IEICE TRANS. FUNDAMENTALS, VOL.E91–A, NO.11 NOVEMBER 2008

PAPER *Special Section on Wideband Systems*

Exact Error Rate Analysis for Pulsed DS- and Hybrid DS/TH-CDMA in Nakagami Fading*

Mohammad Azizur RAHMAN[†a)], ***Nonmember***, **Shigenobu SASAKI**[††], ***Member***, **Hisakazu KIKUCHI**[††], ***Fellow***, **Hiroshi HARADA**[†], ***Member***, ***and*** **Shuzo KATO**[†], ***Fellow***

SUMMARY Exact bit error probabilities (BEP) are derived in closed-form for binary pulsed direct sequence (DS-) and hybrid direct sequence time hopping code division multiple access (DS/TH-CDMA) systems that have potential applications in ultra-wideband (UWB) communications. Flat Nakagami fading channel is considered and the characteristic function (CF) method is adopted. An exact expression of the CF is obtained through a straightforward method, which is simple and good for any arbitrary pulse shape. The CF is then used to obtain the exact BEP that requires less computational complexity than the method based on improved Gaussian approximation (IGA). It is shown under identical operating conditions that the shape of the CF, as well as, the BEP differs considerably for the two systems. While both the systems perform comparably in heavily faded channel, the hybrid system shows better BEP performance in lightly-faded channel. The CF and BEP also strongly depend on chip length and chip-duty that constitute the processing gain (PG). Different combinations of the parameters may result into the same PG and the BEP of a particular system for a constant PG, though remains nearly constant in a highly faded channel, may vary substantially in lightly-faded channel. A comparison of the results from the exact method with those from the standard Gaussian approximation (SGA) reveals that the SGA, though accurate for both the systems in highly-faded channel, becomes extremely optimistic for low-duty systems in lightly-faded channel. The SGA also fails to track several other system trade-offs.

key words: *ultra-wideband, code division multiple access, error rate analysis, Nakagami fading*

1. Introduction

Recently, multiple access communications using ultra-wideband (UWB) signals have received considerable attention [1]. Several works have been presented on the topic [2]–[13]. The UWB technology usually transmits signal with low duty-cycle, whereas, conventional communication systems like direct sequence code division multiple access (DS-CDMA) transmit signal continuously i.e. with full duty [4], [12], [13]. Motivated by the emergence of UWB, new ideas of pulsed DS- and hybrid DS time hopping code division multiple access (DS/TH-CDMA) with low chip-duty have evolved [4]–[7], [11]–[13]. Envisioning the potential application of such signals in UWB, those have received much interest from the UWB community [4]–[7], [11]–[13]. While these pulsed CDMA can be thought as the generalization of conventional DS-CDMA with the scope for signal transmission with an arbitrary chip-duty, require considerably different treatment for developing analytical framework for performance evaluation [4], [12].

In [4] Chernoff bounds on bit error probabilities (BEP) were given for an episodic DS-CDMA system. Later, [5]–[11] presented approximate BEP for impulse radio systems with pulse based polarity randomization. More recently, [12] presented approximate BEP for pulsed DS- and hybrid DS/TH-CDMA systems considering additive white Gaussian noise (AWGN) channel. The standard, improved and simplified improved Gaussian approximations: abbreviated as SGA, IGA and SIGA respectively were considered in [5], [6], [10]–[12]. It was shown that the simple method SGA is not accurate for low-duty systems in general. The other simple method SIGA that was previously accurately used for conventional DS-CDMA systems [14], has restrictive region of validity for low-duty systems. However, the IGA [15] provided accurate results at the expense of much higher computational complexity. Presently, the method based on IGA presents the only-available reliable closed-form BEP expression for such systems.

Although there have been several studies [5]–[12] presenting approximate BEP of low-duty CDMA systems in AWGN channel, studies considering fading channel has not yet received much attention. Since the Nakagami fading can model a wide variety of fading scenarios, it can be used to model various practical channel conditions. Hence, studies of low-duty CDMA systems in Nakagami fading is of practical interest. In addition, most of the approximate models discussed above are inaccurate in a wide region of practical interest [12]. And since practical BEP requirement for system evaluation is low, simulations become extremely time-consuming. Hence, what we really need is a reliable model for performance evaluation that can be obtained from an exact analysis. Since exact analysis is usually limited by the complexity, the search for a simple exact analysis is universal. To the best knowledge of the authors, the exact error analysis hasn't been addressed for any low-duty system in Nakagami fading channel, though [2], [3] presented exact error analysis for several TH-UWB systems and a DS-UWB system with chip-duty of unity in AWGN channel.

In looking back to works on conventional DS-CDMA,

Manuscript received March 10, 2008.
Manuscript revised June 6, 2008.
[†]The authors are with the National Institute of Information and Communications Technology (NICT), Yokosuka-shi, 239-0847 Japan.
[††]The authors are (also the first author was) with the Department of Electrical and Electronic Engineering, Niigata University, Niigata-shi, 950-2181 Japan.
*This paper was presented in part in IEEE Wireless Comm. and Net. Conf. 2007 and IEEE Vehi. Tech. Conf. 2007 Spring.
a) E-mail: aziz@nict.go.jp
DOI: 10.1093/ietfec/e91–a.11.3150

[16] presents the most well known exact error analysis in AWGN channel. Since the complexity of exact analysis increases in a fading channel, [17], [18] presented the exact analysis and [19] presented a precise (but not exact) analysis for simply a binary phase shift keying (BPSK) link in the presence of co-channel interference (CCI) without considering any spreading (hence not CDMA) for mathematical tractability. At this moment, the exact BEP for conventional binary DS-CDMA employing an arbitrary pulse shape in flat Nakagami fading channel doesn't either exist in literature (see [14]–[25] and the references there in). Although an exact error analysis for conventional DS-CDMA in flat Nakagami fading channel is given recently in [24] by extending a previous work for Rayleigh channel [25], the analysis is solely for a rectangular pulse shape. Additionally, the computational complexity of the method is so high that even the authors presented numerical examples in the paper using an approximate model.

In this paper, we present exact error analysis for the pulsed DS- and hybrid DS/TH-CDMA systems with arbitrary pulse shape and chip-duty. We consider a flat Nakagami fading channel that is a single path channel with no delay spread. Since the analysis is performed for arbitrary chip-duty, it is also applicable for conventional DS-CDMA. The characteristic function (CF) method is adopted where the exact expression for CF is obtained in closed-form following a simple and straightforward procedure. Finally, standard methods are followed to obtain the exact expression of the BEP, the computational complexity of which is even simpler than the IGA. Taking advantage of the simplicity, the exact method is employed to investigate trade-offs among several system parameters. The inability of the SGA to track many such trade-offs is also pointed.

2. System Model

A typical representation of the signal of an arbitrary user $k \in (1, 2, \ldots, K)$ in the binary pulsed DS- and hybrid DS/TH-CDMA has the following form at the transmission side

$$s_k(t) = \sqrt{\frac{E_k}{N_s}} \sum_{l=-\infty}^{+\infty} b^{(k)}_{\lfloor l/N_s \rfloor} a^{(k)}_l \psi(t - lT_c - c^{(k)}_l T_p) \times \cos(2\pi f_c t + \theta_k) \tag{1}$$

where t is time and $\psi(t)$ is the unit-energy baseband communication pulse with duration T_p. Here, f_c is the carrier frequency and θ_k is the phase of transmission for user k, which is uniform in $[0, 2\pi]$. The rest of the signal structure is described as follows:

- E_k is the bit energy of user k.
- N_s is the number of chips or pulses per information bit. T_c is the chip duration and hence the bit duration is $T_b = N_s T_c$.
- $\{b^{(k)}_i\}$ is the i-th bit of user k, which is a random variable (RV) uniform on $\{+1, -1\}$. Here, $i = \lfloor l/N_s \rfloor$ and $\lfloor . \rfloor$ represents floor function.
- $\{a^{(k)}_l\}$ is the random polarity code for user k, which is also an RV uniform on $\{+1, -1\}$ and is periodic with period N_s. Note that various other sequences including m-sequences and orthogonal sequences are popular in literature. However, we consider random sequences for analytical tractability, especially in asynchronous environment [11]–[25].
- For the DS system, $T_p \le T_c$ and $\{c^{(k)}_l\} = 0$. Hence, the pulse width can be any fraction of the chip duration and it is placed in the beginning of each chip. The chip-duty is given by $\delta = T_p/T_c$, $0 < \delta \le 1.0$.
- For the hybrid DS/TH system, the chip duration T_c is divided into $N_c (\ge 2)$ equally spaced slots of duration T_p giving $T_c = N_c T_p$. $\{c^{(k)}_l\}$ is the TH code, which an RV uniform on $\{0, 1, \ldots, N_c - 1\}$ and is periodic with period N_s. The chip-duty is given by $\delta = 1/N_c$.
- The processing gain (PG) and the average pulse repetition rate (PRR) for both the systems are given by $PG = N_s/\delta$ and $PRR = 1/T_c$ respectively.

3. Multiple Access Interference Modeling

Considering a time-invariant slow flat Nakagami faded channel, the received signal can be given by

$$r(t) = \sum_{k=1}^{K} \sqrt{\frac{E_k}{N_s}} \sum_{l=-\infty}^{+\infty} b^{(k)}_{\lfloor l/N_s \rfloor} a^{(k)}_l \beta_k \times \psi(t - lT_c - c^{(k)}_l T_p - \tau_k) \cos(2\pi f_c t + \vartheta_k) + n(t) \tag{2}$$

where τ_k is the delay of the k-th user signal with respect to the signal of the desired user 1 ($\tau_1 = 0$) and $n(t)$ is AWGN noise with two-sided power spectral density of $N_o/2$. β_k is the fading amplitude having a Nakagami distributed probability density function (pdf) given by

$$f_{\beta_k}(\nu) = \frac{2m_k^{m_k} \nu^{2m_k - 1}}{\Omega_k^{m_k} \Gamma(m_k)} \exp\left(-\frac{m_k \nu^2}{\Omega_k}\right), \nu \ge 0 \tag{3}$$

where m_k ($0.5 \le m_k \le \infty$) is the Nakagami fading parameter for user k, $\Gamma(.)$ is gamma function and $\Omega_k = E[\beta_k^2]$, $E[.]$ representing mean value. ϑ_k is the phase of the k-th user's received signal that includes the effects of phase of transmission θ_k and phase alteration due to delay and fading effect, and is uniformly distributed over $[0, 2\pi]$. Let $\tau_k = \Delta_k + \gamma_k T_c$, where γ_k is an RV uniform on $\{0, 1, \ldots, N_s - 1\}$ and $0 < \Delta_k \le T_c$.

A coherent correlation receiver is considered. The received signal is correlated with a template of the form $s^{(1)}_{temp}(t) = \sum_{l=iN_s}^{(i+1)N_s - 1} a^{(1)}_l \psi(t - lT_c - c^{(k)}_l T_p) \cos(2\pi f_c t + \vartheta_1)$. Hence, the decision statistics, while detecting the the i-th bit of the desired user 1, can be given by

$$y^{(1)} = \sqrt{E_1 N_s} b^{(1)}_i \beta_1 + \sum_{k=2}^{K} I_k + \eta. \tag{4}$$

The right side of (4) has three parts, of which the first part is the desired signal component, the second part is the multiple access interference (MAI) component and the third part η is the AWGN component having variance of $\sigma_\eta^2 = N_o N_s/2$. Here, I_k is the MAI from user k and is defined next independently for the two systems.

3.1 Pulsed DS-CDMA

In the pulsed DS-CDMA system, transmitted pulses of duration T_p are uniformly spaced in time repeating at every T_c ($T_p \leq T_c$) interval. Hence, I_k can be given by

$$I_k = \sum_{l=iN_s}^{(i+1)N_s-1} \sqrt{E_k/N_s}\beta_k \cos\phi_k W_l^{(k)} \tag{5}$$

where $\phi_k = \vartheta_k - \vartheta_1$. $W_l^{(k)}$ is the MAI component in the l-th chip of the desired user 1 from user k, given by

$$W_l^{(k)} = \begin{cases} L_l^{(k)}\hat{R}_\psi(\alpha_k), & 0 < \Delta_k \leq T_p \\ 0, & T_p < \Delta_k \leq T_c - T_p \\ M_l^{(k)}R_\psi(\alpha_k), & T_c - T_p < \Delta_k \leq T_c \end{cases} \tag{6}$$

for $0 < \delta \leq 0.5$, where $W_l^{(k)}$ assumes any of the three values of right side with probability δ, $1 - 2\delta$ and δ respectively from top to bottom. For $0.5 < \delta \leq 1.0$,

$$W_l^{(k)} = \begin{cases} L_l^{(k)}\hat{R}_\psi(\alpha_k), & 0 < \Delta_k \leq T_c - T_p \\ L_l^{(k)}\hat{R}_\psi(\alpha_k) & \\ +M_l^{(k)}R_\psi(\alpha_k), & T_c - T_p < \Delta_k \leq T_p \\ M_l^{(k)}R_\psi(\alpha_k), & T_p < \Delta_k \leq T_c \end{cases} \tag{7}$$

where $W_l^{(k)}$ assumes any of the three values of right side with probability $1 - \delta$, $2\delta - 1$ and $1 - \delta$ respectively from top to bottom. Here, $L_l^{(k)}$ and $M_l^{(k)}$ are independent RVs uniform on $\{+1, -1\}$ and $\hat{R}_\psi(\alpha_k)$, $R_\psi(\alpha_k)$ are the partial autocorrelation functions of the baseband pulse waveform given by $\hat{R}_\psi(\alpha_k) = \int_{\alpha_k}^{T_p} \psi(t)\psi(t-\alpha_k)dt$ and $R_\psi(\alpha_k) = \hat{R}_\psi(T_p - \alpha_k)$. Note that α_k in relation to $\hat{R}_\psi(\alpha_k)$ and $R_\psi(\alpha_k)$ are given by $\alpha_k = \Delta_k, 0 < \Delta_k \leq T_p$ and $\alpha_k = \Delta_k - (T_c - T_p), T_c - T_p < \Delta_k \leq T_c$ respectively, while in both cases $0 < \alpha_k \leq T_p$ [12]. Figure 1 shows a pictorial representation of MAI scenario in asynchronous DS system. Note that for $0 < \delta \leq 0.5$, if there is a collision between the signals of user 1 and user k, then only one of the two partial autocorrelations of the baseband pulse contribute to the MAI at a time. Also, it is possible that there is no collision and hence no MAI. However, for $0.5 < \delta \leq 1.0$, collisions always occur, and depending on α_k, any one or both of $\hat{R}_\psi(\alpha_k)$ and $R_\psi(\alpha_k)$ may come into play. As a special case, while $\delta = 1$, both $\hat{R}_\psi(\alpha_k)$ and $R_\psi(\alpha_k)$ effect together, which is the case of conventional DS-CDMA.

3.2 Pulsed Hybrid DS/TH-CDMA

In pulsed hybrid DS/TH-CDMA system, let us assume that $\Delta_k = \alpha_k + \lambda_k T_p$, where λ_k is an RV uniform on $\{0, 1, \ldots, N_c - 1\}$ and $0 < \alpha_k \leq T_p$. We further assume that there are n partial pulse collisions per bit while the signals of user 1 and user k collide. Due to random selection of the position of the pulse in each chip of each user, note that n is an RV uniform on $\{0, 1, \ldots, N_s\}$. Hence, we can write

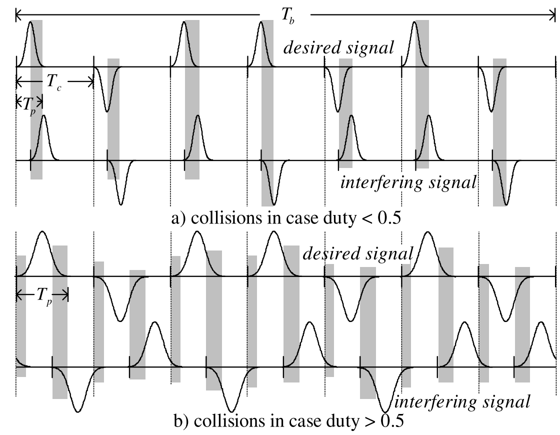

Fig. 1 Example collision scenarios for asynchronous DS system: a) duty < 0.5 b) duty > 0.5. Colliding parts are shown in shades.

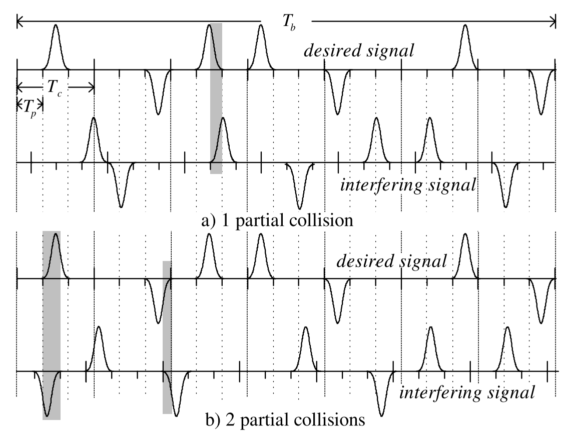

Fig. 2 Example collision scenarios for asynchronous DS/TH system: a) 1 partial collision b) 2 partial collisions. Colliding parts are shown in shades.

$$I_k = \begin{cases} 0, & n = 0 \\ \sqrt{E_k/N_s}\beta_k \cos\phi_k \sum_{n=1}^{1} W_n^{(k)}, & n = 1 \\ \sqrt{E_k/N_s}\beta_k \cos\phi_k \sum_{n=1}^{2} W_n^{(k)}, & n = 2 \\ \vdots & \vdots \\ \sqrt{E_k/N_s}\beta_k \cos\phi_k \sum_{n=1}^{N_s} W_n^{(k)}, & n = N_s \end{cases} \tag{8}$$

where $\phi_k = \vartheta_k - \vartheta_1$ as before. $W_n^{(k)}$ is the MAI component from user k in the n-th colliding chip of the desired user 1, given by $W_n^{(k)} = L_l^{(k)}\hat{R}_\psi(\alpha_k)$ or $M_l^{(k)}R_\psi(\alpha_k)$. Here $W_n^{(k)}$ assumes any of the two values with equal probability of 0.5. Note that n is nothing but a collision counter, however, the actual collision(s) may occur in an arbitrary chip l, which is an RV uniform on $[iN_s, iN_s + 1, \ldots, (i+1)N_s - 1]$. Figure 2 shows a pictorial representation of MAI scenario in asynchronous DS/TH system for 1 and 2 partial pulse collisions. In (8), I_k assumes any of the values of right side

$$\begin{aligned}\Phi_{I_k|\alpha_k,\beta_k,\phi_k,L_l^{(k)},M_l^{(k)}}(\omega) &= (1-2\delta)+\delta\exp\left(j\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k\hat{R}_\psi(\alpha_k)\textstyle\sum_{l=iN_s}^{(i+1)N_s-1}L_l^{(k)}\right)\\ &+\delta\exp\left(j\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k R_\psi(\alpha_k)\textstyle\sum_{l=iN_s}^{(i+1)N_s-1}M_l^{(k)}\right),\ 0<\delta\le 0.5\end{aligned} \tag{10}$$

$$\begin{aligned}\Phi_{I_k|\alpha_k,\beta_k,\phi_k,L_l^{(k)},M_l^{(k)}}(\omega) &= (1-\delta)\exp\left(j\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k\hat{R}_\psi(\alpha_k)\textstyle\sum_{l=iN_s}^{(i+1)N_s-1}L_l^{(k)}\right)\\ &+(2\delta-1)\exp\left(j\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k\textstyle\sum_{l=iN_s}^{(i+1)N_s-1}\left(L_l^{(k)}\hat{R}_\psi(\alpha_k)(\alpha_k)+M_l^{(k)}R_\psi(\alpha_k)\right)\right)\\ &+(1-\delta)\exp\left(j\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k R_\psi(\alpha_k)\textstyle\sum_{l=iN_s}^{(i+1)N_s-1}M_l^{(k)}\right),\ 0.5<\delta\le 1.0\end{aligned} \tag{11}$$

$$\begin{aligned}\Phi_{I_k|\alpha_k,\beta_k,\phi_k}(\omega) &= (1-2\delta)\\ &+\delta\left[\cos^{N_s}\left(\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k\hat{R}_\psi(\alpha_k)\right)+\cos^{N_s}\left(\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k R_\psi(\alpha_k)\right)\right],\ 0<\delta\le 0.5\end{aligned} \tag{12}$$

$$\begin{aligned}\Phi_{I_k|\alpha_k,\beta_k,\phi_k}(\omega) &= (1-\delta)\left[\cos^{N_s}\left(\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k\hat{R}_\psi(\alpha_k)\right)+\cos^{N_s}\left(\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k R_\psi(\alpha_k)\right)\right]\\ &+(2\delta-1)\cos^{N_s}\left(\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k\hat{R}_\psi(\alpha_k)\right)\cos^{N_s}\left(\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k R_\psi(\alpha_k)\right),\ 0.5<\delta\le 1.0.\end{aligned} \tag{13}$$

according to total number of partial pulse collisions per bit, n. Let us assume that the probability that there will be a partial pulse collision in any chip is p. Due to the TH nature of the position of the pulse in each chip, the number of pulse collisions per bit will be binomially distributed [11], [12]. Hence, the probability of occurrence of a situation with n collisions per bit is $\begin{pmatrix} N_s \\ n \end{pmatrix} p^n(1-p)^{N_s-n}$, where $p = 2/N_c$ for asynchronous case and $p = 1/N_c$ for synchronous case [11], [12]. Note that in asynchronous case, the interfering pulse may be behind or ahead of the desired pulse. In each case, the probability of partial overlap is $T_p/T_c = 1/N_c$. Hence the total probability is $2/N_c$.

4. The Exact CF of MAI for Pulsed DS-CDMA

4.1 Asynchronous System

For an asynchronous system, α_k is uniformly distributed over $[0, T_p]$. The conditional CF of MAI from an arbitrary user k can be given by

$$\begin{aligned}&\Phi_{I_k|\alpha_k,\beta_k,\phi_k,L_l^{(k)},M_l^{(k)}}(\omega)\\ &= E\left[\exp(j\omega I_k)\,|\,\alpha_k,\beta_k,\phi_k,L_l^{(k)},M_l^{(k)}\right]\end{aligned} \tag{9}$$

where $j = \sqrt{-1}$. Note that (9) presents the exact CF conditioned on the independent RVs $\alpha_k, \beta_k, \phi_k$, $L_l^{(k)}$ and $M_l^{(k)}$. Putting the expression of I_k from (5) in (9) and taking help from (6), (7), one can rewrite (9) as (10) for $0 < \delta \le 0.5$ and as (11) for $0.5 < \delta \le 1.0$ as shown at the top of the page. Now we recall that both $L_l^{(k)}$ and $M_l^{(k)}$ are uniform on $\{+1, -1\}$. Hence, there are two possibilities: either $L_l^{(k)} = M_l^{(k)}$ or $L_l^{(k)} = -M_l^{(k)}$. Counting for all possible cases and using the identity $\cos z = \{\exp(jz) + \exp(-jz)\}/2$, we obtain the following two expressions given in (12) and (13) from (10) and (11) respectively.

Here note that developing of (12) and (13) from (10) and (11), respectively, follows from the fact that random polarity sequences are considered in this paper. Hence, the chip polarities for a particular user are selected independently. This is a key observation taken into account in this paper that facilitates obtaining simple expression for the exact CF.

The expressions (12), (13) at a first sight may seem not to be of convenient forms, since both involve cosines having power N_s and (13) further includes a multiplication of two cosines each having a power of N_s. However, this is not the fact. As shown in the appendix (see Eqs. (A· 1)–(A· 4)), the cosines having power N_s of (12), (13) and the multiplication of two cosines each having a power N_s of (13) can be expressed as sum of cosines having unity power. After so doing, we take the following two actions in sequel. We first integrate the expressions obtained from (12), (13) over the density of ϕ_k, which is uniform over $[0, 2\pi]$ and use the identity

$$J_0(z) = \frac{1}{2\pi}\int_0^{2\pi}\cos(z\cos\phi)d\phi \tag{14}$$

to obtain $\Phi_{I_k|\alpha_k,\beta_k}(\omega)$ in terms of J_0, where J_0 is the zeroth order Bessel function of the first kind. Next, we integrate $\Phi_{I_k|\alpha_k,\beta_k}(\omega)$ over the Nakagami density of β_k to obtain $\Phi_{I_k|\alpha_k}(\omega)$. Again, by employing an identity given by

$$\int_0^\infty J_0(z\nu)f_{\beta_k}(\nu)d\nu = {}_1F_1\left(m_k;1;-\frac{\Omega_k z^2}{4m_k}\right)\hat{=}\,\mathcal{F}(z^2) \tag{15}$$

from [19, Eqs. (41), (42)], where $f_{\beta_k}(\nu)$ is the Nakagami density of β_k and ${}_1F_1(:;:;:)$ is the confluent hypergeometric function, $\Phi_{I_k|\alpha_k}(\omega)$ can finally be given for $0 < \delta \le 0.5$ by (16) for N_s being even and by (17) for N_s being odd as shown above. For $0.5 < \delta \le 1.0$, $\Phi_{I_k|\alpha_k}(\omega)$ takes the form

$$\Phi_{I_k|\alpha_k}(\omega) = (1-2\delta) + \frac{\delta}{2^{N_s-1}}\left[\sum_{q=0}^{\frac{N_s}{2}-1}\binom{N_s}{q}\left\{\mathcal{F}\left(\omega^2\hat{x}^2(q)\right)+\mathcal{F}\left(\omega^2 x^2(q)\right)\right\}+\binom{N_s}{\frac{N_s}{2}}\right], \quad \begin{matrix} N_s \text{ even} \\ 0<\delta\le 0.5 \end{matrix} \tag{16}$$

$$\Phi_{I_k|\alpha_k}(\omega) = (1-2\delta) + \frac{\delta}{2^{N_s-1}}\left[\sum_{q=0}^{\frac{N_s-1}{2}}\binom{N_s}{q}\left\{\mathcal{F}\left(\omega^2\hat{x}^2(q)\right)+\mathcal{F}\left(\omega^2 x^2(q)\right)\right\}\right], \quad \begin{matrix} N_s \text{ odd} \\ 0<\delta\le 0.5 \end{matrix} \tag{17}$$

$$\begin{aligned}\Phi_{I_k|\alpha_k}(\omega) = \; & \frac{2\delta-1}{2^{2N_s}}\left[\sum_{q_1=0}^{\frac{N_s}{2}-1}\sum_{q_2=0}^{\frac{N_s}{2}-1}2\binom{N_s}{q_1}\binom{N_s}{q_2}\left\{\mathcal{F}\left(\omega^2[\hat{x}(q_1)+x(q_2)]^2\right)+\mathcal{F}\left(\omega^2[\hat{x}(q_1)-x(q_2)]^2\right)\right\}\right. \\ & \left.+\binom{N_s}{\frac{N_s}{2}}\sum_{q=0}^{\frac{N_s}{2}-1}2\binom{N_s}{q}\left\{\mathcal{F}\left(\omega^2\hat{x}^2(q)\right)+\mathcal{F}\left(\omega^2 x^2(q)\right)\right\}+\binom{N_s}{\frac{N_s}{2}}^2\right] \\ & +\frac{1-\delta}{2^{N_s-1}}\left[\sum_{q=0}^{\frac{N_s}{2}-1}\binom{N_s}{q}\left\{\mathcal{F}\left(\omega^2\hat{x}^2(q)\right)+\mathcal{F}\left(\omega^2 x^2(q)\right)\right\}+\binom{N_s}{\frac{N_s}{2}}\right], \quad \begin{matrix} N_s \text{ even} \\ 0.5<\delta\le 1.0 \end{matrix}\end{aligned} \tag{18}$$

$$\begin{aligned}\Phi_{I_k|\alpha_k}(\omega) = \; & \frac{2\delta-1}{2^{2N_s-1}}\left[\sum_{q_1=0}^{\frac{N_s-1}{2}}\sum_{q_2=0}^{\frac{N_s-1}{2}}\binom{N_s}{q_1}\binom{N_s}{q_2}\left\{\mathcal{F}\left(\omega^2[\hat{x}(q_1)+x(q_2)]^2\right)+\mathcal{F}\left(\omega^2[\hat{x}(q_1)-x(q_2)]^2\right)\right\}\right] \\ & +\frac{1-\delta}{2^{N_s-1}}\left[\sum_{q=0}^{\frac{N_s-1}{2}}\binom{N_s}{q}\left\{\mathcal{F}\left(\omega^2\hat{x}^2(q)\right)+\mathcal{F}\left(\omega^2 x^2(q)\right)\right\}\right], \quad \begin{matrix} N_s \text{ odd} \\ 0.5<\delta\le 1.0 \end{matrix}\end{aligned} \tag{19}$$

as (18) for N_s being even and as (19) for N_s being odd as shown above. Note that in (18) and (19),

$$\hat{x}(q) = (N_s - 2q)\sqrt{E_k/N_s}\hat{R}_\psi(\alpha_k), \tag{20}$$

$$x(q) = (N_s - 2q)\sqrt{E_k/N_s}R_\psi(\alpha_k), \tag{21}$$

$\binom{u}{v}$ is the binomial coefficient and $\mathcal{F}(.)$ is defined in (15). Finally, the unconditional exact CF is obtained by integrating (16)–(19) over the density of α_k. Since α_k is uniformly distributed over $[0, T_p]$, the exact CF of MAI from an arbitrary user k is given by

$$\Phi_{I_k}(\omega) = \frac{1}{T_p}\int_0^{T_p}\Phi_{I_k|\alpha_k}(\omega)d\alpha_k. \tag{22}$$

Note that in developing the exact CF according to our method, we did't actually need the density of collision statistics of the spreading sequences, since the sequences are random. This facilitates obtaining the simple expressions of exact CF for a general pulse shape and chip-duty involving confluent hypergeometric functions that are readily computed by Matlab. Previously, [24] developed the exact CF for conventional DS-CDMA considering a rectangular pulse shape. The authors followed a method where the density of collision statistics of the spreading sequences were required that eventually led to a complex expression for the CF. In evaluating the exact CF derived by us, which is also applicable for conventional DS-CDMA as a special case, an integration over the chip duration is required. This is due to the fact that a general pulse shape is considered here. Finally, assuming that the interference from different users are independent, the exact CF of total MAI takes the form

$$\Phi_I(\omega) = \prod_{k=2}^{K}\Phi_{I_k}(\omega). \tag{23}$$

Figures 3 and 4 show the exact CFs of the MAI considering a 1) rectangular and a 2) Gaussian pulse shape respectively. By defining the pulse shape as $\psi(t) = v(t)/\sqrt{(\mathcal{E})}$, 1) the rectangular pulse shape is represented by

$$v(t) = \begin{cases} 1, & 0 < t \le T_p \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

and 2) the Gaussian pulse shape is represented by

$$v\left(t + \frac{T_p}{2}\right) = e^{-\pi(\frac{t}{t_m})^2} \tag{25}$$

with $t_m = 0.4T_p$. Here, $\mathcal{E}$ is the energy of $v(t)$ defined by $\mathcal{E} = \int_{-\infty}^{+\infty} v^2(t)dt$. The partial autocorrelations of the baseband pulse $\psi(t)$ are given by 1) $\hat{R}_\psi(\alpha_k) = (T_p - \alpha_k)/\mathcal{E}$ and 2) $\hat{R}_\psi(\alpha_k) = e^{-0.5\pi(\frac{\alpha_k}{t_m})^2}$ respectively and $R_\psi(\alpha_k) = \hat{R}_\psi(T_p - \alpha_k)$ in both cases. Both the Figs. are for $K = 2$, implying one interfering user. The other parameters are $N_s = 7$ and

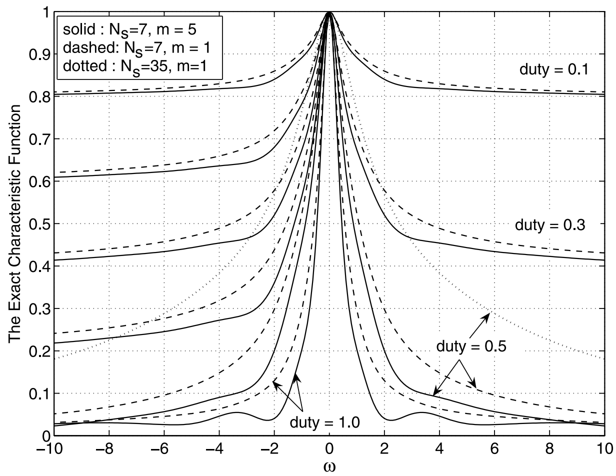

Fig. 3 The exact characteristic function of MAI for $K = 2$ in pulsed DS-CDMA considering rectangular pulse shape. Here, $E_b/N_o = 20$ dB.

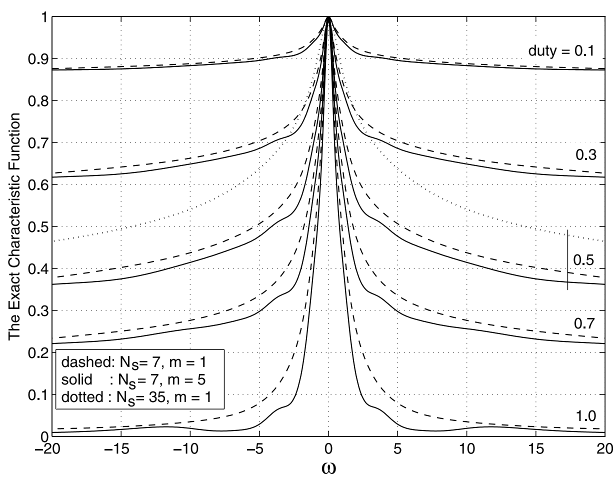

Fig. 4 The exact characteristic function of MAI for $K = 2$ in pulsed DS-CDMA considering Gaussian pulse shape. Here, $E_b/N_o = 20$ dB.

$\delta = 0.1, 0.3, 0.5$ and 1.0. Figure 4 further shows the CF for $N_s = 7$, $\delta = 0.7$. Same Nakagami parameter ($m = m_1 = m_2 = 1, 5$) and same bit energy ($E_b = \Omega_1 E_1 = \Omega_2 E_2$) are considered for the two users and $E_b/N_o = 20$ dB is assumed. Figures 3, 4 also show the CF for $N_s = 35, \delta = 0.5$ with $m = 1$. As seen, the shape of the CF varies considerably with the parameters pulse shape, chip length and chip-duty. Interestingly, the shape of the CF is also different for $(N_s, \delta) = (7, 0.1), (35, 0.5)$, despite the *PG* being the same 70 in the two cases.

4.2 Synchronous System

A synchronous system can either be bit synchronous ($\tau_k = \gamma_k = \Delta_k = 0$) or chip synchronous ($\Delta_k = 0$). In either case $\alpha_k = 0$ that results in $\hat{R}_\psi(\alpha_k) = 1$ and $R_\psi(\alpha_k) = 0$. Hence, a simplified expression of $\Phi_{I_k|\alpha_k,\beta_k,\phi_k}(\omega)$ is obtained for $0 < \delta \leq 1.0$ applicable for both the cases, given by

$$\Phi_{I_k|\alpha_k,\beta_k,\phi_k}(\omega) = \cos^{N_s}\left(\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k\right). \tag{26}$$

Here note that the right side of (26) is not a function of pulse shape or chip-duty, nor it is a function of α_k. Hence, we can write $\Phi_{I_k|\alpha_k,\beta_k,\phi_k}(\omega) = \Phi_{I_k|\beta_k,\phi_k}(\omega)$. Using the first two identities shown in the appendix and following the method just presented for asynchronous system, $\Phi_{I_k}(\omega)$ applicable for both bit and chip synchronous systems can be given by

$$\Phi_{I_k}(\omega) = \frac{1}{2^{N_s}}\left[\sum_{q=0}^{\frac{N_s}{2}-1} 2\binom{N_s}{q} \times \mathcal{F}\left(\frac{E_k(N_s-2q)^2\omega^2}{N_s}\right) + \binom{N_s}{\frac{N_s}{2}}\right] \tag{27}$$

for $0 < \delta \leq 1.0$, N_s being even and by

$$\Phi_{I_k}(\omega) = \frac{1}{2^{N_s-1}}\left[\sum_{q=0}^{\frac{N_s-1}{2}} \binom{N_s}{q}\mathcal{F}\left(\frac{E_k(N_s-2q)^2\omega^2}{N_s}\right)\right] \tag{28}$$

for $0 < \delta \leq 1.0$, N_s being odd. Note that since the unconditional exact CF of (27) and (28) are also independent of pulse shape and chip-duty, so is the BEP expected.

5. The Exact CF of MAI for Pulsed Hybrid DS/TH-CDMA

5.1 Asynchronous System

The conditional CF of MAI from an arbitrary user k in pulsed hybrid DS/TH-CDMA can be given in a similar fashion by (9). While putting the expression of I_k from (8) in (9), we discover that $\Phi_{I_k|\alpha_k,\beta_k,\phi_k,L_l^{(k)},M_l^{(k)}}(\omega)$ takes specific form $\Phi^{(n)}_{I_k|\alpha_k,\beta_k,\phi_k,L_l^{(k)},M_l^{(k)}}(\omega)$ for a specific number of partial pulse collisions per bit, n. Finally, considering all possible collisions $0 \leq n \leq N_s$,

$$\Phi_{I_k|\alpha_k,\beta_k,\phi_k,L_l^{(k)},M_l^{(k)}}(\omega) = \sum_{n=0}^{N_s}\binom{N_s}{n} \times p^n(1-p)^{N_s-n}\Phi^{(n)}_{I_k|\alpha_k,\beta_k,\phi_k,L_l^{(k)},M_l^{(k)}}(\omega) \tag{29}$$

where the probability of a pulse collision $p = 2/N_c$ for an asynchronous system. Taking help from (8), it can now be shown that for no pulse collision per bit (i.e. $n = 0$),

$$\Phi^{(0)}_{I_k|\alpha_k,\beta_k,\phi_k,L_l^{(k)},M_l^{(k)}}(\omega) = 1. \tag{30}$$

While there is only one partial collision per bit (i.e. $n = 1$), it may contribute MAI in two ways: either through $\hat{R}_\psi(\alpha_k)$ or $R_\psi(\alpha_k)$. Hence, we can write

$$\Phi^{(1)}_{I_k|\alpha_k,\beta_k,\phi_k,L_l^{(k)},M_l^{(k)}}(\omega) = \left[\exp\left(j\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k L_l^{(k)}\hat{R}_\psi(\alpha_k)\right) + \exp\left(j\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k M_l^{(k)}R_\psi(\alpha_k)\right)\right]/2. \tag{31}$$

Now recalling that both $L_l^{(k)}$ and $M_l^{(k)}$ are uniform on

$$\Phi_{I_k|\alpha_k,\beta_k,\phi_k}(\omega) = \sum_{n=0}^{N_s} \binom{N_s}{n} p^n (1-p)^{N_s-n} \cos^n\left(\omega\sqrt{E_k/N_s}\beta_k \cos\phi_k[\hat{R}_\psi(\alpha_k) + R_\psi(\alpha_k)]/2\right) \times \cos^n\left(\omega\sqrt{E_k/N_s}\beta_k \cos\phi_k[\hat{R}_\psi(\alpha_k) - R_\psi(\alpha_k)]/2\right). \quad (34)$$

$$\begin{aligned}\Phi_{I_k|\alpha_k}(\omega) = (1-p)^{N_s} &+ \sum_{n=1:2:N_{so}} \binom{N_s}{n} \frac{p^n(1-p)^{N_s-n}}{2^{2n-1}} \times\left[\sum_{q_1=0}^{\frac{n-1}{2}}\sum_{q_2=0}^{\frac{n-1}{2}} \binom{n}{q_1}\binom{n}{q_2}\left\{\mathcal{F}\left(\omega^2[\hat{x}(q_1)+x(q_2)]^2\right)+\mathcal{F}\left(\omega^2[\hat{x}(q_1)-x(q_2)]^2\right)\right\}\right] \\ &+ \sum_{n=2:2:N_{se}} \binom{N_s}{n} \frac{p^n(1-p)^{N_s-n}}{2^{2n}} \times\left[\sum_{q_1=0}^{\frac{n}{2}-1}\sum_{q_2=0}^{\frac{n}{2}-1} 2\binom{n}{q_1}\binom{n}{q_2}\left\{\mathcal{F}\left(\omega^2[\hat{x}(q_1)+x(q_2)]^2\right)+\mathcal{F}\left(\omega^2[\hat{x}(q_1)-x(q_2)]^2\right)\right\}\right. \\ &\left. + \binom{n}{\frac{n}{2}}\sum_{q=0}^{\frac{n}{2}-1} 2\binom{n}{q}\left\{\mathcal{F}\left(\omega^2\hat{x}^2(q)\right)+\mathcal{F}\left(\omega^2 x^2(q)\right)\right\} + \binom{n}{\frac{n}{2}}^2\right] \quad (35)\end{aligned}$$

$\{+1, -1\}$ and using the identity $\cos z = \{\exp(jz) + \exp(-jz)\}/2$ as before, we obtain

$$\Phi^{(1)}_{I_k|\alpha_k,\beta_k,\phi_k}(\omega) = \left[\cos\left(\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k\hat{R}_\psi(\alpha_k)\right) + \cos\left(\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k R_\psi(\alpha_k)\right)\right]/2. \quad (32)$$

Note again that the assumption of independently selected random polarity sequences helps develop (32) from (31). While there are two pulse collisions (i.e. $n = 2$), those may contribute MAI through any of the following 2^2 arrangements of the partial autocorralations: $\hat{R}_\psi(\alpha_k)\hat{R}_\psi(\alpha_k)$, $\hat{R}_\psi(\alpha_k)R_\psi(\alpha_k)$, $R_\psi(\alpha_k)\hat{R}_\psi(\alpha_k)$ or $R_\psi(\alpha_k)R_\psi(\alpha_k)$. For a general case with $0 \le n \le N_s$, the number of different collision arrangements can be similarly shown to be 2^n. Hence, it can be shown easily that

$$\Phi^{(n)}_{I_k|\alpha_k,\beta_k,\phi_k}(\omega) = \left[\cos\left(\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k\hat{R}_\psi(\alpha_k)\right) + \cos\left(\omega\sqrt{E_k/N_s}\beta_k\cos\phi_k R_\psi(\alpha_k)\right)\right]^n/2^n. \quad (33)$$

Now, performing simple trigonometric manipulations on the expression of $\Phi^{(n)}_{I_k|\alpha_k,\beta_k,\phi_k}(\omega)$ given in (33) and putting it in (29), we obtain (34) as shown above. The expression (34) again involves multiplication of two cosines each having a power of n, $0 \le n \le N_s$. As mentioned earlier and shown in the appendix (see Eqs. (A·3), (A·4)), the multiplication of two cosines each having the same integer power can be expressed as sum of cosines having unity power. Applying this in (34) and following the procedure shown in Sect. 4, $\Phi_{I_k|\alpha_k}(\omega)$ can be given by (35) as shown above, where, for N_s being even, $N_{se} = N_s$, $N_{so} = N_s - 1$ and for N_s being odd, $N_{so} = N_s$, $N_{se} = N_s - 1$. Note that the right side of (35) has three parts of which the first part is due to the case of no collisions, i.e. $n = 0$, and the second and third parts are due to all odd and even number of collisions respectively. Also in (35),

$$\hat{x}(q) = (n-2q)\sqrt{E_k/N_s}[\hat{R}_\psi(\alpha_k) + R_\psi(\alpha_k)]/2, \quad (36)$$

$$x(q) = (n-2q)\sqrt{E_k/N_s}[\hat{R}_\psi(\alpha_k) - R_\psi(\alpha_k)]/2 \quad (37)$$

and $\mathcal{F}(.)$ is defined in (15). Note that in developing (35), we employed the information of collision statistics of the hybrid DS/TH spreading sequences in a smart way, which facilitated obtaining the simple expression. Once $\Phi_{I_k|\alpha_k}(\omega)$ is obtained, the unconditional exact CF of MAI from user k can be obtained using (22). Finally, using (23) we obtain the unconditional exact CF of total MAI.

Figure 5 shows the exact CF of the MAI in a pulsed hybrid DS/TH-CDMA system considering a Gaussian pulse shape. We use $K = 2$ implying one interfering user. The other parameters are $(N_s, N_c) = (2, 15), (3, 10), (7, 2)$ and $(7, 10)$. Same Nakagami parameter ($m = m_1 = m_2 = 1, 5$) and same bit energy ($E_b = \Omega_1 E_1 = \Omega_2 E_2$) are considered for the two users, and $E_b/N_o = 20$ dB is assumed as before. As seen, the shape of the CF vary considerably both with chips per bit N_s and slots per chip N_c. The CF also assumes different shape for $(N_s, N_c) = (2, 15), (3, 10)$, despite the *PG* being fixed at 30. To present a comparison of the shapes of CF for pulsed DS and hybrid DS/TH systems, we compare the shapes for $(N_s, \delta) = (7, 0.1), (7, 0.5)$ in Fig. 4 with those for $(N_s, N_c) = (7, 10), (7, 2)$ respectively in Fig. 5. De-

$$\Phi_{I_k}(\omega) = (1-p)^{N_s} + \sum_{n=1:2:N_{so}} \begin{pmatrix} N_s \\ n \end{pmatrix} \frac{p^n(1-p)^{N_s-n}}{2^{2n-1}} \left[\sum_{q=0}^{\frac{2n-1}{2}} \begin{pmatrix} 2n \\ q \end{pmatrix} \mathcal{F}\left(\frac{E_k(n-q)^2\omega^2}{N_s} \right) \right]$$
$$+ \sum_{n=2:2:N_{se}} \begin{pmatrix} N_s \\ n \end{pmatrix} \frac{p^n(1-p)^{N_s-n}}{2^{2n}} \left[\sum_{q=0}^{n-1} 2 \begin{pmatrix} 2n \\ q \end{pmatrix} \mathcal{F}\left(\frac{E_k(n-q)^2\omega^2}{N_s} \right) + \begin{pmatrix} 2n \\ n \end{pmatrix} \right] \quad (38)$$

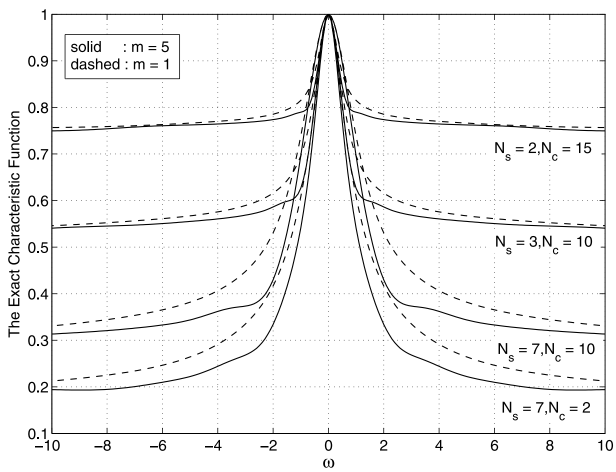

Fig. 5 The exact characteristic function of MAI for $K = 2$ in pulsed hybrid DS/TH-CDMA considering Gaussian pulse shape. Here, $E_b/N_o = 20$ dB.

spite the system parameters being identical, the shapes of CF largely differ for the two systems.

5.2 Synchronous System

Since a synchronous system is independent of α_k with $\hat{R}_\psi(\alpha_k) = 1$ and $R_\psi(\alpha_k) = 0$, $\Phi_{I_k|\beta_k,\phi_k}(\omega) = \Phi_{I_k|\alpha_k,\beta_k,\phi_k}(\omega)$ can be obtained from (34) just by putting the two autocorrelation values therein. The unconditional CF of MAI from user k, applicable for both bit synchronous and chip synchronous systems, is finally obtained as (38) shown above, where the probability of pulse collision, $p = 1/N_c$. Note that though similar to synchronous pulsed DS-CDMA, the CF of pulsed hybrid DS/TH-CDMA is independent of pulse shape, unlike the former, the latter is chip-duty dependent.

6. The Exact Error Probabilities

The conditional exact BEP conditioned on the fading amplitude of the desired user 1, β_1, can be given by

$$P_{e|\beta_1} = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\sin(\sqrt{E_1 N_s}\beta_1\omega)}{\omega}\Phi_I(\omega)\Phi_\eta(\omega)d\omega \quad (39)$$

where $\Phi_\eta(\omega) = \exp(-\sigma_\eta^2\omega^2/2)$ is the CF of AWGN. Integrating (39) over the Nakagami density of β_1, and using an identity given in [26, Eqs. (3)–(23)] we obtain the unconditional exact BEP expression as

$$P_e = \frac{1}{2} - \frac{1}{\pi}\frac{\Gamma(m_1+\frac{1}{2})}{\Gamma(m_1)}\sqrt{\frac{E_1 N_s \Omega_1}{m_1}}\int_0^\infty \Phi_I(\omega)$$
$$\times \Phi_\eta(\omega)_1F_1\left(m_1+\frac{1}{2};\frac{3}{2};-\frac{E_1N_s\Omega_1\omega^2}{4m_1}\right)d\omega. \quad (40)$$

Here note that the exact BEP of (40) depends on the pulse shape and chip-duty, only through $\Phi_I(\omega)$. Hence, the shape of $\Phi_I(\omega)$ for specific pulse shape and chip-duty governs the error rate accordingly. Also note that evaluating the exact BEP requires evaluation of only two integrations. One of those is the outer integration shown in (40) and the other is the integration associated with the calculation of CF as given in (22). However, evaluation of approximate BEP based on the IGA [5], especially for an arbitrary pulse shape, requires multiple in-folded integrations and is more complicated.

7. Error Probabilities Based on Gaussian Approximation

Under the SGA of MAI, the conditional BEP conditioned on the fading amplitude of the desired user 1, β_1, can be given by

$$P_{e|\beta_1}^{SGA} = 0.5\,\mathrm{erfc}\left(\beta_1\sqrt{\frac{0.5E_1N_s}{\sigma_\eta^2+\mu_I}}\right) \quad (41)$$

where erfc(.) is complimentary error function and μ_I is the mean of total MAI variance. For asynchronous case, μ_I assumes the same expression for both pulsed DS and hybrid DS/TH systems, given by $\mu_I = \sum_{k=2}^K \Omega_k E_k \rho_\psi \delta$ [12], where $\rho_\psi = \frac{1}{T_p}\int_0^{T_p}\hat{R}_\psi^2(\alpha_k)d\alpha_k = \frac{1}{T_p}\int_0^{T_p}R_\psi^2(\alpha_k)d\alpha_k$. For synchronous case, $\mu_I = \lambda\sum_{k=2}^K \Omega_k E_k/2$, where $\lambda = 1$ for pulsed DS system and $\lambda = \delta$ for pulsed hybrid DS/TH system [12]. Now integrating $P_{e|\beta_1}^{SGA}$ over the density of β_1 and employing the identity from [27, p.20, Eq. (45)], the unconditional BEP under SGA can be given by

$$P_e^{SGA} = \frac{\Lambda^{m_1}}{2\sqrt{\pi}}\frac{\Gamma(m_1+\frac{1}{2})}{m_1\Gamma(m_1)}\,{}_2F_1\left(m_1,\frac{1}{2};m_1+1;\Lambda\right) \quad (42)$$

where $\Lambda = \left(1+\frac{E_1N_s\Omega_1}{2m_1(\sigma_\eta^2+\mu_I)}\right)^{-1}$ and ${}_2F_1$ is Gauss hypergeometric function.

8. Numerical Examples

In this section, specific numerical examples are presented on error performance of the systems. Only asynchronous systems are considered. The results in Fig. 6 are given for a rectangular pulse shape (see (24)) and rest of the results are given for a Gaussian pulse shape (see (25)). Theoretical results are confirmed by Monte Carlo simulations. Same Nakagami parameter and bit energy are consider for all users, ($m = m_k$, and $E_b = \Omega_k E_k$, $k = 1, 2, \ldots, K$). Figures 6, 7 show the BEP versus E_b/N_o for total users, $K = 2$ for rectangular and Gaussian pulse shape respectively. Figure 8 shows the BEP versus K for $E_b/N_o = 20$ dB considering Gaussian pulse shape. In all three Figs., $N_s = 7$, $\delta = 0.05$, 0.5 with $m = 1$ and 5 are considered. An absolute match between the BEP from the exact method and simulations can be seen. As seen from Figs. 6, 7 similar system behavior and comparable performance can be noticed for both pulse shapes. In Figs. 6, 7 as well as in most of the subsequent Figs. we consider small number of users. This is because, for such range of simultaneous users, the two systems show wide performance difference. Also, in Figs. 6–8, we consider $\delta = 0.05$, 0.5 with the view to investigate system performance in two different cases, namely with low and high chip-duty. Two important observations that can be made from the Figs. 7 and 8 are as follows:

- For a heavily faded channel with $m = 1$ (Rayleigh fading), both pulsed DS and hybrid DS/TH systems perform comparably irrespective of chip-duty, E_b/N_o and number of simultaneous users. The SGA provides reasonably accurate approximation in this case.
- For a lightly faded channel with $m = 5$, the two systems show comparable performance only while the E_b/N_o is low and/or the number of simultaneous users is large. The hybrid system shows better performance for high E_b/N_o values and small number of users. The performance difference between the two systems increases considerably with increasing E_b/N_o and lowering chip-duty. In contrary, the SGA predicts the two systems to perform identically as long as the system parameters are identical. The SGA also becomes highly optimistic at high E_b/N_o, especially for small number of users.

Figure 9 shows BEP versus $PG = N_s/\delta$ for $m = 1$ and 5 considering Gaussian pulse shape while one interfering user is present. $E_b/N_o = 16$ dB is assumed. While the role of PG has been extensively studied in the context of conventional communication systems, such as DS-CDMA [14], [15], the same in the context of low-duty systems is somewhat unexplored. The PG in conventional DS-CDMA equals the chip-length (N_s), whereas, is composed of two parameters namely chip length and chip-duty (δ) in low-duty systems. Hence, the PG can be increased either by increasing N_s or by decreasing δ. Performance improvement is obvious while the PG increases. As shown, the PG is increased by 1) increasing N_s keeping δ fixed at 0.2 and 2) decreasing δ keeping N_s fixed at 5. The effect of increasing PG either by increasing N_s or by decreasing δ on the multiple access performance looks similar from the view point of SGA. Although this is true for $m = 1$, actual BEP performance for both

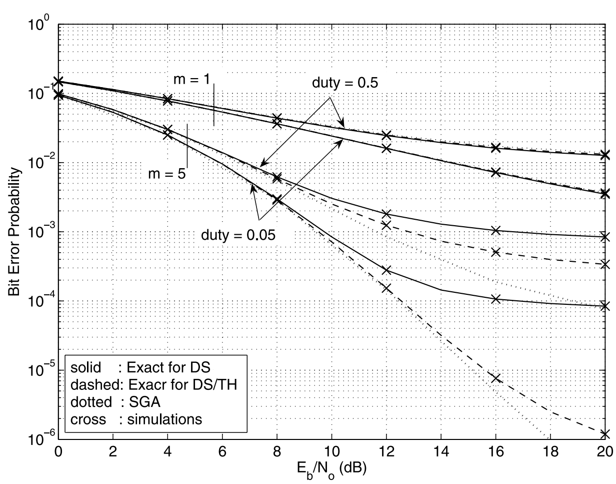

Fig. 6 BEP versus E_b/N_o for $K = 2$ with $N_s = 7$, $\delta = 0.05$, 0.5 and $m = 1, 5$. Rectangular pulse.

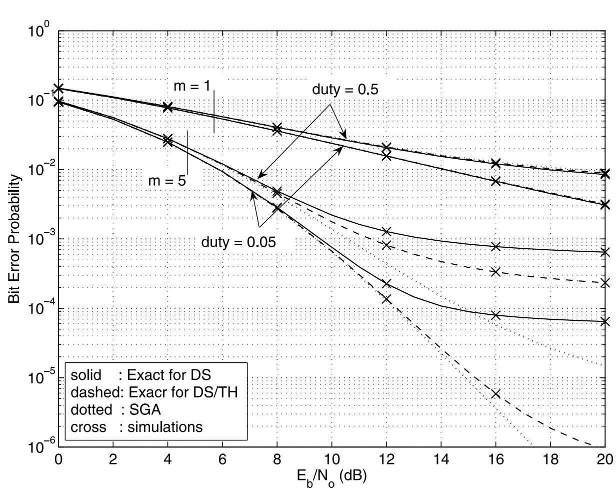

Fig. 7 BEP versus E_b/N_o for $K = 2$ with $N_s = 7$, $\delta = 0.05$, 0.5 and $m = 1, 5$. Gaussian pulse.

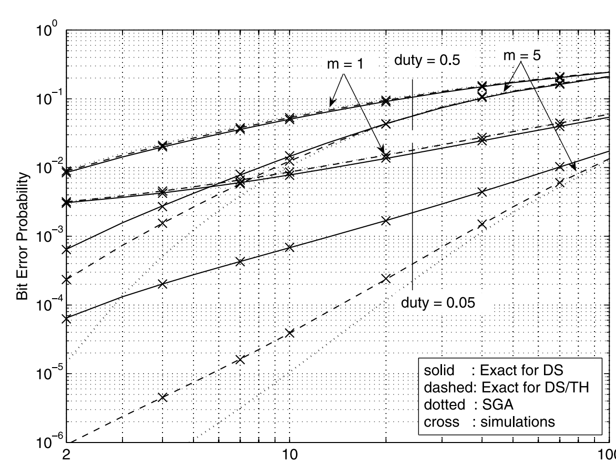

Fig. 8 BEP versus total users K for $N_s = 7$, $\delta = 0.05$, 0.5 and $m = 1, 5$ with $E_b/N_o = 20$ dB. Gaussian pulse.

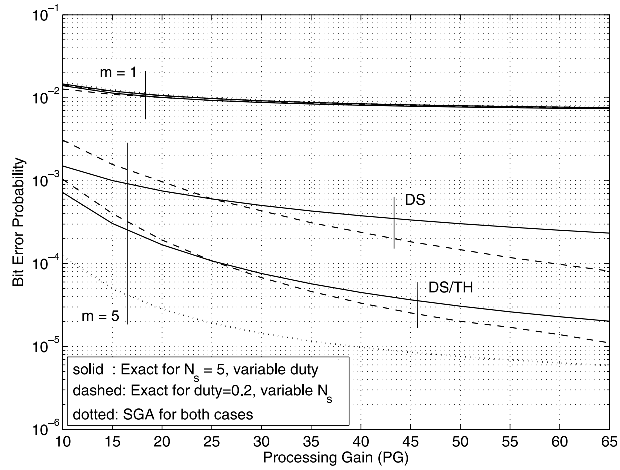

Fig. 9 BEP versus $PG = N_s/\delta$ for various N_s and chip-duty with $K = 2$, $E_b/N_o = 16$ dB and $m = 1, 5$. Gaussian pulse.

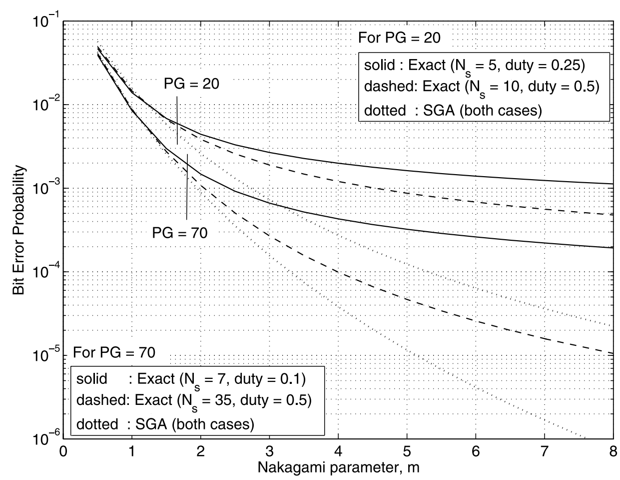

Fig. 10 BEP versus Nakagami parameter m for pulsed DS-CDMA with $K = 3$. $PG = 20$: $(N_s, \delta) = (5, 0.25)$, (10, 0.5) and $PG = 70$: $(N_s, \delta) = (7, 0.1)$, (35, 0.5). $E_b/N_o = 16$ dB. Gaussian pulse.

types of the systems improves with a steeper slope while the PG is increased by increasing N_s for $m = 5$. This is interesting to notice such similar behavior of the two systems, though the hybrid system shows better performance for all values of the PG while $m = 5$. This also reveals the fact that unlike conventional DS-CDMA, the performance of pulsed DS- and hybrid DS/TH-CDMA for a constant PG may not be a constant, especially while the channel is lightly faded.

In Figs. 10 and 11 we present BEP versus Nakagami parameter, m for pulsed DS and hybrid DS/TH system respectively while $K = 3$ and Gaussian pulse shape is used in both cases. Figure 10 shows performance of pulsed DS system for $PG = 20$, with $(N_s, \delta) = (10, 0.5), (5, 0.25)$ and $PG = 70$, with $(N_s, \delta) = (35, 0.5), (7, 0.1)$. E_b/N_o dB is set to 16 dB. Figure 11 shows performance of pulsed hybrid DS/TH system for $PG = 50$, with $(N_s, N_c) = (1, 50), (2, 25)$, (5, 10) and (10, 5). E_b/N_o dB is set to 20 dB. As seen, the SGA approximates the BEP for a specific fading level to be

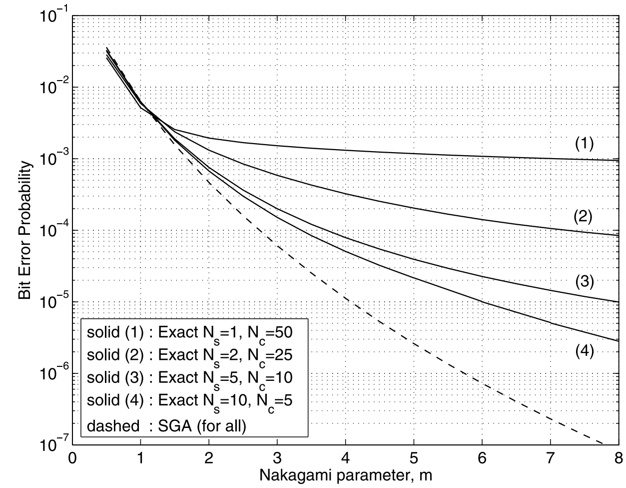

Fig. 11 BEP versus Nakagami parameter m for pulsed hybrid DS/TH-CDMA with $K = 3$. $PG = 50$: $(N_s, N_c) = (1, 50)$, (2, 25), (5, 10) and (10, 5). $E_b/N_o = 20$ dB. Gaussian pulse.

a constant as long as the PG is constant. This is true and the SGA is also found accurate for $0.5 \leq m \leq 1.5$ in both cases. However, while the value of m increases beyond 1.5, the SGA becomes optimistic and the actual system performance is also found to vary for different combinations of N_s and δ, despite the $PG = N_s/\delta$ being fixed. In such a case, systems with longer chip length and higher chip-duty tend to perform better, a trend that increases with increasing m. In contrary, the contribution of N_s and δ in system performance may be considered interchangeable for a highly faded channel.

9. Conclusion

In this paper, we have developed an exact yet simple analytical framework for performance study of general pulsed DS-CDMA systems with arbitrary chip-duty and chip waveform in flat Nakagami-m fading channel. The study has been further extended for a hybrid DS/TH-CDMA system. Both the systems are presently considered for UWB applications. The exact method presented here can be a powerful tool to investigate many system trade-offs in practical applications, guaranteeing both credibility and simplicity. Though this study has been performed considering a flat faded channel, we believe that this may be the first but is very important step toward developing a framework for performance study in a frequency-selective channel, which a UWB channel is more likely to be.

References

[1] R.C. Qiu, H. Liu, and X. Shen, "Ultra-wideband for multiple access communications," IEEE Commun. Mag., vol.43, no.2, pp.80–87, Feb. 2005.

[2] B. Hu and N.C. Beaulieu, "Accurate performance evaluation of time-hopping and direct-sequence UWB systems in multi-user interference," IEEE Trans. Commun., vol.53, no.6, pp.1053–1062, June 2005.

[3] B. Hu and N.C. Beaulieu, "Accurate evaluation of multiple-access performance in TH-PPM and TH-BPSK UWB systems," IEEE Trans. Commun., vol.52, no.10, pp.1758–1766, Oct. 2004.
[4] B.M. Sadler and A. Swami, "On the performance of episodic UWB and direct-sequence communications systems," IEEE Trans. Wireless Commun., vol.3, no.6, pp.2246–2255, Nov. 2004.
[5] S. Gezici, H. Kobayashi, H.V. Poor, and A.F. Molisch, "Performance evaluation of impulse radio UWB systems with pulse-based polarity randomization in asynchronous multiuser environment," Proc. WCNC 2004, vol.2, pp.908–913, March 2004.
[6] S. Gezici, H. Kobayashi, H.V. Poor, and A.F. Molisch, "Performance evaluation of impulse radio UWB systems with pulse-based polarity randomization," IEEE Trans. Signal Process., vol.53, no.7, pp.2537–2549, July 2005.
[7] S. Niranjayan, A. Nallanathan, and B. Kannan, "Modeling of multiple access interference and BER derivation for TH and DS UWB multiple access systems," IEEE Trans. Wireless Commun., vol.5, no.10, pp.2794–2804, Oct. 2006.
[8] S. Gezici, Z. Sahinoglu, H. Kobayashi, and H.V. Poor, "Ultra-wideband impulse radio systems with multiple pulse types," IEEE J. Sel. Areas Commun., vol.24, no.4, pp.892–898, April 2006.
[9] M. Sabattini, E. Masry, and L.B. Milstein, "A non-Gaussian approach to the performance analysis of UWB TH-BPPM systems," Proc. IEEE. Conf. Ultra-Wideband Sys. 2003, pp.52–55, Nov. 2003.
[10] E. Fisher and H.V. Poor, "On the tradeoff between two types of processing gains," IEEE Trans. Commun., vol.53, no.10, pp.1744–1753, Oct. 2005.
[11] M.A. Rahman, S. Sasaki, J. Zhou, and H. Kikuchi, "Simple-to-evaluate error probabilities for impulse radio UWB multiple access system with pulse-based polarity randomization," IEEE Commun. Lett., vol.9, no.9, pp.772–774, Sept. 2005.
[12] M.A. Rahman, S. Sasaki, and H. Kikuchi, "Error analysis for ultra-wideband DS- and hybrid DS/TH-CDMA with arbitrary chip-duty," IEICE Trans. Fundamentals, vol.E89-A, no.6, pp.1668–1679, June 2006.
[13] M.A. Rahman, S. Sasaki, and H. Kikuchi, "An exact error analysis for low-duty pulsed DS-CDMA systems in flat nakagami fading," IEEE Commun. Lett., vol.11, no.4, pp.295–297, April 2007.
[14] J.M. Holtzman, "A simple, accurate method to calculate spread spectrum multi-access error probabilities," IEEE Trans. Commun., vol.40, no.3, pp.461–464, March 1992.
[15] R.K. Morrow, Jr. and J.S. Lehnert, "Bit-to-bit error dependence in slotted DS/SSMA packet systems with random signature sequences," IEEE Trans. Commun., vol.37, no.10, pp.1052–1061, Oct. 1989.
[16] E. Gereniotis and B. Ghaffari, "Performance of binary and quaternary direct-sequence spread-spectrum multiple-access systems with random signature sequences," IEEE Trans. Commun., vol.39, no.5, pp.713–724, May 1991.
[17] K. Sivanesan and N.C. Beaulieu, "Exact BER analysis of Nakagami/Nakagami CCI BPSK and Nakagami/Rayleigh CCI QPSK systems in slow fading," IEEE Commun. Lett., vol.8, no.1, pp.45–47, Jan. 2004.
[18] K.A. Hamdi, "Exact probability of error of BPSK communication links subject to asynchronous interference in Rayleigh fading environment," IEEE Trans. Commun., vol.50, no.10, pp.1577–1579, Oct. 2002.
[19] N.C. Beaulieu and J. Cheng, "Precise error-rate analysis of bandwidth efficient BPSK in Nakagami fading and cochannel interference," IEEE Trans. Commun., vol.52, no.1, pp.149–158, Jan. 2004.
[20] T. Eng and L.B. Milstein, "Coherent DS-CDMA performance in Nakagami multipath fading," IEEE Trans. Commun., vol.43, no.2/3/4, pp.1134–1143, Feb.-April 1995.
[21] G.P. Efthymoglou, V.A. Aalo, and H. Helmken, "Performance analysis of corerent DS-CDMA systems in a Nakagami fading channels with arbitrary parameters," IEEE Trans. Veh. Technol., vol.46, no.2, pp.289–297, May 1997.
[22] M.K. Simon and M.S. Aluini, Digital Communications over Fading Channels: A Unified Approach to Performance Analysis, John Wiley & Sons, New York, 2000.
[23] K. Sivanesan and N.C. Beaulieu, "Performance analysis for bandlimited DS-CDMA systems in Nakagami fading," Proc. 2004 IEEE Intl. Conf. Commun., vol.1, pp.400–404, June 2004.
[24] J. Cheng and N.C. Beaulieu, "Error rate of asynchronous DS-CDMA in Nakagami fading," IEEE Trans. Wireless Commun., vol.4, no.6, pp.2674–2676, Nov. 2005.
[25] J. Cheng and N.C. Beaulieu, "Accurate DS-CDMA bit-error probability calculation in Rayleigh fading," IEEE Trans. Wireless Commun., vol.1, no.1, pp.3–15, Jan. 2002.
[26] A. Papoulis, Fourier Integral and its Applications, McGraw-Hall, New York, 1962.
[27] S. Okui, Denki Tsushin Kogaku no Tame no Tokushu Kansu to Sono Ohyou (Special Functions and Their Applications for Telecommunication Engineering), Morikita Publishing Co., Tokyo, 1997.
[28] R.L. Peterson, R.Z. Zimer, and D.E. Borth, Introduction to Spread Spectrum Communications, Prentice Hall, New Jersey, 1995.

Appendix

From [28, appendix G], we obtain the following two trigonometric identities for integer $n \geq 1$:

$$\cos^{2n} z = \frac{1}{2^{2n}} \left\{ \sum_{q=0}^{n-1} 2 \binom{2n}{q} \cos[2(n-q)z] + \binom{2n}{n} \right\} \tag{A$\cdot$1}$$

$$\cos^{2n-1} z = \frac{1}{2^{2n-2}} \left\{ \sum_{q=0}^{n-1} \binom{2n-1}{q} \cos[(2n-2q-1)z] \right\}. \tag{A$\cdot$2}$$

Using these two identities and with simple mathematical manipulation, we easily obtain the following two expressions shown in (A$\cdot$3) and (A$\cdot$4) where $\hat{z}_1(q) = 2(n-q)z_1$ and $\hat{z}_2(q) = 2(n-q)z_2$.

$$\cos^{2n} z_1 \cos^{2n} z_2 = \frac{1}{2^{4n}} \left[\sum_{q_1=0}^{n-1} \sum_{q_2=0}^{n-1} 2 \binom{2n}{q_1} \binom{2n}{q_2} \{\cos[\hat{z}_1(q_1) + \hat{z}_2(q_2)] + \cos[\hat{z}_1(q_1) - \hat{z}_2(q_2)]\} \right. \\ \left. + \binom{2n}{n} \sum_{q=0}^{n-1} 2 \binom{2n}{q} \{\cos[\hat{z}_1(q)] + \cos[\hat{z}_2(q)]\} + \binom{2n}{n}^2 \right] \quad (\text{A}\cdot 3)$$

$$\cos^{2n-1} z_1 \cos^{2n-1} z_2 = \frac{1}{2^{4n-3}} \sum_{q_1=0}^{n-1} \sum_{q_2=0}^{n-1} \binom{2n-1}{q_1} \binom{2n-1}{q_2} \{\cos[\hat{z}_1(q_1) - z_1 + \hat{z}_2(q_2) - z_2] \\ + \cos[\hat{z}_1(q_1) - z_1 - \hat{z}_2(q_2) + z_2]\} \quad (\text{A}\cdot 4)$$

Mohammad Azizur Rahman received his Bachelor's degree from Bangladesh University of Engineering and Technology (BUET), Dhaka, Bangladesh in August 2001 and Master's and Ph.D. degrees from Niigata University, Niigata, Japan in March 2005 and 2008 respectively all in Electrical and Electronic Engineering (EEE). From August 2001 to March 2002 he served as a Lecturer in the Department of EEE of BUET. From April 2002 through March 2008, he was attached with Niigata University, Niigata, Japan as a Japanese Government scholar. From October 2007 to March 2008, he was an intern at National Institute of Information and Communications Technology (NICT), Yokosuka, Japan where is presently working as an expert researcher focusing on 60 GHz millimeter wave project. Dr. Rahman was a recipient of *2005 Japan Telecommunication Advancement Foundation (TAF) Technology Award for Students* for outstanding Master's degree thesis. His research interests include wideband communications, CDMA and other multiple access systems, fading channels, diversity receivers etc. He is a member of the IEEE.

Shigenobu Sasaki (M'93) received B.E., M.E. and Ph.D. degrees from Nagaoka University of Technology, Nagaoka, Japan, in 1987, 1989 and 1993, respectively. Since 1992, he has been with Niigata University, where he is now an Associate Professor in the Department of Electrical and Electronic Engineering. From 1999 to 2000, he was a visiting scholar at the Department of Electrical and Computer Engineering, University of California, San Diego. From 2003 to 2006, he was with the UWB technology institute, National Institute of Information and Communication Technology (NICT) as an expert researcher. His research interests are in the area of digital communications with special emphasis on spread spectrum communication systems, ultra-wideband communication systems, cognitive radio technology and wireless communications. He is a member of IEEE and SITA.

Hisakazu Kikuchi (M'84) received the B.E. and M.E. degrees from Niigata University in 1974 and 1976, respectively, and Dr. Eng. degree from Tokyo Institute of Technology in 1988. From 1976 to 1979 he worked at Information Processing Systems Laboratory, Fujitsu Ltd., Tokyo. Since 1979 he has been with Niigata University, where he is a professor of electrical engineering. He was a visiting scholar at Electrical Engineering Department, University of California, Los Angeles during a year of 1992 to 1993. He holds a visiting professorship at Chongqing University of Posts and Telecommunications and Nanjing University of Information Science and Technology, both in China, since 2002 and 2005, respectively. His research interests are in the areas of image/video processing and digital signal processing as well as ultra wideband systems. Dr. Kikuchi is a fellow of IEICE, and is a member of ITE (Institute of Image Information and Television Engineers), IIEEJ (Institute of Image Electronics Engineers of Japan), JSIAM (Japan Society for Industrial and Applied Mathematics), RISP (Research Institute of Signal Processing), IEEE, and SPIE. He served the chair of Circuits and Systems Group, IEICE, in 2000 and the general chair of Digital Signal Processing Symposium, IEICE, in 1988 and Karuizawa Workshop on Circuits and Systems, IEICE, in 1996.

Hiroshi Harada received the Ph.D. degree from Osaka University, Osaka, Japan, in 1995. From 1995, he joined Communications Research Laboratory (CRL), Ministry of Posts and Telecommunications (MPT), Japan (currently National Institute of Information and Communications Technology (NICT)). From 1996 to 1997, he was a postdoctoral fellow of Delft University of Technology, The Netherlands. He is currently the director of ubiquitous mobile communications group of NICT. He was the chair of Technical Committee on Software Radio, IEICE (The Institute of Electronics, Information and Communication Engineers) Communication Society from May 2005 to May 2007 and he is currently a visiting professor of University of Electro-Communications, and a board of directors of Software Defined Radio (SDR) forum. He also has joined many standardization committees in United States as well as in Japan and fulfilled important roles for them. Especially He serves as the vice chair of IEEE SCC41 and IEEE P1900.4. His current research interests include digital-signal-processing based mobile telecommunication systems, e.g. SDR, cognitive radio, dynamic spectrum access network, and broadband wireless access (BWA) systems. He received the Young Scientist Award from the Minister of Education, Culture Sports Science and Technology (MEXT), Japan and the achievement award of IEICE in 2006 on the topic of software defined radio. He is co-author of “Simulation and Software Radio for Mobile Communications” (Artech House, 2002).

Shuzo Kato received his Ph.D. degree in electrical and communications engineering from Tohoku University, Sendai Japan in 1977. From 1977 to 1995, he worked at NTT (Nippon Telegraph and Telephone) Research Laboratories in Japan, specializing personal and satellite communications systems R&D. These include core technology developments for TDMA equipment, modems, and forward error correction schemes in addition to ASIC implementations of PHS (Personal Handy Phone) handsets and many satellite communication terminals. He has managed to develop 39 kinds of ASICs so far without re-spins including the world first TDMA chip set in 1986, the world fastest Viterbi decoder chip in 1987 and 1993, lowest power consumption ADPCM codec (500 μW) in 1994, best receiver sensitivity (6 dB improvement) and the world first 2 V operating CMOS SOC PHS baseband chip (deploying first coherent demodulator for 384 kb/s mobile terminals) and many others. He founded Pacific Communications Research Corp. focusing on ASIC, SW and system design for PCS In 1995, at the same time he served as Senior Executive Vice President, and later as President of Uniden Corporation. From January 1999 to July 2001, he served as Executive Vice President, Mitsubishi Wireless Communications Inc (MWCI) in USA, as well as President, Mobile Communications Technology Center of MWCI in San Diego, CA responsible for mobile phone technology development up to real/sellable and high yield products with all certificates (FCC, CTIA and inter-operability). From 2002 to 2005, he served as Executive Vice President of Teradyne Japan responsible for P/L, Engineering, Production and Global Marketing as well as President and CEO of Omni Wireless Inc., in California, USA. He currently is Professor, Research Institute of Electrical Communications, Tohoku University, Japan, Program Coordinator, Ubiquitous Mobile Communications at NICT (National Institute of Information and Communications Technology) of Japan as well as President of Pacific Star Communications, Inc., and an Affiliate Faculty Member, Electrical Engineering, University of Hawaii, USA. He has published over 200 technical papers, held over 75 patents (including a patent which became DOD (Department of Defense, USA) standard in 1998), co-founded International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC). He is a Fellow of the IEEE and IEICE and served as an Editor of IEEE Transaction on Communications, Chairman of Satellite and Space Communications Committee, COMSOC IEEE.