. The standard Gaussian approximation (SGA) for the DS system is shown to become extremely optimistic as the chip-duty becomes low. Though the hybrid system performs better, the SGA still remains optimistic. To obtain accurate results, Holtzman's simplified improved Gaussian approximation (SIGA) and Morrow and Lehnert's improved Gaussian approximation (IGA) are used. A shortcoming of the SIGA is rediscovered that renders it unusable for low-duty DS systems, especially, at high signal-to-noise ratio. However, for the hybrid system, the SIGA works as an excellent tool. The IGA is used to get accurate results for the low-duty DS systems. It is shown that lowering of chip-duty by keeping chip rate and chip length unchanged improves performance for asynchronous DS and both asynchronous and synchronous hybrid systems. However, under the same processing gain, a high-duty system performs better than a low-duty system. Performance of synchronous DS system remains independent of chip-duty.
key words: *ultra-wideband, code division multiple access, error analysis, standard and improved Gaussian approximations*

1. Introduction

The newly emerging ultra-wideband (UWB) communication is one of the most prospective technologies for future wireless multiple access communications. Since from the inception of UWB [1], the time hopping (TH) impulse radio (IR) UWB systems combined with pulse position modulation (PPM) have been on the focus for last few years. Multiple access analysis of such IR UWB systems has been widely studied (i.e. see [1]–[8] and the references therein). One drawback of TH-IR PPM system is that it transmits unipolar pulses that cause spectral lines in the frequency spectrum [9]. The use of bipolar signaling and bipolar modulation as binary phase shift keying (BPSK) are the right remedy for this problem.

Though many succeeding works on UWB have considered TH-IR PPM systems following the pioneer works of [1], [2], multiple access analysis of UWB systems using bipolar signaling has't received much attention. For instance, the well-known concept of conventional direct-sequence code division multiple access (DS-CDMA) communication can be a key base idea to develop UWB systems with bipolar signaling. One basic difference of conventional DS-CDMA from a UWB system is that the UWB system usually transmits signals of low duty-cycle, whereas, the conventional DS-CDMA transmits signal continuously rendering the signal duty-cycle to be unity. As a consequence, while viewing the conventional DS-CDMA from the perspective of UWB multiple access systems, UWB DS-CDMA systems can be designed by lowering the chip-duty of spreading sequences that is unity in conventional DS-CDMA. While the chip-duty is low, it additionally provides a new option for randomizing the position of the pulse within each chip giving the UWB hybrid DS/TH-CDMA system. Error analysis of such UWB DS- and hybrid DS/TH-CDMA systems are of practical interest, however, haven't yet been addressed, except in some of our recent works [10]–[12].

As the signal duty goes low, the probability that the signals of simultaneous interfering users will collide with that of the desired user decreases, decreasing the probability of interference [1]. Hence, the multiple access performance of a system is supposed to depend on the collision property of the signal, however though the dependency is not studied yet. Notably, the collision property of signals depend on its structure, which is considerably different for DS and DS/TH. The collision property is of importance both under perfect and imperfect power control and is supposed to bring about a tradeoff in the choice of signaling [1]. However, any study comparing the collision properties and hence the multiple access performances of low-duty DS and hybrid DS/TH sequences is not yet known.

The error analysis for conventional DS-CDMA using random sequences is present in current literature [13]–[15], which is for systems using chip-duty of unity. In [14] Morrow and Lehnert showed the standard Gaussian approximation (SGA) to be optimistic and proposed an accurate improved Gaussian approximation (IGA) with greater computational complexity. Later, Holtzman in [15] proposed a simplified improved Gaussian approximation (SIGA) that provides good accuracy with computational complexity comparable to the SGA. Following the SGA method, error probabilities for some UWB systems were presented in [1], [2], [16]–[18]. However, the SGA for UWB may not be accurate in most cases of practical interest [3]–[7]. How different UWB system parameters including chip-duty affects

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[†]The authors are with the Department of Electrical and Electronic Engineering, Niigata University, Niigata-shi, 950-2181 Japan.
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a) E-mail: aziz@telecom0.eng.niigata-u.ac.jp
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the accuracy of SGA needs investigation. In addition, since the UWB signaling structure is considerably different from that of conventional DS-CDMA, whether the simple method SIGA can be applied for UWB is not clear. Otherwise, employing the IGA would be necessary, the use of which for UWB hasn't yet been investigated either.

In this paper, we present a simple error analysis applicable for both UWB DS- and hybrid DS/TH-CDMA. We consider a UWB DS-CDMA system with arbitrary chip-duty δ of the spreading sequences used, $0 < \delta \leq 1$ and a UWB hybrid DS/TH-CDMA system with $\delta = 1/I$, I being an integer ≥ 2. The error analysis is presented from the perspective of collision properties of the sequences. This facilitates to obtain error probabilities following SGA, SIGA and IGA, along with giving us way for investigating the validity of the methods for UWB. Effects of various system parameters including chip-duty on system performance are investigated and the multiple access capabilities of arbitrary-duty DS and DS/TH sequences are compared. To make the analysis tractable, rectangular pulse shape and random sequences are considered. The assumption of rectangular pulse helps us obtain closed form error probabilities. Note that these are standard assumptions in literature [6], [8], [11], [13]–[15], [19], [20]. Monte Carlo simulations are also presented to support the theory.

2. System Model

The binary UWB DS- and UWB hybrid DS/TH-CDMA systems under consideration has K simultaneous users. The signal of the kth user can be given by

$$s_k(t-\alpha_k) = \sum_{l=-\infty}^{+\infty} \sqrt{2P_k}a_k(t-lT_b-\alpha_k) \times b_k(t-lT_b-\alpha_k)\cos(\omega_c t+\phi_k) \quad (1)$$

where P_k is the average signal power of user k, ω_c is the carrier frequency and $b_k(t)$, $a_k(t)$ are the data sequence and the spreading chip sequence respectively that are independent of each other. Here l represents l-th bit and assume $k \in [0, 1, \ldots, K-1]$. α_k and ϕ_k are random delay and phase respectively relative to the desired signal of the 0th user (i.e., $\alpha_0 = 0, \phi_0 = 0$). α_k and ϕ_k are independent and uniformly distributed over $[0, T_b]$ and $[0, 2\pi]$ respectively, where $T_b$is the bit duration. $b_k(t)$ is a random process having outcomes uniform on $\{+1, -1\}$. $a_k(t)$ over each bit duration is defined as

$$a_k(t) = \sum_{i=0}^{N-1} a_{k,i}\psi(t-iT_c-C_{i,k}T_p) \quad (2)$$

where $\psi(t)$ is the chip waveform, T_c is the chip duration and T_p is the duration of signal transmission over each chip. $C_{i,k}$ is a random variable (RV) controlling the time hopping in the ith chip of the kth user in the hybrid DS/TH system that equals to zero for the DS system. Assuming a rectangular pulse shape $\psi(t) = 1$, $0 \leq t < T_p$ and $\psi(t) = 0$, otherwise.

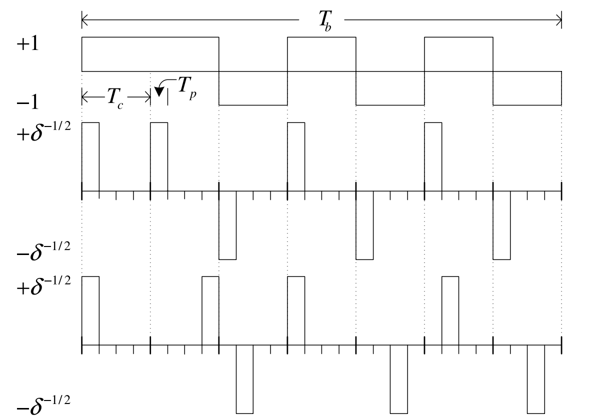

Fig. 1 Spreading chip sequences for conventional DS-CDMA (top), UWB DS-CDMA (middle) and UWB hybrid DS/TH-CDMA (bottom) with $N = 7$ and $a_{i,k} = \{+1, +1, -1, +1, -1, +1, -1\}\delta^{-1/2}$. $\delta = T_p/T_c$ is the chip-duty which is unity for the conventional DS-CDMA and 0.25 for the UWB systems shown. For the UWB hybrid system, $C_{i,k} = \{0, 3, 1, 0, 2, 1, 2\}$, which is zero for the UWB DS system.

In both the systems, the chip-duty is defined as $\delta = T_p/T_c$. For the DS system we consider $0 \leq \delta < 1$ and for the hybrid system $\delta = 1/I$, I being an integer ≥ 2. As a result, $C_{i,k}$ in the hybrid system is considered as a random process uniform on $\{0, 1, \ldots, I-1\}$. The energy of $\psi(t)$ is T_c and so, $a_{k,i}$ is a random process uniform on $\{+\delta^{-1/2}, -\delta^{-1/2}\}$. Additionally, $a_{k,i}$ and $C_{i,k}$ in (2) are periodic with period $N = T_b/T_c$ and hence the processing gain $PG = N/\delta$. A diagram of the signaling structure is shown in Fig. 1.

Here note that a term pulse repetition frequency (PRF) is sometime used in UWB literature. The average PRF for both types of the systems introduced above equals $1/T_c$.

3. Multiple Access Interference Modeling

Considering an additive white Gaussian noise (AWGN) channel, the received signal can be given by

$$r(t) = \sum_{k=0}^{K-1} s_k(t-\alpha_k) + n(t) \quad (3)$$

where $n(t)$ is AWGN noise with two sided power spectral density of $N_o/2$. The time asynchronism of the random sequence of the kth user can be written as $\alpha_k = \tau_k + \gamma_k T_c$. Here, γ_k is an integer within $\{0, 1, \ldots, N-1\}$ and $0 \leq \tau_k < T_c$. Considering a template of the form

$$s_0^{temp}(t) = \sum_{i=0}^{N-1} a_{0,i}\psi(t-iT_c-C_{i,0}T_p)\cos(\omega_c t) \quad (4)$$

the decision statistics for detecting an arbitrary bit of user 0 by a coherent correlation receiver in the UWB DS- or hybrid DS/TH-CDMA system can be given by

$$\lambda_0 = \sqrt{P_0/2}b_0T_b + \sum_{i=0}^{N-1}\sum_{k=1}^{K-1} \sqrt{P_k/2}W_{i,k}\cos\phi_k + n \quad (5)$$

where the right side of (5) has three parts, of which the first part is the desired signal component, the second part is the multiple access interference (MAI) component and the third part n is the AWGN component having variance of $\sigma_n^2 \approx N_o T_b/4$. Here, $W_{i,k}$ is the MAI component on the ith chip of user 0 contributed by user k, given by

$$W_{i,k} = L_{i,k}\hat{R}_\psi(\tau_k) + M_{i,k}R_\psi(\tau_k) \tag{6}$$

where $\hat{R}_\psi(\tau_k)$ and $R_\psi(\tau_k)$ are continuous-time normalized partial autocorrelation functions of the chip waveform $\psi(t)$ defined as $\hat{R}_\psi(\tau_k) = \frac{1}{\delta}\int_{\tau_k}^{T_c}\psi(t)\psi(t-\tau_k)dt$ and $R_\psi(\tau_k) = \hat{R}_\psi(T_c - \tau_k)$. The independent RVs $L_{i,k}$ and $M_{i,k}$ are uniform on $\{+1, -1\}$, have zero mean and variances $E[(L_{i,k})^2] = E[(M_{i,k})^2] = 1$ where $E[.]$ represents the mean value.

The MAI variance

$$\Psi = \sum_{k=1}^{K-1} Z_k \tag{7}$$

is an RV which is a function of the chip delays $\alpha = [\alpha_0, \alpha_1, \ldots, \alpha_{K-1}]$ and carrier phases $\phi = [\phi_0, \phi_1, \ldots, \phi_{K-1}]$. Here, Z_k is the MAI variance component contributed by user k, given by

$$Z_k = \sum_{i=0}^{N-1} Z_{i,k} \tag{8}$$

where $Z_{i,k}$ is the variance of MAI component on the ith chip of user 0 coming from user k, given by

$$Z_{i,k} = \frac{P_k}{2}\cos^2\phi_k E[W_{i,k}^2|\alpha] \tag{9}$$

3.1 The UWB DS-CDMA System

For ease of presentation, we first start with the synchronous system and then eventually go for the asynchronous system.

Synchronous system. The system is chip synchronous while $\tau_k = 0$ and bit synchronous while both $\tau_k = \gamma_k = 0$. Because, the rectangular pulse of duration T_p is placed within 0 to T_p of each chip of duration T_c for all users, we can write, $\hat{R}_\psi(0) = \frac{1}{\delta}\int_0^{T_p} dt = T_p/\delta$ and $R_\psi(0) = 0$ for both cases. Here note that the $(i-\gamma_k)$th chip of user k collides with the ith chip of user 0, and a collision of chips results in collision of pulses causing MAI. Hence, for both chip and bit synchronous cases, we can write

$$Z_{i,k} = \frac{P_k}{2}T_c^2\cos^2\phi_k \tag{10}$$

Since there are N similar collisions over the bit duration, we get

$$Z_k = \frac{P_k}{2}NT_c^2\cos^2\phi_k \tag{11}$$

Note that Z_k here is independent of chip-duty.

Asynchronous system. For an asynchronous system, τ_k is an RV uniformly distributed over $[0, T_c]$. Hence, there will usually be partial collisions. Note that $\hat{R}_\psi(\tau_k) = \frac{1}{\delta}\int_{\tau_k}^{T_p} dt = (T_p - \tau_k)/\delta$, $0 \le \tau_k < T_p$; $\hat{R}_\psi(\tau_k) = 0$, $T_p \le \tau_k < T_c$ and $R_\psi(\tau_k) = 0$, $0 \le \tau_k < T_c - T_p$; $R_\psi(\tau_k) = (\tau_k + T_p - T_c)/\delta$, $T_c - T_p \le \tau_k < T_c$. Here, the $(i-\gamma_k)$th and $(i-\gamma_k-1)$th chips of user k may partially collide with the ith chip of user 0. However, MAI may occur if there is some collision of pulses in the respective chips. Since, the MAI depends on the collision of pulses, the collision property of the signaling scheme is of importance. Investigation reveals that in asynchronous mode, the DS sequences have different collision properties for $0 < \delta \le 0.5$ and $0.5 < \delta \le 1.0$. In each case, three different situations can be observed. For $0 < \delta \le 0.5$, the MAI component on the ith chip of user 0 contributed by user k, $W_{i,k}$ shown in (6) simplifies to $L_{i,k}\hat{R}_\psi(\tau_k)$ for $0 \le \tau_k < T_p$, 0 for $T_p \le \tau_k < T_c - T_p$ and $M_{i,k}R_\psi(\tau_k)$ for $T_c - T_p \le \tau_k < T_c$. Hence, we get

$$Z_{i,k} = \begin{cases} \frac{P_k}{2\delta^2}T_c^2\cos^2\phi_k(\delta - S_{11})^2, & 0 \le \tau_k < T_p \\ 0, & T_p \le \tau_k < T_c - T_p \\ \frac{P_k}{2\delta^2}T_c^2\cos^2\phi_k(S_{13} - 1 + \delta)^2, & T_c - T_p \le \tau_k < T_c \end{cases} \tag{12}$$

for $0 < \delta \le 0.5$ where S_{11} and S_{13} are independent RVs uniformly distributed over $[0, \delta]$, $[1-\delta, 1]$ respectively. For $0.5 < \delta \le 1.0$, $W_{i,k}$ shown in (6) simplifies to $L_{i,k}\hat{R}_\psi(\tau_k)$ for $0 \le \tau_k < T_c - T_p$, $M_{i,k}R_\psi(\tau_k)$ for $T_p \le \tau_k < T_c$ and remains as it is for $T_c - T_p \le \tau_k < T_p$. Hence,

$$Z_{i,k} = \begin{cases} \frac{P_k}{2\delta^2}T_c^2\cos^2\phi_k(\delta - S_{21})^2, & 0 \le \tau_k < T_c - T_p \\ \frac{P_k}{2\delta^2}T_c^2\cos^2\phi_k\left[2(S_{22}^2 - S_{22}) + 2(\delta^2 - \delta) + 1\right], & \\ & T_c - T_p \le \tau_k < T_p \\ \frac{P_k}{2\delta^2}T_c^2\cos^2\phi_k(S_{23} - 1 + \delta)^2, & T_p \le \tau_k < T_c \end{cases} \tag{13}$$

for $0.5 < \delta \le 1.0$ where S_{21}, S_{22} and S_{23} are independent RVs, uniformly distributed over $[0, 1-\delta]$, $[1-\delta, \delta]$ and $[\delta, 1]$ respectively.

Since the pulse in each chip of each user is placed in the same location for the DS sequences, as is synchronous case, here also we note that a collision means a collision in all N chips of the bit. Hence,

$$Z_k = NZ_{i,k} \tag{14}$$

for $0 < \delta \le 1.0$.

3.2 The UWB Hybrid DS/TH-CDMA System

For the hybrid DS/TH-CDMA system, we notice more interesting collision property of the hybrid DS/TH sequences. For ease of presentation, we again start with the synchronous system.

Synchronous system. Due to the random time hopping patterns, the pulse in any chip of any user may be placed in any of the I slots. Here note that, the $(i-\gamma_k)$th chip of user k coincides with the ith chip of user 0. However, MAI may occur only if the pulses in the respective chips coincide (or, collide). Hence, by noting that $\hat{R}_\psi(\tau=0) = T_p/\delta$ and $R_\psi(\tau=0) = 0$, $\tau = (C_{i-\gamma_k,k} - C_{i,0})T_p$, we get

$$Z_{i,k} = \begin{cases} \frac{P_k}{2} T_c^2 \cos^2\phi_k, & C_{i,0} = C_{i-\gamma_k,k} \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

Note that a coincidence in any chip occurs with a probability δ, the probability of no coincidence being $1-\delta$. Because the position of pulse in any chip of any user is selected randomly and independently, we can write

$$Z_k = \frac{P_k}{2} T_c^2 \cos^2\phi_k J_s \tag{16}$$

where J_s is an RV binomially distributed over $\{0, 1, 2, \ldots, N\}$ [21]. Here note that J_s represents the number of chips within a bit of user 0 where the individual pulses from the colliding chips of user k coincide with the pulses of user 0 and hence contribute to MAI. The probability density function of J_s is given by [21]

$$f_J(j) = \sum_{j=0}^{N} \binom{N}{j} p^j (1-p)^{N-j} \delta_D \tag{17}$$

with $p = \delta$ where δ_D is Dirac delta function.

Asynchronous system. For a hybrid asynchronous system, note that the interference on the ith chip of user 0 may come partially from either $(i-\gamma_k)$th or $(i-\gamma_k-1)$th or from both the chips of any user k. Let us define, $t_{i-\gamma_k} = (C_{i-\gamma_k,k}T_p + \tau_k) - C_{i,0}T_p$ and $t_{i-\gamma_k-1} = (C_{i-\gamma_k-1,k}T_p + \tau_k - T_c) - C_{i,0}T_p$. Any one of the following four possible cases may occur. Considering that only one pulse either from the $(i-\gamma_k)$th or $(i-\gamma_k-1)$th chip of user k contributes to interference, we get *case a:* if $0 \le \tau < T_p$, where $\tau = t_{i-\gamma_k}$ or $t_{i-\gamma_k-1}$, the partial correlations $\hat{R}_\psi(\tau) = (T_p - \tau)/\delta$ and $R_\psi(\tau) = 0$; and *case b:* if $-T_p \le \tau < 0$, where $\tau = t_{i-\gamma_k}$ or $t_{i-\gamma_k-1}$, one gets $R_\psi(\tau) = (T_p+\tau)/\delta$ and $\hat{R}_\psi(\tau) = 0$. However, when the pulses in $(i-\gamma_k)$th and $(i-\gamma_k-1)$th chips are adjacent and both of them contribute to interference, which may occur *case c:* while $0 \le t_{i-\gamma_k} < T_p$ giving $\hat{R}_\psi(\tau) = (T_p - \tau)/\delta, R_\psi(\tau) = \tau/\delta$, where $\tau = t_{i-\gamma_k}$. The final possibility is that *case d:* there may be no collision at all. Accumulating the concept developed above, $Z_{i,k}$ can be given by

$$Z_{i,k} = \begin{cases} ^{(a)}\frac{P_k}{2\delta^2} T_c^2 \cos^2\phi_k (\delta - S_{11})^2, & 0 \le \tau < T_p \\ ^{(b)}\frac{P_k}{2\delta^2} T_c^2 \cos^2\phi_k (\delta + S_{12})^2, & -T_p \le \tau < 0 \\ ^{(c)}\frac{P_k}{2\delta^2} T_c^2 \cos^2\phi_k [\delta^2 - 2\delta S_{11} + 2S_{11}^2], & \\ & 0 \le t_{i-\gamma_k} < T_p \\ ^{(d)}0, & \text{otherwise} \end{cases} \tag{18}$$

where $\tau = t_{i-\gamma_k}$ or $t_{i-\gamma_k-1}$ and S_{12} is an RV, uniformly distributed over $[-\delta, 0]$. Note that the average probability of occurrence of the events c, d are δ^2, $1-2\delta$ respectively and that of a, b are $(2-\delta)\delta/2$ each.

Here note that $Z_{i,k}$ in (18) gives the exact statistical representation of the possible MAI on the ith chip of user 0 from user k. However, use of (18) in the succeeding sections seems cumbersome. Hence, we look for an easy but still accurate approximation of $Z_{i,k}$. Due to the TH nature of the position of pulse within any chip, we note an interesting property that if there is a partial coincidence (of pulse) of amount τ, $0 \le \tau < T_p$ in any chip, the partial coincidences in other chips of the bit will be either τ or $T_p - \tau$ or there will be no coincidences at all. In addition, there is one pulse in average to interfere with an arbitrary pulse of the desired user. So, $Z_{i,k}$ can be approximately given by

$$Z_{i,k} = \begin{cases} \frac{P_k}{2} \cos^2\phi_k \left(\frac{\hat{R}_\psi^2(\tau) + R_\psi^2(\tau)}{2} \right) \\ = \frac{P_k}{4\delta^2} T_c^2 \cos^2\phi_k \left[\delta^2 - 2\delta S_{11} + 2S_{11}^2\right], \\ \qquad \text{partial coincidence} \\ 0, \qquad \text{otherwise} \end{cases} \tag{19}$$

Note that a coincidence in any chip occurs with a probability 2δ, the probability of no coincidence being $1-2\delta$. Using this information and those developed in the synchronous case above, Z_k for $I \ge 3$ can be approximately given by

$$Z_k = \frac{P_k}{4\delta^2} T_c^2 \cos^2\phi_k \left[\delta^2 - 2\delta S_{11} + 2S_{11}^2\right] J_{as} \tag{20}$$

where J_{as} is an RV binomially distributed over $\{0, 1, \ldots, N\}$. As before, J_{as} represents the number of partial collisions within a bit of user 0 where the pulses of user 0 are struck partially by pulses from the signal of user k. The probability density function of J_{as} is given by (17) with $p = 2\delta$. For $I = 2$, using (18) in (8) is recommended.

4. Error Probabilities

With the representations of the MAI variance derived in the previous section, we are now ready to develop the bit error probability (BEP) equations for the systems under consideration. Because of its simplicity and wide acceptance in literature, let us start with the standard Gaussian approximation (SGA).

4.1 Standard Gaussian Approximation (SGA)

The SGA is simple in the sense that once we get the mean of the MAI variance, we can obtain the BEP.

4.1.1 The UWB DS-CDMA System

Synchronous system. For synchronous system, the mean of the MAI variance Ψ can be given by,

$$\mu = \sum_{k=1}^{K-1} E[Z_k] = \frac{NT_c^2}{4} \sum_{k=1}^{K-1} P_k \tag{21}$$

for $0 < \delta \leq 1$. The BEP under SGA is then given by

$$P_e^{SGA} = Q\left(\sqrt{\frac{P_0 T_b^2/2}{\mu + \sigma_n^2}}\right) = Q\left(\left[\left(\frac{\chi N P_0}{\sum_{k=1}^{K-1} P_k}\right)^{-1} + \left(\frac{2E_b}{N_o}\right)^{-1}\right]^{-\frac{1}{2}}\right) \tag{22}$$

where $Q(x) = (2\pi)^{-1/2}\int_x^{\infty} \exp(-u^2/2)du$, energy per bit of the desired user $E_b = P_0 T_b$ and $\chi = 2$, $0 < \delta \leq 1$.

Asynchronous system. According to Appendix A, by noting that

$$\begin{aligned}
&E[S_{11}] = \delta/2,\\
&E[S_{11}^2] = \delta^2/3,\\
&E[S_{13}] = 1 - \delta/2,\\
&E[S_{13}^2] = 1 - \delta + \delta^2/3,\\
&E[S_{21}] = (1 - \delta)/2,\\
&E[S_{21}^2] = (1 - \delta)^2/3,\\
&E[S_{22}] = 1/2,\\
&E[S_{22}^2] = (1 - \delta + \delta^2)/3,\\
&E[S_{23}] = (1 + \delta)/2,\\
&E[S_{23}^2] = (1 + \delta + \delta^2)/3,
\end{aligned}$$

the mean of Ψ can be given by,

$$\mu = \frac{NT_c^2\delta}{6}\sum_{k=1}^{K-1} P_k \tag{23}$$

for $0 < \delta \leq 1$. Finally, the BEP is given by (22) with $\chi = 3/\delta$, $0 < \delta \leq 1$.

4.1.2 The UWB Hybrid DS/TH-CDMA System

Synchronous system. For a hybrid synchronous system, by noting that $E[J_s] = N\delta$ [21], the mean of Ψ is given by

$$\mu = \frac{NT_c^2\delta}{4}\sum_{k=1}^{K-1} P_k \tag{24}$$

The BEP can be given by (22) with $\chi = 2/\delta$, $\delta = 1/I$, $I = 2, 3\ldots$

Asynchronous system. In a similar fashion, using the fact $E[J_{as}] = 2N\delta$ [21] in (20), or (18) in (8), the mean in this case takes the form as shown in (23). The BEP is given by (22) with $\chi = 3/\delta$, $\delta = 1/I$, $I = 2, 3, \ldots$ Here note that in asynchronous case, both DS and DS/TH systems have the same expression for the mean of MAI variance, which results in the same BEP expression for the systems under SGA.

4.1.3 Remarks

Because of its simplicity, the SGA has been widely used in literature to obtain BEP for conventional DS-CDMA in early works [13]–[15], [19], [20]. Many recent works on UWB multiple access systems have also been performed under the SGA [1], [2], [16], [17], however, without due justification. Moreover, the effects of UWB system parameters, especially, chip-duty and power levels of the users including the effect of the signaling scheme (DS or DS/TH) on the accuracy of the SGA has not yet been investigated. Though the validity of the SGA for UWB has not yet been fully explored, some works based on exact analysis [4] that are usually unwieldy and simulation [7] that are not reusable, show the inefficiency of the SGA. At this situation, a simple to evaluate and accurate method of error analysis is demanding for UWB multiple access system. Hence, in the next stage we proceed for an accurate method for BEP evaluation without surrendering the simplicity.

4.2 Simplified Improved Gaussian Approximation (SIGA)

An acceptably accurate, simple-to-evaluate approximation, known as simplified improved Gaussian approximation (SIGA) was first presented in [15]. This requires the standard deviation σ other than the mean μ of Ψ to be determined. The BEP then becomes

$$P_e^{SIGA} = \frac{2}{3}Q\left(\sqrt{\frac{P_0 T_b^2/2}{\mu + \sigma_n^2}}\right) + \frac{1}{6}Q\left(\sqrt{\frac{P_0 T_b^2/2}{\mu + \sqrt{3}\sigma + \sigma_n^2}}\right) + \frac{1}{6}Q\left(\sqrt{\frac{P_0 T_b^2/2}{\mu - \sqrt{3}\sigma + \sigma_n^2}}\right) \tag{25}$$

4.2.1 The UWB DS-CDMA System

Synchronous system. By neglecting the covariance term of Z_k (that we do all throughout this section) [20], one can show that for a synchronous DS system

$$\sigma = \left[\sum_{k=1}^{K-1}[E(Z_k^2) - E(Z_k)^2]\right]^{1/2} = \frac{NT_c^2}{4\sqrt{2}}\left(\sum_{k=1}^{K-1} P_k^2\right)^{1/2} \tag{26}$$

for $0 < \delta \leq 1$ which is also independent of δ like the mean.

Asynchronous system. It can be shown that (see Appendix A),

$$\begin{aligned}
&E[S_{11}^3] = \delta^3/4,\\
&E[S_{11}^4] = \delta^4/5,\\
&E[S_{13}^3] = 1 - 3\delta/2 + \delta^2 - \delta^3/4,\\
&E[S_{13}^4] = 1 - 2\delta + 2\delta^2 - \delta^3 + \delta^4/5,\\
&E[S_{21}^3] = (1 - \delta)^3/4,\\
&E[S_{21}^4] = (1 - \delta)^4/5,
\end{aligned}$$

$$E[S_{22}^3] = (1 - 2\delta + 2\delta^2)/4,$$
$$E[S_{22}^4] = (1 - 3\delta + 4\delta^2 - 2\delta^3 + \delta^4)/5,$$
$$E[S_{23}^3] = (1 + \delta + \delta^2 + \delta^3)/4,$$
$$E[S_{23}^4] = (1 + \delta + \delta^2 + \delta^3 + \delta^4)/5.$$

Further by noting that $R_\psi(\tau)$ and $\hat{R}_\psi(\tau)$ and hence S_{11} and S_{13} are mutually exclusive for $0 < \delta \leq 0.5$ and neglecting the terms not including N^2 [20] for $0.5 < \delta \leq 1$, one can show

$$\sigma = \frac{NT_c^2}{2}\left[\left(\frac{3}{20}\delta - \frac{1}{9}\delta^2 + \frac{3}{4}\hat{w}_\psi\right)\sum_{k=1}^{K-1} P_k^2\right]^{1/2} \tag{27}$$

where $\hat{w}_\psi = 0$ for $0 < \delta \leq 0.5$, and $\hat{w}_\psi = (2\delta - 1)E[(\delta - S_{22})^2(S_{22} - 1 + \delta)^2] = -1/30 + \delta/3 - 4\delta^2/3 + 8\delta^3/3 - 8\delta^4/3 + 16\delta^5/15$ for $0.5 < \delta \leq 1$.

4.2.2 The UWB Hybrid DS/TH-CDMA System

Synchronous system. By noting that $E[J_s^2] = N\delta(N\delta - \delta + 1)$ [21], for the hybrid synchronous system σ can be given by

$$\sigma = \frac{T_c^2}{4\sqrt{2}}\left[\left(N^2\delta^2 - 3N\delta^2 + 3N\delta\right)\sum_{k=1}^{K-1} P_k^2\right]^{1/2} \tag{28}$$

Asynchronous system. Similarly, by noting that $E[J_{as}^2] = 2N\delta(2N\delta - 2\delta + 1)$ [21], σ for the hybrid asynchronous system with $I \geq 3$ can be given by

$$\sigma = \frac{T_c^2}{4}\left[\left(\frac{46N^2\delta^2 - 126N\delta^2 + 63N\delta}{180}\right)\sum_{k=1}^{K-1} P_k^2\right]^{1/2} \tag{29}$$

Using (18) and (8) for $I = 2$ the σ becomes

$$\sigma = \frac{T_c^2}{2}\left[N\left(\frac{3}{20}\delta - \frac{1}{90}\delta^2\right)\sum_{k=1}^{K-1} P_k^2\right]^{1/2}, I = 2 \tag{30}$$

4.2.3 Region of Validity

As we will show later, the SIGA is valid and works as an excellent tool for synchronous UWB DS and both synchronous and asynchronous UWB hybrid systems in almost all cases of practical importance. However, the third term of (25) can be evaluated only for $\mu \geq \sqrt{3}\sigma - \sigma_n^2$. For the hybrid system, we may not be able to evaluate that part for very small number of users, especially while N is small, chip-duty is low and SNR is very high. Interestingly, we observe that the contribution from the third part is infinitesimally small for $\mu < \sqrt{3}\sigma - \sigma_n^2$ and can be neglected. However, for the asynchronous UWB DS system, use of the third part remains critical and so (25) can be evaluated accurately for number of users which is a solution of

$$\overbrace{\left[\frac{E_b}{N_o}\right]^{-1}}^{\text{LHS}} \geq \overbrace{\frac{2\sqrt{3}}{N}\left[\left(\frac{3}{20}\delta - \frac{1}{9}\delta^2 + \frac{3}{4}\hat{w}_\psi\right)\times\sum_{k=1}^{K-1}\left(\frac{P_k}{P_0}\right)^2\right]^{\frac{1}{2}} - \frac{2\delta}{3N}\sum_{k=1}^{K-1}\frac{P_k}{P_0}}^{\text{RHS}} \tag{31}$$

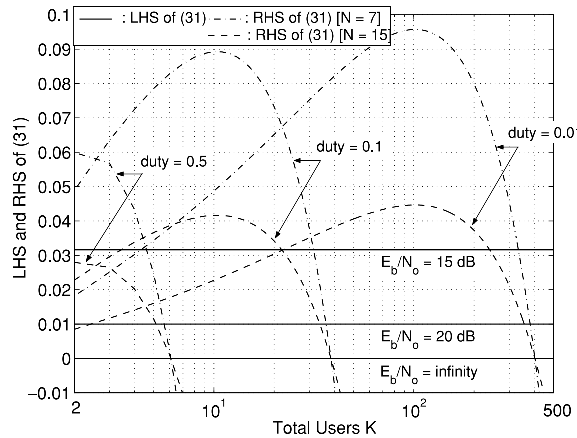

Fig. 2 Region of validity of the SIGA for asynchronous UWB DS-CDMA with equal powers of all users. The LHS and RHS shown in the figure stands for the left hand side and right hand side respectively of Eq. (31).

Table 1 Chip-duty vs. users showing the region of validity where (25) can be evaluated for asynchronous UWB DS system (Equal powers of all users). (a) $E_b/N_o = \infty$, $N > 0$, (b) $E_b/N_o = 20$ dB, $N = 15$, (c) $E_b/N_o = 15$ dB, $N = 15$.

δ	1.0	0.5	0.1	0.01
(*a*) K	≥ 3	≥ 7	≥ 39	≥ 403
(*b*) K	≥ 3	≥ 6	≥ 34	≤ 2 and ≥ 357
(*c*) K	≥ 2	≥ 2	≤ 3 and ≥ 22	≤ 22 and ≥ 240

To obtain a clear picture of the matter, we plot the left hand side (LHS) and right hand side (RHS) of the $\geq$ sign of (31) in Fig. 2. The region of validity for SIGA is the area where (31) holds, which depends on four parameters namely SNR, chip length, chip-duty and power levels of the users. Whenever LHS < RHS, we wouldn't be able to use the SIGA. Some results extracted from Fig. 2 are also shown in Table 1 for clarity. Both Fig. 2 and Table 1 show the inappropriateness of the SIGA for asynchronous UWB DS systems, especially for small N and at moderate to high SNR. This necessitates for the accurate IGA for the system.

4.3 Improved Gaussian Approximation (IGA)

The IGA first introduced in [14] for DS-CDMA is based on the observation that the MAI is approximately Gaussian conditioned on the delays and phases of all the interfering signals. IGA requires the distribution of Ψ to be known, and the BEP is then given by

$$P_e^{IGA} = \int_0^\infty Q\left(\sqrt{\frac{P_0 T_b^2/2}{\Psi + \sigma_n^2}}\right) f_\Psi(\zeta)d\zeta \tag{32}$$

where $f_\Psi(\zeta)$ is the distribution of Ψ.

Finding $f_\Psi(\zeta)$ requires that the distribution of Z_k be determined for all interfering users and then convolved together [14], [20]. It is done with the underlying assumption that the interference from different users are independent.

4.3.1 The UWB DS-CDMA System

Synchronous system. For a synchronous UWB DS system with any chip-duty $0 < \delta \leq 1$, finding the distribution of Z_k is easy (see Appendix B), which is given by

$$f_{Z_k}(z) = \frac{1}{\pi\sqrt{z(\aleph_k - z)}}, \quad 0 < z < \aleph_k \tag{33}$$

where $\aleph_k = NP_kT_c^2/2$. Note that Eq. (33) is independent of δ.

Asynchronous system. Since the collision and interference properties of UWB DS sequence are different for $0 < \delta \leq 0.5$ and $0.5 < \delta \leq 1.0$ in asynchronous mode, the density of MAI variance takes different forms in the two regions. The density of Z_k for $0 < \delta \leq 0.5$ can be shown to be (see Appendix B)

$$f_{Z_k}(z) = \begin{cases} \beta\delta_D(z), & z = 0 \\ \frac{(1-\beta)\delta}{\pi\sqrt{z\aleph_k}} \log_e\left[\frac{\sqrt{\aleph_k}+\sqrt{\aleph_k - z}}{\sqrt{\aleph_k}-\sqrt{\aleph_k - z}}\right], & 0 < z < \aleph_k \end{cases} \tag{34}$$

where δ_D is Dirac delta function and $\beta = 1 - 2\delta$. For $0.5 < \delta \leq 1$, we get (see Appendix B)

$$f_{Z_k}(z) = \begin{cases} \frac{\beta\delta}{\pi\sqrt{z\aleph_k}} \log_e\left|\frac{\sqrt{\aleph_k(1+D)/2}+\sqrt{\aleph_k(1+D)-z\delta^2}}{\sqrt{\aleph_k(1+D)/2}-\sqrt{\aleph_k(1+D)-z\delta^2}}\right|, \\ \qquad 0 < z < \frac{\aleph_k(1+D)}{2\delta^2} \,\&\, \frac{\aleph_k(1+D)}{2\delta^2} < z \leq \frac{\aleph_k(1+D)}{\delta^2} \\ \frac{(1-\beta)\delta}{\pi\sqrt{z\aleph_k}} \log_e\left[\frac{\sqrt{\aleph_k}+\sqrt{\aleph_k - z}}{\sqrt{\aleph_k(1+D)/\delta^2}+\sqrt{\aleph_k(1+D)/\delta^2 - z}}\right], \\ \qquad 0 < z \leq \frac{\aleph_k(1+D)}{\delta^2} \\ \frac{(1-\beta)\delta}{\pi\sqrt{z\aleph_k}} \log_e\left[\frac{\sqrt{\aleph_k}+\sqrt{\aleph_k - z}}{\sqrt{\aleph_k}-\sqrt{\aleph_k - z}}\right], \\ \qquad \frac{\aleph_k(1+D)}{\delta^2} < z \leq \aleph_k \end{cases} \tag{35}$$

where $D = 4(\delta^2 - \delta)$ and $\beta = 2\delta - 1$.

Here, it is notable that for asynchronous low-duty DS system, the density of MAI variance shown in (34) becomes highly impulsive, which is far away from Gaussian density.

4.3.2 The UWB Hybrid DS/TH-CDMA System

Synchronous system. As we have described in appendix B, the density of Z_k in a UWB hybrid DS/TH synchronous system can be given by

$$f_{Z_k}(z) = \begin{cases} (1-\delta)^N \delta_D(z), & z = 0 \\ \sum_{j=1}^{N} \binom{N}{j} \frac{\delta^j (1-\delta)^{N-j}}{\pi\sqrt{z(j\aleph_k/N - z)}}, & 0 < z < \frac{j\aleph_k}{N} \end{cases} \tag{36}$$

Asynchronous system. For the hybrid asynchronous system, finding the actual density of Z_k using (8) and (18) is a tough task. Finding the density of $Z_{i,k}$ first and then convolving that $N-1$ times with itself to get the density of Z_k, a method that was used in [5], doesn't provide accurate results in our case. The reason is that the interferences occurring at different chips of the desired user are not independent; rather interferences at adjacent chips are dependent. Accurate density of Z_k cannot be obtained even from (20), because (20) is an expression that can only approximate the mean and the variance of the distribution accurately, not the distribution as a whole. The SIGA works excellent and we don't need the density.

5. Numerical Examples

In this section, the results derived in the previous sections are illustrated by providing specific examples. We present results for equal and unequal power levels of the users. Here recall that the SNR, E_b/N_o, mentioned in the next subsections represents the SNR of the desired user 0 with $E_b = P_0T_b$.

5.1 Equal Powers

5.1.1 Performance

With the assumption of perfect power control, in this subsection, we set $P_k = P$ for all users, $k = 0, 1, \ldots, K-1$. Because the asynchronous systems are more practical, we first present results for such systems. Figure 3 shows BEP vs. SNR, E_b/N_o for asynchronous UWB DS- and hybrid DS/TH-CDMA systems with $N = 20$, chip-duty $\delta = 0.5, 0.25$ and total users $K = 8$. As seen, lowering of chip-duty provides better performance, since $PG = N/\delta$ is obtained. The results are presented from SGA, SIGA and Monte Carlo simulations. The results from the SGA tend to tell us that with similar system parameters such as, chip length N, chip-duty δ etc, both the DS and DS/TH systems

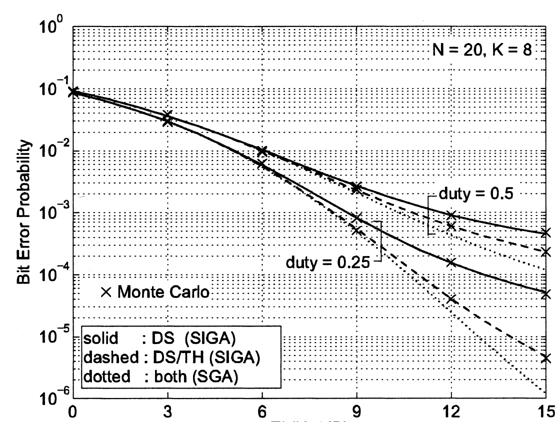

Fig. 3 BEP vs. SNR, E_b/N_o for asynchronous UWB DS- and hybrid DS/TH-CDMA for $N = 20$ and chip-duty $\delta = 0.5, 0.25$ with $K = 8$.

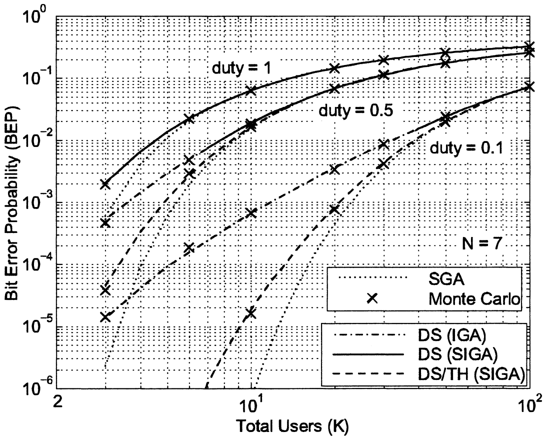

Fig. 4 BEP vs. total number of users for the asynchronous UWB DS- and hybrid DS/TH-CDMA for $N = 7$ and chip-duty $\delta = 1.0$ (DS only), 0.5 and 0.1 as shown. The processing gain in each case is $PG = N/\delta$. AWGN is neglected, $E_b/N_o = \infty$.

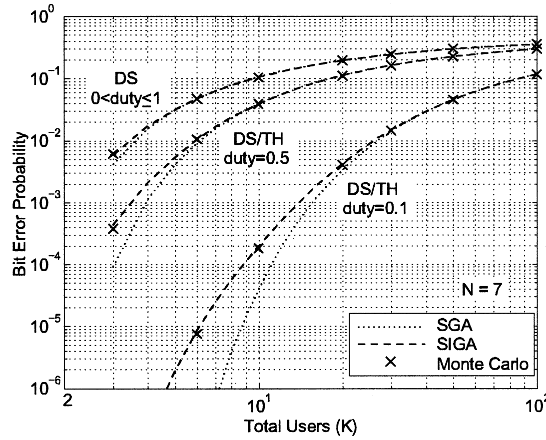

Fig. 5 BEP vs. total number of users for the synchronous UWB DS- and hybrid DS/TH-CDMA for $N = 7$ and chip-duty $0 < \delta \leq 1$ (DS), $\delta = 0.5$ and 0.1 (DS/TH) as shown. The processing gain in each case is $PG = N/\delta$. AWGN is neglected, $E_b/N_o = \infty$.

should show similar performance. However, though this is true at low SNR, the two systems are actually found to perform differently at high SNR. The results from the SIGA match well with those from simulation and can be considered as reliable approximations. Here note that at high SNR, the hybrid system shows better performance than the DS counterpart.

Figure 4 shows BEP vs. total users K for the asynchronous UWB DS- and hybrid DS/TH-CDMA systems with $N = 7$ and chip-duty $\delta = 1.0$ (DS only), 0.5 and 0.1. The SGA and SIGA are used for both types of the systems. To focus the applicability of SIGA at high SNR, the AWGN is neglected here. In other words, infinite SNR is assumed. As seen, the SIGA though excellent for the hybrid system, can only be evaluated beyond a number of users in DS (Table 1) that limits its use for low-duty DS signals. The IGA is used in such cases. The results obtained from the IGA and SIGA are very accurate and match well with those from Monte Carlo simulations. As before, performances of both the systems improve with lowering of chip-duty, because PG is obtained. However, the hybrid system performs better than the DS system for small to medium number of users, and the performance difference between them increases with lowering of chip-duty. In other words, the hybrid system performs closer to the SGA. In contrast, the SGA, which cannot distinguish between DS and DS/TH, becomes increasingly optimistic for the DS system with lowering of chip-duty.

Figure 5 shows BEP vs. K for the synchronous UWB systems. Performance of the DS system remains independent of chip-duty even though PG is obtained at low duty. The reason can be understood intuitively. Since, lowering of chip-duty brings no change in the collision property of synchronous DS spreading sequences, no change in the multiple access performance is seen. So, providing PG by lowering chip-duty is not the right way for a synchronous DS system. However, performance of the synchronous hybrid system

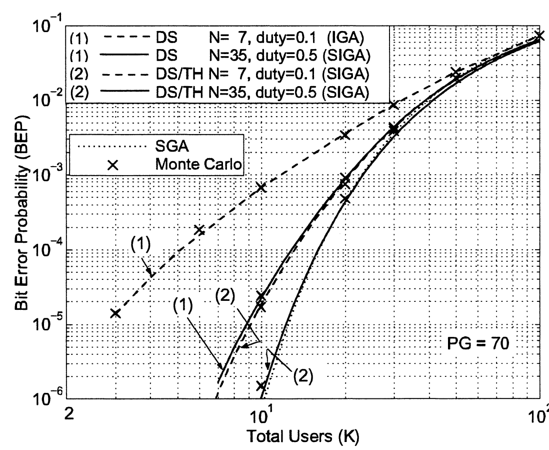

Fig. 6 Comparisons of performances of low-duty and high-duty systems under same processing gain ($PG = N/\delta = 70$). Asynchronous systems are considered. AWGN is neglected, $E_b/N_o = \infty$.

improves with lowering of chip-duty. This is because the collision probability for DS/TH sequences decreases with lowering of chip-duty even in synchronous case.

Because of practical importance, let us now come back to asynchronous system again. Recall that the processing gain of the system is defined as $PG = N/\delta$. Hence, systems may have same PG with different combinations of N and δ. How N and δ affect the multiple access performance is investigated next. In Fig. 6, we compare the performance of low-duty and high-duty UWB systems under the same $PG = 70$, with $(N, \delta) = (7, 0.1)$, $(35, 0.5)$ and $E_b/N_o = \infty$. For small to medium number of users, a high-chip-duty system performs better than a similar low-duty system.

However, the performance difference is larger in the case of DS system. Figure 7 shows the BEP vs. PG for $K = 4$ with $E_b/N_o = 15$ dB. As expected, performance improvement is seen with increase in the PG. The effects of N and δ on the multiple access performance looks similar from the view point of SGA. However, except in small PG,

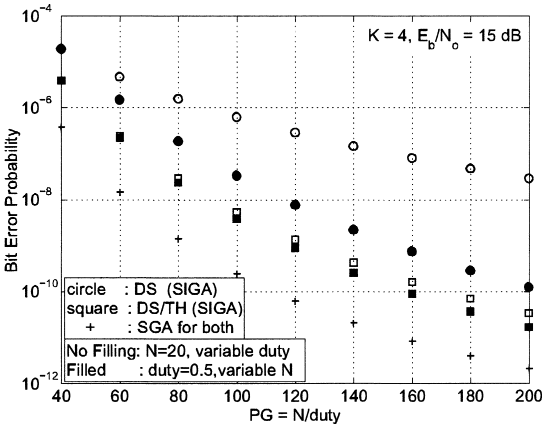

Fig. 7 BEP vs. processing gain $PG = N/\delta$ for asynchronous UWB DS- and hybrid DS/TH-CDMA with $K = 4$ and $E_b/N_o = 15$ dB.

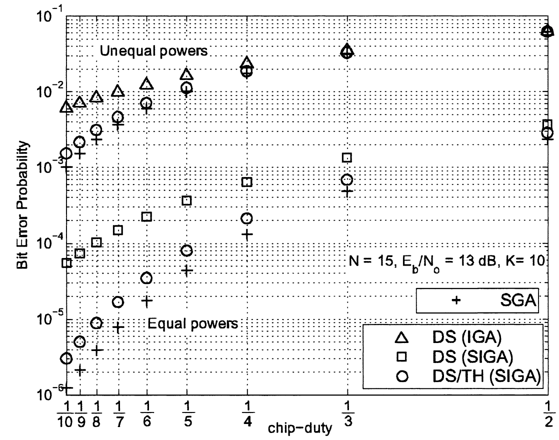

Fig. 8 BEP vs. chip-duty with equal ($P/P_0 = 0$ dB) and unequal ($P/P_0 = 6$ dB) power levels of the users, where $P_k = P$, $k = 1, 2, \ldots, K-1$. Asynchronous systems are considered with $N = 15$, $E_b/N_o = 13$ dB and $K = 10$.

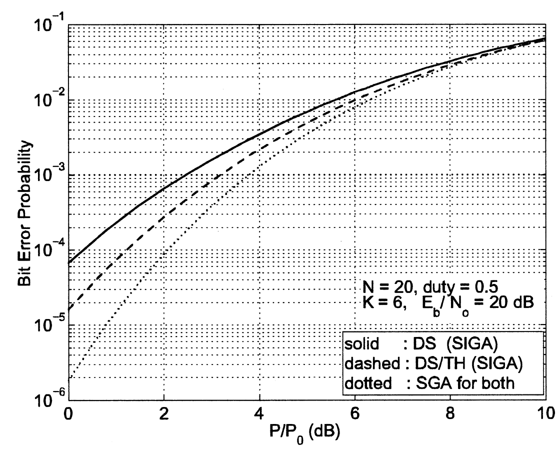

Fig. 9 BEP vs. P/P_0 where $P_k = P$, $k = 1, 2, \ldots, K-1$ for total users, $K = 6$. Here, SNR, $E_b/N_o = 20$ dB, $N = 20$ and $\delta = 0.5$. Asynchronous systems are considered.

the actual system performance becomes different for different combinations of N and δ, despite having the same PG. The PG is increased by 1) decreasing δ keeping N fixed at 20 and 2) increasing N keeping δ fixed at 0.5. As seen, better performance is obtained in the second case. This is because, between N and δ, N effects performance more dominantly than δ. Also, notably, the SGA which becomes increasingly optimistic as compared to the SIGA, can neither differentiate between the DS and DS/TH systems, nor can it model the effect of N and δ under the same PG. As seen from SIGA, the use of long repetition code brings more performance improvement than decreasing signal duty-cycle ($= \delta$) to increase the PG.

In the results presented so far, understanding the role of the PRF of the systems could be of interest. It can be shown that for both the systems, the average PRF$= 1/T_c = N/T_b$. Hence, a fixed data rate implies $N \propto$ PRF. So, the effect of PRF on multiple access performance is proportional to that of N.

5.1.2 Remarks on Collision Properties

As we have indicated in the introduction, modeled in the subsequent sections and just evaluate above, the multiple access performance with low-duty signals largely depends on the collision properties of the signals. To investigate the matter further, let us define a term *probability of collision (PoC)*, which represents the probability that the signal of any interfering user collides in some part with that of the desired user. We consider asynchronous systems. The PoC for DS signaling equals 2δ. However, note that a collision in this case means a collision in all N pulses of the bit. Unlike this, the probability that there may be collision in any chip equals 2δ for the DS/TH signal. Because the collisions in different chips are independent, the probability that there is no collision at all over the bit duration is $(1-2\delta)^N$, giving us the PoC equal to $1-(1-2\delta)^N$. It can be seen that for $\delta < 0.5$, $\mathrm{PoC}_{DS/TH} \gg \mathrm{PoC}_{DS}$. This more random collisions makes the central limit theorem (CLT) [22] more applicable for DS/TH system. Hence, its performance under perfect power control approaches to the lower bound given by the SGA, resulting in much better performance than that of DS for small to medium number of users. Note that the phenomena described above becomes possible because in binary system, a collision may randomly either aid or degrade the desired user's signal.

5.2 Unequal Powers

Figures 8 and 9 show the effects of unequal power levels of the users on error performance of the two types of asynchronous systems and on the validity of the SGA. We consider a case where the interfering users (except the desired user 0) use the same power $P_k = P$, $k = 1, 2, \ldots, K-1$. Figure 8 presents the BEP versus chip-duty δ for the systems with equal ($P/P_0 = 0$ dB) and unequal ($P/P_0 = 6$ dB) powers of the users. Because for the DS/TH system $\delta = 1/I$, where I can take only integer values ≥ 2, we present only discrete values in Fig. 8 as we did in Fig. 7. We set $N = 15$

and SNR, $E_b/N_o = 13$ dB and $K = 10$. As seen, while the interfering users have higher power level, system performance is degraded. However, still decreasing chip-duty brings performance improvement in both cases of equal and unequal powers. Here note that while we have presented most of the accurate results of Fig. 8 using SIGA, we were unable to use it for the DS system with $P/P_0 = 6$ dB (see Eq. (31)). The IGA, though a bit more involved, was useful in this case.

From Fig. 8, it is also notable that the performance difference between DS and DS/TH systems, especially at low duty, decreases considerably at unequal power case. To investigate the effect of unequal power levels on the two types of systems further, we present BEP vs. P/P_0 in Fig. 9. We set $K = 6$ and SNR, $E_b/N_o = 20$ dB and vary the ratio $\frac{P}{P_0}$ from 0 dB to 10 dB. Here note that the MAI increases with increasing $\frac{P}{P_0}$, and $\frac{P}{P_0} = 0$ dB stands for the special case of equal powers for all users. Figure 9 shows results both from SIGA and SGA for both the systems with $N = 20$ and $\delta = 0.5$. Though for small values of $\frac{P}{P_0}$, the SGA is optimistic, it's interesting to see that the SGA is reasonably accurate for $\frac{P}{P_0} > 6$ dB. Figure 9 also shows that though the DS/TH system performs better for small $\frac{P}{P_0}$, performance of such system is degraded faster as $\frac{P}{P_0}$ increases, until the SIGA touches SGA where both DS and DS/TH systems perform comparably. As a reason for this, the collision properties of the signals are again responsible. As explained in the preceding subsection, the 'more collision' and $\mathrm{PoC}_{DS/TH} \gg \mathrm{PoC}_{DS}$ was the underlying reason for better performance of DS/TH in case of equal powers of all users. Unfortunately, the same 'more collision' and $\mathrm{PoC}_{DS/TH} \gg \mathrm{PoC}_{DS}$ in DS/TH make it more vulnerable to powers coming from other users, a factor that becomes critical in the absence of power control, especially while the interfering users have power levels higher than the desired user.

6. Conclusions

Bit error probabilities have been presented for UWB DS- and hybrid DS/TH-CDMA systems that use spreading sequences with arbitrary chip-duty. The multiple access performances of the systems were compared and the effect of chip-duty on the validity of various approximations used for deriving closed form expressions was investigated. It was shown that under the same processing gain, a high-duty system performs better than a low-duty system. It can be concluded from this paper that if perfect power control can be maintained, a hybrid DS/TH should be the better choice over DS, especially for low-duty signaling. However, if the perfect power control cannot be maintained, use of the hybrid system over DS may not be justified, especially, while the interfering users use higher power levels. A situation with no multipath is considered in this paper. Future research includes analysis over multipath channels.

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Appendix A

Finding the means of $S_{11}^n \sim S_{23}^n$, n being an integer:
Since S_{11} is an RV uniformly distributed over $[0, \delta]$,

$$E\left[S_{11}^n\right] = \delta^n E\left[(x_1)^n\right] = \frac{\delta^n}{n+1} \tag{A·1}$$

where x_1 is an RV uniformly distributed over [0, 1]. In a similar fashion, since S_{21} is an RV uniformly distributed over $[0, 1-\delta]$,

$$E\left[S_{21}^n\right] = (1-\delta)^n E\left[x_1^n\right] = \frac{(1-\delta)^n}{n+1} \tag{A·2}$$

By knowing that S_{13}, S_{22} and S_{23} are RVs uniformly distributed over $[1-\delta, 1]$, $[1-\delta, \delta]$ and $[\delta, 1]$ respectively, one can write

$$E\left[S_{13}^n\right] = E\left[(1-\delta+S_{11})^n\right] \tag{A·3}$$

$$E\left[S_{22}^n\right] = E\left[(1-\delta+x_2)^n\right] \tag{A·4}$$

where x_2 is an RV uniformly distributed over $[0, 2\delta-1]$ and

$$E\left[S_{23}^n\right] = E\left[(\delta+S_{21})^n\right] \tag{A·5}$$

those give the results shown in Sects. 4.1.1 and 4.2.1.

Appendix B

Finding the densities of Z_k:
Let us define $\aleph_k = NP_kT_c^2/2$.

Synchronous UWB DS system. (11) gives $Z_k = \aleph_k \cos^2\phi$ and (33) is easily obtained [21].

Asynchronous UWB DS system. Let us consider that the RV, $Z_k = UV$ where U and V are two independent RVs. The density of Z_k is then given by [14], [21]

$$f_{Z_k}(z) = \int_{v_1}^{v_2} \frac{1}{|v|} f_U\left(\frac{z}{v}\right) f_V(v) dv \tag{A·6}$$

with $v_1 = -\infty$ and $v_2 = +\infty$ where $f_U(u)$ and $f_V(v)$ are the densities of U and V respectively. Using (12) in (14), Z_k for $0 < \delta \le 0.5$ is defined by defining, $U = 2\cos^2\phi$ and $V = \aleph_k(1-S_{11}/\delta)^2/2$ for $0 \le \tau_k < T_p$, $V = 0$ for $T_p \le \tau_k < T_c - T_p$ and $V = \aleph_k(1-1/\delta+S_{13}/\delta)^2/2$ for $T_c - T_p \le \tau_k < T_c$. Following [21], it can be shown that the density of U is given by, $f_U(u) = \pi^{-1}[u(2-u)]^{-1/2}$; $0 < u < 2$ and that of V is given by $f_V(v) = \delta[\aleph_k v/2]^{-1/2}$; $0 < v < \aleph_k/2$ for $0 \le \tau_k < T_p$ and $T_c - T_p \le \tau_k < T_c$, and $f_V(v) = \delta_D(v)$; $v = 0$ for $T_p \le \tau_k < T_c - T_p$ where $\delta_D$is Dirac delta (impulse) function of unit area. Finally, using (A·6) with $v_1 = 0$, $v_2 = \aleph_k/2$ we get (34).

For $0.5 < \delta \le 1$, using (13) in (14) shows that U remains as above and V becomes $V = \aleph_k(1-S_{21}/\delta)^2/2$ for $0 \le \tau_k < T_c - T_p$, $V = \aleph_k[(S_{22}^2 - S_{22}) + (\delta^2 - \delta) + 1/2]/\delta^2$ for $T_c - T_p \le \tau_k < T_p$ and $V = \aleph_k(1 - 1/\delta + S_{23}/\delta)^2/2$ for $T_p \le \tau_k < T_c$. Again following [21], the densities can be given by $f_V(v) = \delta[\aleph_k v/2]^{-1/2}$; $\aleph_k(D+1)/(2\delta^2) < v < \aleph_k/2$ with $D = 4(\delta^2 - \delta)$ for $0 \le \tau_k < T_c - T_p$ and $T_p \le \tau_k < T_c$, and $f_V(v) = 2\delta^2[4\aleph_k\delta^2 v - \aleph_k^2(D+1)]^{-1/2}$; $\aleph_k(D+1)/(4\delta^2) < v < \aleph_k(D+1)/(2\delta^2)$ for $T_c - T_p \le \tau_k < T_p$. Finally, following [14] we get (35).

Synchronous UWB DS/TH system. (16) gives $Z_k = UV = (\aleph_k/N)\cos^2\phi J_s$. Density of $V = J_s$ is given by (17) with $p = \delta$, and of $U = (\aleph_k/N)\cos^2\phi$ by $f_U(u) = \pi^{-1}[u(\aleph_k/N - u)]^{-1/2}$; $0 < u < \aleph_k/N$ [21]. Then using (A·6) we get (36).

Mohammad Azizur Rahman received his Bachelor's degree in Electrical and Electronic Engineering (EEE) from Bangladesh University of Engineering and Technology (BUET), Dhaka, Bangladesh in August 2001. From August 2001 to March 2002 he served as a Lecturer in the Department of EEE of the same university. From April 2002, he is attached with Niigata University, Niigata, Japan as a Japanses Goverment (Monbukagakusho) scholar. In October 2002, he entered the Department of EEE of the university as a research student, where he later in April 2003 entered as a Master's degree student. He is presently working toward a Doctoral degree in the same place. His research interests include Spread Spectrum and Ultra-wideband Communications, Communication Theory, Rake and Diversity Receivers, Signaling Design, Multiple Access Communications etc. Mr. Rahman was a recipient of the 2005 Telecommunication Advancement Foundation Technology Award for Students for outstanding Master's degree thesis. He is a student member of the IEEE.

Shigenobu Sasaki received B.E., M.E. and Ph.D. degrees from Nagaoka University of Technology, Nagaoka, Japan, in 1987, 1989 and 1993, respectively. Since 1992, he has been with Niigata University, where he is now an Associate Professor in the Department of Electrical and Electronic Engineering. From 1999 to 2000, he was a visiting scholar at the Department of Electrical and Computer Engineering, University of California, San Diego. Since 2003, he has been with the UWB technology institute, National Institute of Information and Communication Technology (NICT) as an expert researcher. His research interests are in the area of digital communications with special emphasis on spread spectrum communication systems, ultra-wideband communication systems and wireless communications. He is a member of the IEEE and SITA.

Hisakazu Kikuchi received the B.E. and M.E. degrees from Niigata University in 1974 and 1976, respectively, and Dr. Eng. degree from Tokyo Institute of Technology in 1988.From 1976 to 1979 he worked at Information Processing Systems Laboratory, Fujitsu Ltd., Tokyo. Since 1979 he has been with Niigata University, where he is a professor of electrical engineering. He was a visiting scholar at Electrical Engineering Department, University of California, Los Angeles during a year of 1992 to 1993. He holds a visiting professorship at Chongqing University of Posts and Telecommunications, China, since 2002. His research interests are in the areas of image/video processing and digital signal processing as well as ultra wideband systems. Dr. Kikuchi is a member of ITE (Institute of Image Information and Television Engineers), IIEEJ (Institute of Image Electronics Engineers of Japan), JSIAM (Japan Society for Industrial and Applied Mathematics), RISP (Research Institute of Signal Processing), IEEE, and SPIE. He served the chair of Circuits and Systems Group, IEICE, in 2000 and the general chair of Digital Signal Processing Symposium, IEICE, in 1988 and Karuizawa Workshop on Circuits and Systems, IEICE, in 1996.