

# Effects of Localized Distribution of Terminals and Mobility on Performance Improvement by Direct Communication

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**SUMMARY** We investigated performance improvement in a cellular system by introducing direct communication between terminals. Previous research has indicated that direct communication efficiently uses channels; however, this is not always so. We studied two factors that affect how much efficiency improves. One is the distribution of terminals. We defined some typical distributions with localization of terminals and analyzed how the difference between the distributions affected the performance improvement by direct communication. Another factor is the mobility of terminals, because mobility shortens the length of time during which terminals are directly connected. We analyzed how mobility affected performance improvement by direct communication. For the analyses, we used some theoretical techniques.

**key words:** cellular system, direct communication, localized distribution of terminals, mobility

## 1. Introduction

In cellular systems, a mobile terminal is always connected to a base station (BS) during communication even if it communicates with another mobile terminal in the same cell. In [1] and [2], a new type of a cellular system is proposed, in which a mobile terminal is directly connected to another terminal if these terminals are close to each other. Such a direct connection requires only one channel, while a connection relayed by a BS requires two channels, one of which is used to connect a terminal and the BS, and the other is used to connect another terminal to the BS. Hence, the new cellular concept with direct communication efficiently uses channel resources and is suitable for wireless networks for operational mobile robots, because mobile robots tend to be close to each other while they are performing the same operation [1]. Note that this expectation implicitly assumes that the distribution of robots is localized.

Obviously, improvement of the efficiency of channel use depends on how many communications are carried by direct connections. Namely, if a lot of communications are carried by connection via a BS, direct connection does not work well from the viewpoint of efficient channel use. Therefore, knowing in what situations direct connection im-

proves the efficiency of channel use is very important.

In [3], a theoretical technique for analyzing traffic characteristics of circuit switching calls in cellular networks with direct connection is proposed. This technique can be used to compute the blocking probability and the carried traffic in a circular cell, where terminals are independently and uniformly distributed over the cell. Calls are classified into three kinds. The first one is calls between terminals in different cells and requires a channel in each cell. The second one is calls that can be carried by direct communication, and the third one is calls between terminals in a cell that are relayed by a BS. Performance improvement by direct communications is shown to be greatly affected by the ratio of call arrival rate of the first call to that of the sum of the three types of calls. The ratio that does not improve performance can be estimated based on the theoretical technique.

The ratio defined above is one of the factors that affect performance improvement by direct communication. However, other factors exist. In this paper, we focus on two other factors: localization of the distribution of terminals in a cell and mobility.

In [3], terminals are assumed to be uniformly and independently distributed over a cell; however, the distribution of terminals is potentially localized. Specifically, some terminals may gather at one place while other terminals gather at another place. This happens in the networks of operational robots, where the two places correspond to different work places of the robots. Then, frequent use of direct communication between terminals in the same place is expected. At the same time, however, if the two places are far from each other, terminals in different places cannot directly connect easily. Therefore, the efficient channel use depends on the location and the number of the work places. Hence, we theoretically explore the effects of such a localized distribution on the performance improvement by direct communication.

Mobility is another factor. A direct connection between terminals can potentially fail due to mobility because the distance between them may become greater than the communication range for direct communication before the end of communication. In such a case, improvement of the efficiency of channel use depends on how long direct communication is possible before the direct link breaks. Taking these concerns into account, we theoretically investigate the effects of mobility on performance improvement by direct communication.

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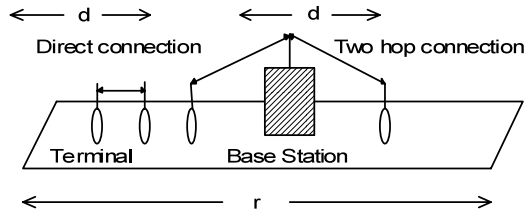


Fig. 1 Model of cell and communication range.

In the process of our investigations, we compute the blocking probability and the carried traffic in a cellular system with direct communication using new theoretical techniques.

This paper consists of five parts. In Sect. 1, we give an introduction, and in Sect. 2, we give some basic definitions and assumptions. In Sects. 3 and 4, we consider the effects of the localized distribution of terminals and mobility on performance improvement by direct communication, respectively. In Sect. 5, we present our conclusion.

## 2. Definitions and Assumptions

We consider a street cellular network and assume that the service area is on a line (street). This assumption is made to clarify the effects of the two factors on the efficiency of channel use and to simplify analysis. A cell is shown in Fig. 1. Let  $r$  be the length of a cell. A base station (BS) exists at the center of the cell. We consider a single cell system.

We assume that two terminals can judge by themselves whether direct communication between them is possible or not based on the received power levels of control signals which are sent to each other [1], [2]. In this paper, we assume that the received power levels are always sufficient for direct communication if the distance between the two terminals is not longer than  $d$ . We also assume that the terminals notify a BS of the result of the judgment. Then, we assume the following: Let  $Y$  be the distance between two terminals. If  $Y$  is not greater than  $d$ , these terminals can directly communicate with each other using one channel. Otherwise, these terminals are relayed by the BS, using two channels. Channel assignment is always done by the BS. The BS assigns a channel to a call which is carried by direct communication and assigns two channels to a call which is relayed by the BS based on the result of the judgment which is sent from the terminals. We assume that a channel is not spatially reused in a cell.

$S$  denotes the total number of channels in a cell. Arriving calls obey a Poisson process with an arrival rate of  $\lambda_0$ . The lifetime of a call obeys an exponential distribution with a mean of  $h_0$ .

## 3. Effects of Localized Distribution

### 3.1 Analysis

In this paper, we consider the effects of the localized distribution of terminals on performance improvement by direct communication by comparing the case where terminals are randomly and uniformly distributed over a cell (Case 1) with other cases where terminals tend to gather at specific places.

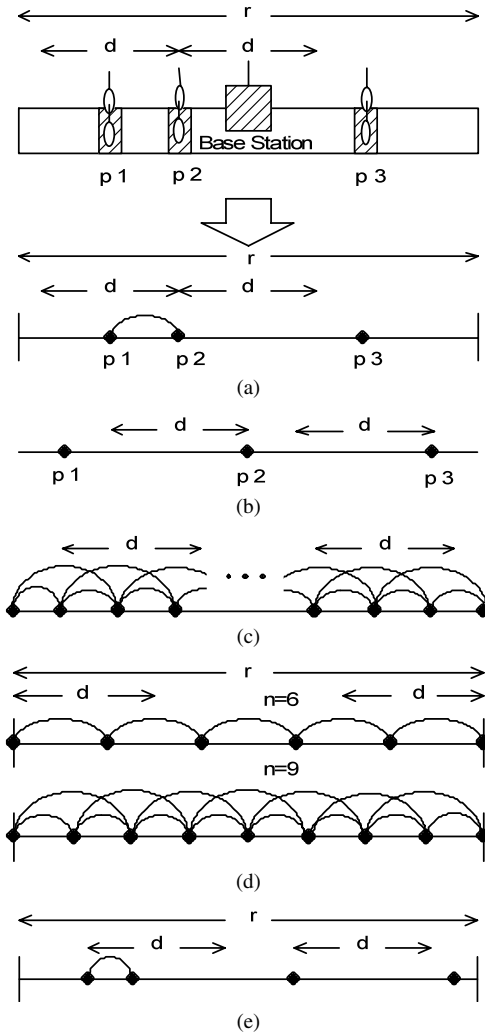


Fig. 2 Points where terminals exist in each case. (a) Example of location of places where terminals exist. (b) Case 2. (c) Case 3 ( $k = 2$ ). (d) Case 4. (e) Case 5.

For simplicity, we assume that terminals in the cell do not move. Assume that every call is between two terminals in the cell.  $P_c$  is defined as the probability that two terminals can be directly connected. Let  $Y$  be the distance between the two terminals. Then, in Case 1, we applied the following equation [4]:

$$P_c = P(Y \leq d) = \frac{2rd - d^2}{r^2}. \quad (1)$$

We consider some cases where a cell has specific places where terminals tend to gather. Assume that each of the places is a point on the one-dimensional cell as represented in Fig. 2(a). Let  $n$  be the number of places, and one terminal exists in each of the  $n$  places. Each place is numbered as shown in Fig. 2(a). Let  $p_i$  be the place  $i$ , where  $i = 1, \dots, n$ . Suppose a terminal exists at one of the  $n$  points randomly

and independently. The cases considered here are as follows:

Case 2: Distance between  $p_i$  and  $p_{i+1}$  is equal to  $L$ , which is a positive constant, for  $1 \leq i \leq n-1$ , and  $L > d$ . Note that the length of a cell is not defined in this case. Here, terminals can be directly connected only if they are at the same point.

Case 3: In this case,  $L \leq d$ . Other assumptions are the same as those for Case 2. Here, terminals can be directly connected even if they are at different points, as opposed to in Case 2.

Case 4: This case is almost the same as Cases 2 and 3; however, the length of a cell  $r$  is given in advance, and  $L$  is determined by  $n$  and  $r$ . A point exists at each edge of the cell, and  $L = r/(n-1)$ .

Case 5: In this case,  $n$  points are distributed uniformly and independently over a cell. This is different from Case 1 in that multiple terminals potentially exist at the same points.

These cases are graphically represented in Figs. 2(b)–(e).

In Case 2,  $P_c$  is equal to the probability that they exist at the same points. Then,

$$P_c = \frac{1}{n}. \quad (2)$$

In Case 3,  $G_i$  denotes the set of points where a node can directly communicate with the node at the point  $p_i$ . Suppose that  $kL \leq d < (k+1)L$ , where  $k$  is a positive integer. Then, the number of points in  $G_i$ , denoted by  $|G_i|$ , is equal to  $2k$  for  $k+1 \leq i \leq n-k$ ,  $|G_1| = k$ ,  $|G_2| = k+1, \dots, |G_k| = 2k-1$ , and  $|G_{n-k+1}| = 2k-1, \dots, |G_n| = k$ . Hence, by computing the number of combinations of places where terminals can be directly connected,

$$P_c = \begin{cases} \frac{n + \sum_{i=1}^k 2(k+i-1) + 2k(n-2k)}{n^2}, & k \leq n, \\ 1, & k > n. \end{cases} \quad (3)$$

In Case 4,  $L = r/(n-1)$ . Hence, if  $r$ ,  $d$ , and  $n$  are given,  $L$  can be determined, and  $k$  can be found such that  $kL \leq d < (k+1)L$ . In addition,  $P_c$  can be computed from Eq. (3) for the given  $r$ ,  $d$  and  $n$ .

In Case 5, direct communication is possible for two terminals in two cases. One is the case where terminals are at the same point. The probability they are at the same point is  $1/n$ . The other is the case where they are at different points but the distance between them is not greater than  $d$ . The probability that direct communication is possible in the second case is given by Eq. (1). Then, in Case 5,

$$P_c = \frac{1}{n} + \frac{n-1}{n} \frac{2rd - d^2}{r^2}. \quad (4)$$

In this paper, we consider a single cell in which every call is between terminals in that cell and assumed the following channel assignment procedure. When a new call arrives and the distance between the terminals is not greater

than  $d$ , the new call is accepted by direct communication if there exists at least one available channel in the cell. Otherwise, the call is blocked. If two channels are available when a call needs to be relayed by the BS, the call is accepted. Otherwise, the call is blocked. The blocking probability in each case can be computed by the method in [3].  $\lambda_1$  and  $\lambda_2$  denote call arrival rates for direct communication and two-hop communication through the BS, respectively. Then,  $\lambda_1 = P_c \lambda_0$  and  $\lambda_2 = (1 - P_c) \lambda_0$ . We define  $a_1 = \lambda_1 h_0$ ,  $a_2 = \lambda_2 h_0$ , and  $m = \lfloor S/2 \rfloor$ , where  $\lfloor x \rfloor$  is the maximum integer that does not exceed  $x$ . Let  $B_1$  and  $B_2$  be the blocking probabilities of direct and two-hop communication, respectively. Then,  $B_1$  and  $B_2$  can be computed by the follow equations [3], [7], [8]:

$$B_1 = P_{0,0} \sum_{j=0}^m \frac{a_1^{S-2j}}{(S-2j)!} \frac{a_2^j}{j!}, \quad (5)$$

and

$$B_2 = P_{0,0} \sum_{j=0}^{m-1} \sum_{i=S-2(j+1)+1}^{S-2j} \frac{a_1^i}{i!} \frac{a_2^j}{j!} + P_{0,0} \frac{a_2^m}{m!} \sum_{i=0}^l \frac{a_1^i}{i!}, \quad (6)$$

where  $l = S \bmod 2$  and  $P_{0,0}^{-1} = \sum_{j=0}^m \sum_{i=0}^{S-2j} \frac{a_1^i}{i!} \frac{a_2^j}{j!}$ . Let  $a_c$  be the traffic carried in the cell. Then,  $a_c = a_1(1 - B_1) + a_2(1 - B_2)$ .

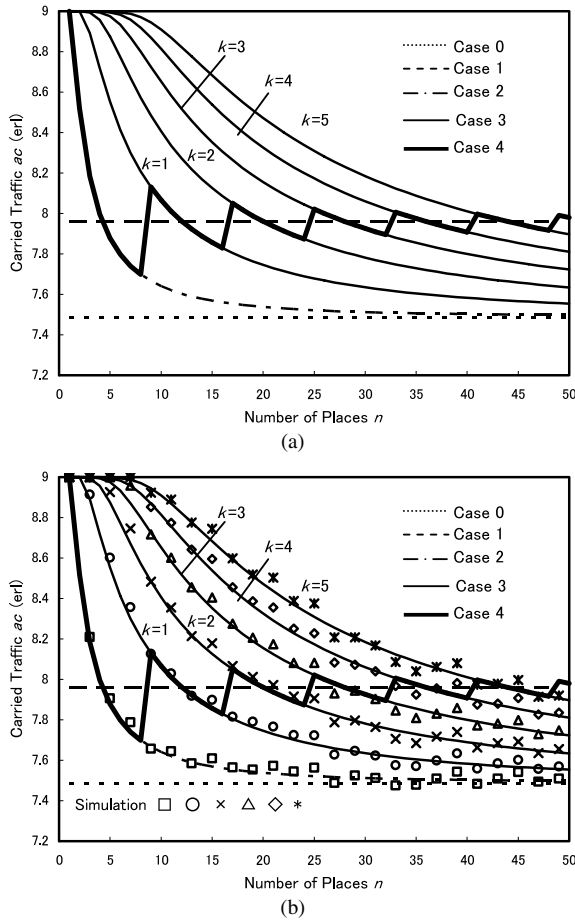
For reference, we also consider a pure cellular system, where a terminal is always connected to a base station during communication (Case 0). We compute the blocking probability by the Erlang B formula, assuming the offered load is  $\lambda_0 h_0$  and the number of channels is  $\lfloor S/2 \rfloor$ , because two channels are required for two-hop communication through a BS.

### 3.2 Results and Discussion

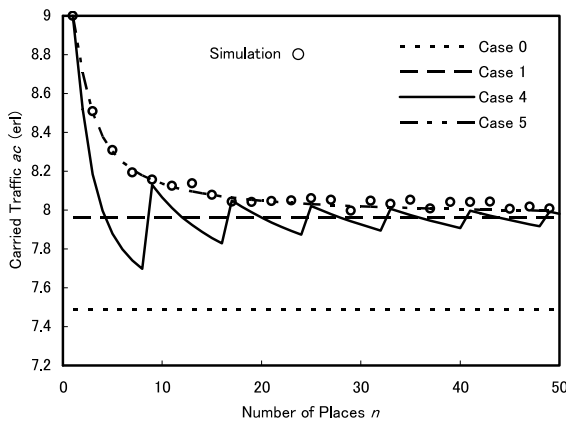
The numerical results of  $a_c$  for Cases 0 to 5 are given in Figs. 3 and 4 as a function of  $n$ , where  $\lambda_0 = 0.1 \text{ s}^{-1}$ ,  $h_0 = 90 \text{ s}$ ,  $r = 400 \text{ m}$ ,  $d = 50 \text{ m}$ , and  $S = 20$ . Although Figs. 3(a) and (b) are basically the same, the results of computer simulations in Cases 2 and 3 are also plotted in Fig. 3(b). The results of computer simulations in Case 5 are plotted in Fig. 4. The theoretical results agree well with the simulation results. From these results, we confirm the validity of the analysis.

In Cases 2 to 5, if  $n = 1$ , every call is carried by direct communication. Therefore, in such cases,  $a_c$  is greater than in Case 1 if  $n = 1$ . However, in Cases 2 and 3,  $a_c$  monotonically decreases as  $n$  increases. In Case 2, only one factor exists to increase  $P_c$  and  $a_c$ , the decrease of  $n$ , which increases the probability that terminals are at the same point. Therefore,  $a_c$  in Case 2 is larger than in Case 1 only in where  $n < r^2/(2rd - d^2)$ . However, the increase of  $n$  is a negative factor which decreases  $a_c$ , and  $a_c$  in Cases 2 and 3 finally converge to that in Case 0, where direct communication is not available.

In Case 3, in addition to the positive factor with Case



**Fig. 3** Traffic carried in Cases 0, 1, 2, 3 and 4. (a) Theoretical results. (b) Theoretical results with simulation results for Cases 2 and 3.



**Fig. 4** Traffic carried in Cases 0, 1, 4 and 5. Results of computer simulation for Case 5 are also plotted.

2, terminals can be directly connected even if they are at different points, and such opportunities increase as  $k$  increases. Therefore,  $a_c$  in Case 3 is larger than in Case 2. In Case 3, however, the size of a cell is not restricted, although it is restricted in actual situations. In Case 4, the size of a cell is restricted, and the increase of  $n$  sometimes increases  $a_c$  due to the restriction, as opposed to in Case 3. In Case

4, the increase of  $n$  is a positive factor from the viewpoint of the probability that terminals at different points can be connected directly. At the same time, it is a negative factor from the viewpoint of the probability that terminals are at the same point. Therefore, as can be observed in Fig. 3,  $a_c$  in Case 4 fluctuates and converges to that in Case 1.

As shown in Fig. 4, in Case 5, a terminal can be directly connected to another terminal at a point randomly distributed in the cell, similar to Case 1. In addition, terminals can be connected if they are at the same point. Hence, as shown in Fig. 4,  $a_c$  in Case 5 is greater than in Cases 1 and 4.

Our results show that direct communication works better in situations like Case 5 than situations like Cases 1 and 4 from the viewpoint of efficient use of channels. A situation like Case 4 will occur as well as Case 5. In this situation, the working places are fixed and the distance between adjacent working places is almost uniform. In such a case, direct communication does not always work well, as compared with Case 1.

Many situations exist beyond those observed here, and the increase of carried traffic by direct communication depends on each situation. These results show that the number of places to which terminals gather and the distribution of the working places should be carefully evaluated to design a cellular system with direct communication hoping increase of the carried traffic.

## 4. Effects of Mobility

### 4.1 Definitions and Assumptions

Here, we consider the effects of mobility on performance improvement by direct communication based on theoretical analysis of the carried traffic. Such an analysis has never been done even in a one dimensional service area. In the case where direct communication is applied to a cellular system with terminals move during communication, various channel assignment procedures can be considered. We consider a simple system that is a combination of a direct communication system and a pure cellular system, where channels are divided into two groups,  $C_1$  and  $C_2$ , which are used for the direct communication and pure cellular systems, respectively. Let  $S_1$  and  $S_2$  be the numbers of channels for direct and two-hop communication, respectively. Then,  $S = S_1 + S_2$ . This system is assumed for simplifying analysis and evaluating the minimum performance improvement by direct communication without ping pong effects, which are frequent transitions between direct and two-hop communication.

The traffic flows in the system are explained in Fig. 5. The traffic flow  $G_1$  represents the arrivals of new calls that are generated by terminals that can be directly connected, and  $G_2$  represents the arrivals of other new calls that are generated by terminals that cannot be directly connected. As depicted in Fig. 5,  $G_1$  first flows into  $C_1$ . If the calls of  $G_1$  encounter the situation where all channels of  $C_1$  are

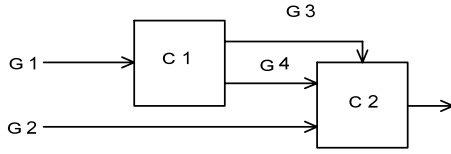


Fig. 5 Traffic flows in system.

used by other calls, these calls form an overflow from  $C_1$  to  $C_2$ . Otherwise, the calls of  $G_1$  are accepted by the direct communication system and are carried by the channels of  $C_1$ . The overflow from  $C_1$  is denoted by  $G_3$ , which flows into  $C_2$ .

A call of  $G_2$  or  $G_3$  must be carried by  $C_2$ . If there are at least two available channels, the call is accepted. Otherwise, the call is blocked.

Direct communication for a call sometimes fails before the end of the call because the distance between the terminals of the call becomes greater than  $d$  due to moving. Suppose that such a call is carried by two channels of  $C_2$  after direct communication fails. The transition from direct communication to two-hop communication forms another traffic flow from  $C_1$  to  $C_2$ , which is denoted by  $G_4$ . The above system does not permit the transition from two-hop communication to direct communication.

We make several other assumptions. We consider a single cell system. Terminals are uniformly and independently distributed over the cell. Terminals move right or left at a constant velocity  $v$ . The probability that a terminal moves right or left is equal to  $1/2$ . Assume a call in two-hop communication via a base station releases channels if one of the two terminals leaves the cell, and a call in direct communication releases a channel if the two terminals leave the cell.

#### 4.2 Offered Load for Direct Communication

We compute the call arrival rate and the mean holding time for direct communication. Let  $\lambda_1$  be the arrival rate of calls using direct communication. From Eq. (1),

$$\lambda_1 = \lambda_0 \cdot \frac{d(2r-d)}{r^2}. \quad (7)$$

Consider the mean of the holding time for direct communication, which is the time interval between the time at which direct communication begins and the time at which direct communication ends. To consider the holding time, we consider three different situations. We define Cases 6, 7, and 8 to classify these situations. In Case 6, terminals in direct communication move in the same direction. In Case 7, terminals approach and pass each other. In Case 8, terminals move away from each other. From the assumptions in the preceding section, direct communication ends in the cell when both terminals leave the cell even if the call is alive in Case 6. Of course, direct communication also ends when the call ends. Before the terminals leave the cell, in Cases 7 and 8, direct communication ends right after the distance between the terminals becomes equal to  $d$ . We use three random variables,  $T_1$ ,  $T_2$ , and  $T_3$ , to compute the mean holding

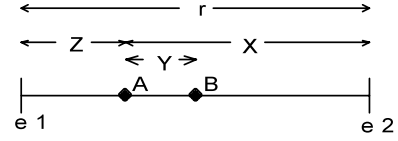


Fig. 6 Case 6.

time. Let  $T_1$  be the time from when direct communication begins to when both terminals leave the cell in Case 6. Let  $T_2$  and  $T_3$  be the times from when direct communication begins to when the distance between the terminals becomes equal to  $d$  in Cases 7 and 8, respectively.

First, we consider  $T_1$  using Fig. 6. Suppose that terminal A and terminal B are directly communicating with each other. The edges of the cell are denoted by  $e_1$  and  $e_2$ . Suppose that terminal A is between  $e_1$  and terminal B, and A and B move toward  $e_2$ . Let  $X$  be a random variable of the distance between A and  $e_2$  at the moment when the call arrives in the system. Here,  $Z = r - X$ . Let  $Y$  be a random variable of the distance between A and B when the new call arrives in the system. Then,  $Y \leq d$ .  $T_1$  is the time from when direct communication begins to when terminal A arrives at  $e_2$  in Case 6.

In the situation represented in Fig. 6,

$$\begin{aligned} P(Z \leq z | Y \leq d) &= \frac{1 - \left\{ \left( \frac{r-z}{r} \right)^2 + \left( \frac{r-d}{r} \right)^2 - \left( \frac{r-d-z}{r} \right)^2 \right\}}{1 - \left( \frac{r-d}{r} \right)^2} \\ &= \frac{2z}{2r-d} \end{aligned} \quad (8)$$

if  $z > r-d$ , where  $z$  is a real number. From the definition,  $T_1 = \frac{X}{v}$ , and  $Z = r - X$ . Then, from Eq. (8),

$$P(T_1 \leq t | Y \leq d) = \frac{2vt-d}{2r-d} \quad (9)$$

if  $\frac{d}{v} < t \leq \frac{r}{v}$ , where  $t$  is a real number. In the same manner,

$$\begin{aligned} P(Z \leq z | Y \leq d) &= \frac{P(Z \leq z, Y \leq d)}{P(Y \leq d)} \\ &= \frac{2rz - z^2 - (r-d)^2}{2r-d-d^2} \end{aligned} \quad (10)$$

if  $r-d \leq z \leq r$ , and

$$P(T_1 \leq t | Y \leq d) = \frac{(vt)^2}{2rd-d^2} \quad (11)$$

if  $0 < t \leq \frac{d}{v}$ . From Eqs. (9) and (11), the density function of  $T_1$  given that  $Y \leq d$  is

$$f_{T_1}(t | Y \leq d) = \begin{cases} \frac{2v^2t}{2rd-d^2}, & 0 \leq t \leq \frac{d}{v}, \\ \frac{2v}{2r-d}, & \frac{d}{v} < t \leq \frac{r}{v}. \end{cases} \quad (12)$$

In Case 6, the mean holding time is the mean time interval between when the call arrives in the system and when the call ends before terminal A arrives at  $e_2$  or the two terminals leave the cell during communication. The mean holding time can be computed by using the results of the analysis of traditional and one-dimensional cellular networks [5], [6]. Consider a mobile terminal in a cell. Suppose that this terminal begins communication with a BS. The time interval from this moment to when this terminal arrives at the cell boundary is denoted by  $T$ . In [5] and [6], the mean holding time, given that  $T = t$ , can be computed by the following equation:

$$h' = h_0 \left\{ 1 - \exp\left(-\frac{t}{h_0}\right) \right\}. \quad (13)$$

This equation can also be used as the mean holding time given that  $T_1 = t$ . From Eq. (13) and the density function of  $T_1$  in Eq. (12), the mean holding time in Case 6, denoted by  $h_{1,1}$ , can be computed as follows:

$$\begin{aligned} h_{1,1} &= \int_0^{\frac{d}{v}} h_0 \left\{ 1 - \exp\left(-\frac{t}{h_0}\right) \right\} \frac{2v^2 t}{2rd - d^2} dt \\ &\quad + \int_{\frac{d}{v}}^{\frac{r}{v}} h_0 \left\{ 1 - \exp\left(-\frac{t}{h_0}\right) \right\} \frac{2v}{2r - d} dt \\ &= h_0 + \frac{2vh_0^2}{2r - d} \left\{ \exp\left(-\frac{r}{vh_0}\right) - h_0 \frac{v}{d} \left( 1 - \exp\left(-\frac{d}{vh_0}\right) \right) \right\}. \end{aligned} \quad (14)$$

In Cases 7 and 8, the density functions of  $T_2$  and  $T_3$ , given that  $Y \leq d$ , can be computed in the same manner as Case 6.

$$f_{T_2}(t|Y \leq d) = \begin{cases} \frac{2v}{d}, & \frac{d}{2v} < t \leq \frac{d}{v}, \\ 0, & t \leq \frac{d}{2v}, \end{cases} \quad (15)$$

and

$$f_{T_3}(t|Y \leq d) = \frac{2v}{d}. \quad (16)$$

$h_{1,2}$  and  $h_{1,3}$  denote the mean holding times in Cases 7 and 8, respectively. From Eq. (13) and the density functions,  $h_{1,2}$  and  $h_{1,3}$  are as follows:

$$h_{1,2} = h_0 + h_0^2 \frac{2v}{d} \left\{ \exp\left(-\frac{d}{vh_0}\right) - \exp\left(-\frac{d}{2vh_0}\right) \right\}, \quad (17)$$

$$h_{1,3} = h_0 - h_0^2 \frac{2v}{d} \left\{ 1 - \exp\left(-\frac{d}{2vh_0}\right) \right\}. \quad (18)$$

Let  $h_1$  be the mean holding time for direct communication in the cell. The probability that terminals in direct communication move toward the same direction is 1/2. The probability that terminals approach and pass each other is 1/4. The probability that they move away from each other is 1/4. Namely, the probabilities that situations defined as Cases 6, 7 and 8 happen are 1/2, 1/4 and 1/4, respectively.  $h_{1,1}$ ,  $h_{1,2}$  and  $h_{1,3}$  are the conditional mean holding times

given that situations defined as Cases 6, 7 and 8 happen, respectively. Hence, from the theorem of total expectation,  $h_1$  is represented as follows:

$$h_1 = \frac{1}{2}h_{1,1} + \frac{1}{4}h_{1,2} + \frac{1}{4}h_{1,3}. \quad (19)$$

We can compute  $h_1$  from Eqs. (14), (17), (18) and (19), and we can compute the offered load of  $G_1$ , denoted by  $a_3$ , as  $\lambda_1 h_1$ .

#### 4.3 Offered Load for $G_2$

$\lambda_2$  denotes the call arrival rate of  $G_2$ . Then,

$$\lambda_2 = \lambda_0 \cdot \frac{(r-d)^2}{r^2}. \quad (20)$$

Consider the mean of the holding time of  $G_2$ , which is the time interval between when two-hop communication begins and when two-hop communication ends. To consider the holding time, we consider three different situations. We define Cases 9, 10, and 11. In Case 9, terminals in two-hop communication move in the same direction. In Case 10, terminals approach each other. In Case 11, terminals move away from each other.

From the assumptions in the preceding section, two-hop communication ends in the cell when one of the two terminals leaves the cell even if the call is alive in Case 9. Of course, two-hop communication ends when the call ends. We use a random variable  $T_4$  to compute the mean holding time. Let  $T_4$  be the time from when two-hop communication begins to when one of the two terminals leaves the cell in Case 9. We consider  $T_4$  using Fig. 7. Suppose that terminal A and terminal B are communicating via a BS with each other. Suppose that terminal A is between  $e_1$  and terminal B, and A and B move toward  $e_1$ . Let  $X$  be the distance between A and  $e_1$  when the call arrives in the system. Let  $Y$  be the distance between A and B. Then,  $Y > d$ .  $T_1$  is the time from when two-hop communication begins to when terminal A arrives at  $e_1$  in Case 9.

The density function of  $T_4$  given that  $Y > d$  can be computed in the same manner as the derivation of Eq. (12):

$$f_{T_4}(t|Y > d) = \frac{2v(r-d-vt)}{(r-d)^2}. \quad (21)$$

From Eqs. (13) and (21), the mean holding time in Case 9, denoted by  $h_{2,1}$ , can be computed as follows:

$$h_{2,1} = h_0 - \frac{2vh_0^2}{r-d} + \frac{2v^2h_0^3}{(r-d)^2} \left\{ 1 - \exp\left(-\frac{r-d}{vh_0}\right) \right\}. \quad (22)$$

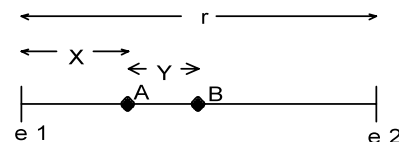


Fig. 7 Case 9.

Let  $h_{2,2}$  and  $h_{2,3}$  be the mean holding times in Cases 10 and 11, respectively. In the same manner as Case 9,

$$h_{2,2} = h_0 - \frac{2vh_0^2}{r-d} \exp\left(-\frac{r}{vh_0}\right) + \frac{2v^2h_0^3}{(r-d)^2} \left\{ \exp\left(-\frac{r}{vh_0}\right) - \exp\left(-\frac{d}{vh_0}\right) \right\}, \quad (23)$$

and

$$h_{2,3} = h_0 - \frac{4vh_0^2}{r-d} + \frac{8v^2h_0^3}{(r-d)^2} \left\{ 1 - \exp\left(-\frac{r-d}{vh_0}\right) \right\}. \quad (24)$$

Let  $h_2$  be the mean holding time for two-hop communication in the cell.  $h_2$  is as follows:

$$h_2 = \frac{1}{2}h_{2,1} + \frac{1}{4}h_{2,2} + \frac{1}{4}h_{2,3}. \quad (25)$$

Then,  $h_2$  can be computed from Eqs. (22), (23), (24), and (25), and the offered load of  $G_2$ , denoted by  $a_4$ , can be computed as  $\lambda_2 h_2$ .

#### 4.4 Offered Loads for $G_3$ and $G_4$

Let  $a_5$  be the offered load of  $G_3$ . We consider that  $a_5$  is only the offered load of an overflow from  $C_1$  and assumed that  $a_5 = B_1 \lambda_1 h_1$ , where  $B_1$  is the blocking probability of  $C_1$ , although the holding time of  $G_3$  is different from that of  $G_1$ . This assumption is an approximation, and we confirm its validity in Sect. 4.6.

Let  $a_6$ ,  $\lambda_4$ , and  $h_4$  be the offered load of  $G_4$ , the call arrival rate of  $G_4$ , and the mean holding time of  $G_4$ , respectively. Then,  $a_6 = \lambda_4 h_4$ . To compute  $\lambda_4$ , we use the probability that two terminals in direct communication would be reconnected by the BS during their communication because of motion. Let  $Y$  be distance between A and B when the new call arrives in the system. Specifically, we use the probability that the distance between the terminals would become greater than  $d$  given that  $Y \leq d$  because of motion. Let  $P_1$  be this probability given that the terminals approach to each other. Let  $P_2$  be the probability given that the terminals move away from each other. These probabilities and  $h_4$  can be computed in a similar manner to the above computation of the mean holding times:

$$P_1 = \frac{2vh_0}{d} \left\{ \exp\left(-\frac{d}{2vh_0}\right) - \exp\left(-\frac{d}{vh_0}\right) \right\}, \quad (26)$$

$$P_2 = \frac{2vh_0}{d} \left\{ 1 - \exp\left(-\frac{d}{2vh_0}\right) \right\}, \quad (27)$$

$$h_4 = h_0 - \frac{2vh_0^2}{r-d} \left\{ 1 - \exp\left(-\frac{r-d}{2vh_0}\right) \right\}. \quad (28)$$

In addition, if the terminals move in the same direction, the distance between them does not become greater than  $d$ . The probability that terminals move in the same direction is  $1/2$ . The probability that terminals approach and pass each other is  $1/4$ . The probability that they move away from each other is  $1/4$ . Hence, from the theorem of total probability, the

probability that two terminals in direct communication are reconnected by the BS during their communication is equal to  $\frac{1}{2} \times 0 + \frac{1}{4}P_1 + \frac{1}{4}P_2 = \frac{1}{4}P_1 + \frac{1}{4}P_2$ . Thus,

$$\lambda_4 = (1 - B_1) \frac{\lambda_1}{4} (P_1 + P_2). \quad (29)$$

From Eqs. (28) and (29), we compute  $a_6$ .

#### 4.5 Blocking Probability

Consider the blocking probability and the carried traffic of the system. From the assumptions in Sects. 2 and 4.1, the call arrivals of  $G_1$  and  $G_2$  obey Poisson processes with intensities of  $\lambda_1$  and  $\lambda_2$ , respectively. As assumed in Sect. 4.4, we consider that  $G_3$  represents overflow traffic with an offered load  $a_5$ . With  $G_4$ , we assume that call arrivals form a Poisson process of intensity  $\lambda_4$ . Then, in the system represented in Fig. 5, one traffic flow of a Poisson process flows into  $C_1$ , two flows of Poisson processes flow into  $C_2$ , and overflow traffic flows from  $C_1$  to  $C_2$ . Then, the system is a typical system that can be analyzed by the equivalent random method [7], [8]. Hence, we analyze the system using this method.

$B_1$  is defined as the blocking probability of  $C_1$ , and  $B_2$  is the blocking probability of  $C_2$ .  $E_x(y)$  is defined as the result of computing the Erlang B formula, where  $x$  is the number of channels and  $y$  is the offered load. Then,  $B_1$  can be computed by  $E_{S_1}(a_1)$  directly.

To compute  $B_2$  by the equivalent random method, we computed  $a^*$  and  $S^*$ , the equivalent random load and the number of equivalent channels, respectively. Let  $a_7$  be  $a_4 + a_6$ . Then,

$$a^* = \sigma^* + 3w(w-1), \quad (30)$$

and

$$S^* = \frac{b^* + w}{b^* + w - 1} a^* - b^* - 1, \quad (31)$$

where  $b^* = a_7 + b$ ,  $\sigma^* = a_7 + \sigma$ ,  $w = \sigma^*/b^*$ ,  $b = a_3 E_{S_1}(a_3)$ , and  $\sigma = b \left(1 - b + \frac{a_3}{S_1 + 1 - a_3 + b}\right)$ . From  $S_2$ ,  $S^*$ , and  $a^*$ ,

$$B_2 = \frac{E_{[S_2/2] + S^*}(a^*)}{E_{S^*}(a^*)}. \quad (32)$$

Let  $a_{c1}$  and  $a_{c2}$  be the traffic carried by  $C_1$  and  $C_2$ , respectively. The total carried traffic in a cell is as follows:

$$a_{tc} = a_{c1} + a_{c2} = a_3(1 - B_1) + a^*(1 - B_2). \quad (33)$$

For reference, we consider the blocking probability and the carried traffic in a pure cellular network, where direct communication is not available. The blocking probability  $B_{pure}$  can be computed by the Erlang B formula assuming that the number of channels is  $[S/2]$  and the offered load is  $\lambda_0 h_5$ , where  $h_5$  is the mean holding time in the pure cellular system, and

$$h_5 = h_0 - \frac{2vh_0^2}{r} + \frac{2v^2h_0^3}{r^2} \left\{ 1 - \exp\left(-\frac{r}{vh_0}\right) \right\}. \quad (34)$$

$a_{c,pure}$  denotes the carried traffic in the pure cellular network. Then,  $a_{c,pure} = (1 - B_{pure})\lambda_0 h_5$ .

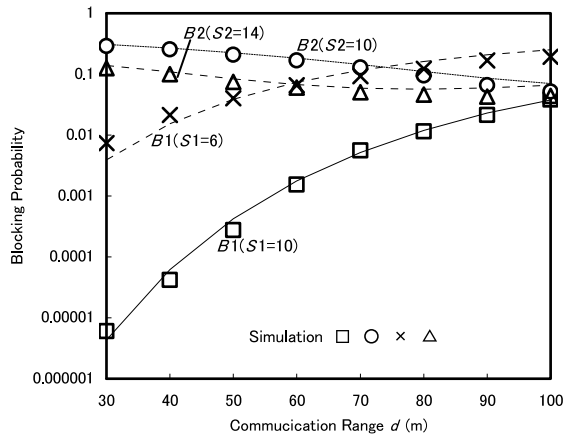


Fig. 8 Blocking probability.

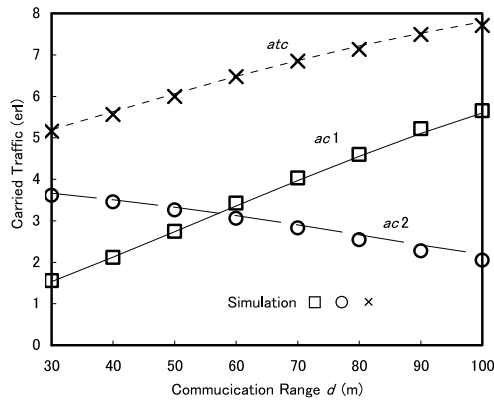


Fig. 9 Carried traffic.

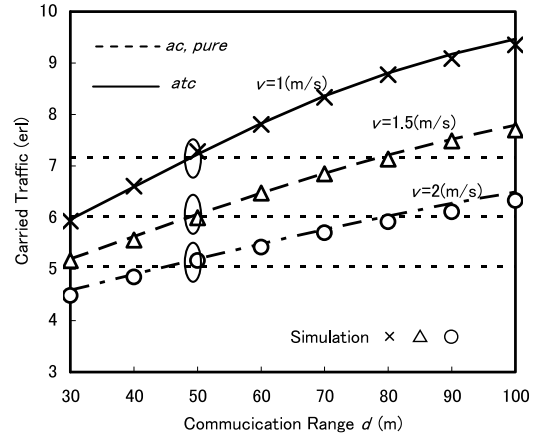
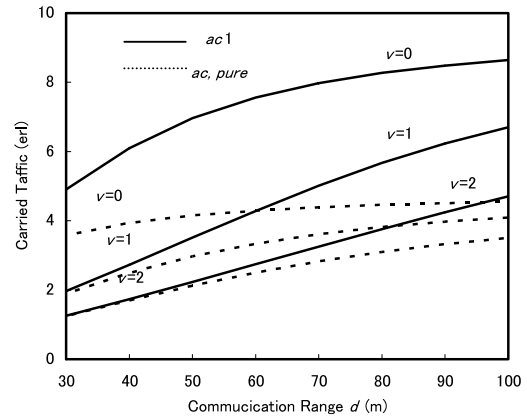
Fig. 10 Carried traffic for different  $v$ .

Fig. 11 Carried traffic.

#### 4.6 Numerical Results and Discussion

We use the following parameters:  $\lambda_0 = 0.2 \text{ s}^{-1}$ ,  $h_0 = 90 \text{ s}$ , and  $r = 200 \text{ m}$ . The numerical results of the theoretical computations of  $B_1$  and  $B_2$  for  $S_1 = S_2 = 10$  and  $S_1 = 6, S_2 = 14$  are shown in Fig. 8.  $a_{c1}$ ,  $a_{c2}$ , and  $a_{tc}$  for  $S_1 = S_2 = 10$  are shown in Fig. 9. We assume that  $v = 1.5 \text{ m/s}$ . The horizontal axis is the communication range  $d$ . The results of computer simulations are also plotted in both figures. The theoretical results agree well with the simulation results, although the analysis was done with some approximations. From these results, we confirm the validity of the analysis. In Fig. 9,  $a_{c1}$  increases and  $a_{c2}$  decreases as  $d$  increases, and  $a_{tc}$  increases as  $d$  increases.

In Fig. 10,  $a_{tc}$  and  $a_{c,pure}$  for different values of  $v$  are shown, where  $S_1 = S_2 = 10$ . We also show  $a_{c,pure}$  for  $S = 20$ . As shown in this figure,  $a_{tc}$  is smaller than  $a_{c,pure}$  for small values of  $d$  and larger than  $a_{c,pure}$  for large values of  $d$ . The system with direct communications we consider is accompanied with a negative effect of the division of channels into two groups and another negative effect of prohibiting reconnection from a two hop connection to a direct connection. From Fig. 10, direct communication potentially

increases the carried traffic even with such negative effects.

In Fig. 11, we show the theoretical results of  $a_{c1}$  with  $S_1 = 10$  and the offered load of  $a_3$  for  $v = 0, 1$ , and  $2 \text{ m/s}$ . In this figure,  $a_{c,pure}$  which is computed with  $S = 10$  and the offered load of  $a_3$ , is also shown. The carried traffic  $a_{c,pure}$  refers to the traffic carried by the pure cellular with  $S$  channels when the offered load is  $a_3$ . Specifically,  $a_{c,pure}$  refers to the capability of a two-hop connection under the same conditions as a direct connection with  $S_1 = 10$ . From this figure,  $a_{c1}$  is always greater than  $a_{c,pure}$ ; however, the difference between  $a_{c1}$  and  $a_{c,pure}$  decreases as  $v$  increases. Therefore, although a direct connection carries more traffic than a two-hop connection, the increase in the traffic is not large when mobile terminals are assumed. Then, a significant increase in the traffic carried by a direct connection cannot be expected in such a case, and the performance of a system with direct communication in a mobile environment must be carefully predicted.

#### 5. Conclusion

We theoretically considered the effects of the localized distribution of terminals on performance improvement by introducing direct communication to an ordinary cellular sys-



tem. We considered some typical distributions where terminals tend to gather at specific points and showed that the performance of direct communication is potentially degraded for some biased distributions while it sometimes improves due to the deviation of terminals. In our considerations, when terminals can be at the same place and the places are randomly distributed, direct communication improved performance.

Next, we theoretically analyzed the performance of a cellular system with direct communication considering effect of mobility. From the results of the analysis, we showed that performance improvement by direct communication is greatly affected by mobility, and significant improvements in a mobile environment cannot always be expected. Direct communication improves performance when the communication range is appropriately large and the velocity is appropriately low. Although our theoretical analysis has been done with some approximations and simple models, such an analysis has never been done before.

In the future, we plan to extend the theoretical methods to two-dimensional service areas, other motion patterns, other channel assignment procedures, etc.

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