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On Array Calibration Technique for Multipath Reference Waves

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SUMMARY High resolution direction-of-arrival (DOA) estimation algorithm for array antennas becomes popular in these days. However, there are several error factors such as mutual coupling among the elements in actual array. Hence array calibration is indispensable to realize intrinsic performance of the algorithm. In the many applications, it is preferable that the calibration can be done in the practical environment in operation. In such a case, the incident wave becomes coherent multipath wave. Calibration of array in the multipath environment is a hard problem, even when DOA of elementary waves is known. To realize array calibration in the multipath environment will be useful for some applications even if reference signals are required. In this report, we consider property of reference waves in the multipath environment and derive a new calibration technique by using the multipath coherent reference waves. The reference wave depends on not only the DOA but also complex amplitude of each elementary wave. However, the proposed technique depends on the DOA only. This is the main advantage of the technique. Simulation results confirm the effectiveness of the proposed technique.

key words: *array antenna, array calibration, direction-of-arrival estimation, subspace technique, multipath reference wave*

1. Introduction

Array calibration is the problem to compensate error due to mutual coupling among the elements of the array and the amplitude/phase imbalance of them. The array calibration is important especially for high-resolution Direction-of-Arrival (DOA) estimation techniques, such as the MUSIC [1] and the ESPRIT [2], to realize their high resolution capability. For precise array calibration, we usually employ reference signals having known DOA in a radio anechoic chamber to make an ideal one-wave incidence environment [3].

Such a calibration experiment is effective to remove static error even when the array environment, which includes the measured array system as well as surrounding coupling objects, are identical to the actual operation environment. However, element pattern of the array is sensitive to structure scattering [4], hence the above mentioned experiment becomes sometimes difficult especially for large system. In addition, the amplitude/phase imbalance often depends on temperature [5]. The practical array calibration should often cope with these difficulties. One of the promising approaches is the array calibration in the actual operation environment, or on-site calibration.

For the array calibration by using reference wave in the actual operation environment, we cannot often ignore multipath propagation. In the multipath environment, the reference wave may contain several elementary waves reflected/scatered by surrounding objects (*ex.* wall). There are few reports on the calibration with multipath waves, which focus on the calibration in a specific multipath environment such as two-path waves for direct and ground/sea-surface reflection (e.g. [6]).

In this paper, we provide a new array calibration technique with multipath reference waves. The term “multipath reference wave” in this paper means that the wave contains several coherent elementary waves of known DOAs. Since we consider the calibration by using reference (transmitting) antennas, it can be assumed that the DOA of direct wave is known precisely. In addition, we can often roughly estimate DOAs of possible multipath waves since we know the propagation environment. Obviously, blind calibration without knowing DOAs is preferable for the practical array calibration. However, such a calibration scheme is extremely difficult especially in the coherent multipath environment. The proposed calibration scheme will be the first step to develop such a kind of the blind calibration technique. Precise calibration is still difficult when the DOAs are known in the coherent multipath environment. This is because phase of each elementary wave affects calibration accuracy in the environment. The proposed technique has the feasible property not to affect complex amplitude of the multipath waves if the DOAs of the waves are correct. In other words, the proposed technique can apply coherent multipath reference wave without estimating the complex amplitude (magnitude and phase) of its elementary wave.

This paper is organized as follows. In Sect. 2, we provide problem formulation with array calibration in the multipath environment. Theoretical derivation of the proposed calibration technique for multipath reference waves is shown in Sect. 3. Numerical study to show accuracy and robustness of the technique is presented in Sect. 4. Finally, the conclusions are reported in Sect. 5.

2. Problem Formulation

In this section, we formulate array calibration problem both in the single-path environment and in the multipath environment to show difficulty of array calibration in the multipath environment.

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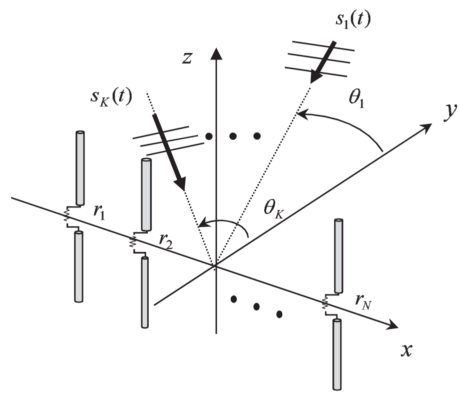

Fig. 1 DOA estimation with N-element array.

2.1 Array Calibration in Single-Path Environment

Consider the array calibration for an N-element array with a narrow-band reference wave using as shown in Fig. 1 (K = 1 for this single-path environment). The output of the N-element array can be written as follows: [3], [7]

$$\boldsymbol{r}(t) = [r_1(t), \cdots, r_N(t)]^T = \boldsymbol{C}\boldsymbol{a}(\theta)s(t) + \boldsymbol{n}(t) \tag{1a}$$
$$= \tilde{\boldsymbol{a}}(\theta)s(t) + \boldsymbol{n}(t), \tag{1b}$$

where $s(t)$ and $\boldsymbol{n}(t) = [n_1(t), n_2(t), \cdots, n_N(t)]^T$ denote the complex amplitude of the reference signal and the noise vector, respectively, and ^{T} denotes transpose. Moreover, $\boldsymbol{a}(\theta)$ is the ideal mode vector of the reference wave with known DOA (θ). The $N \times N$ matrix $\boldsymbol{C}$ is the error matrix or the calibration matrix of the array including the effect of mutual coupling, element imbalance and so forth [8], [9]. The vector $\tilde{\boldsymbol{a}}(\theta)$ is the mode vector with errors, which can be defined by

$$\tilde{\boldsymbol{a}}(\theta) = \boldsymbol{C}\boldsymbol{a}(\theta). \tag{2}$$

In this case, it is obvious that we can easily calibrate, or estimate, $\boldsymbol{C}$ by the above equation when we have several independent reference waves. Typical calibration scheme on this concept can be found in [7]. In the followings, the notation $\tilde{}$ is used for the vector or matrix which contains error to be calibrated.

2.2 Array Calibration in Multipath Environment

In the multipath environment, the reference wave will often become a coherent sum of elementary waves, or multipath wave. Assuming that there exist K elementary waves, the reference wave can be given by

$$\boldsymbol{r}(t) = \boldsymbol{C}\boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{3}$$

where $\boldsymbol{A}$ and $\boldsymbol{s}(t)$ are the mode-matrix and the complex amplitude vector of the waves defined by

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \cdots, \boldsymbol{a}(\theta_K)], \tag{4a}$$
$$\boldsymbol{s}(t) = [s_1(t), \cdots, s_K(t)]^T. \tag{4b}$$

Since the waves are coherent, we only have one signal component which spans the signal subspace. The data vector $\boldsymbol{r}(t)$ in (3) can be rewritten as

$$\boldsymbol{r}(t) = \boldsymbol{C}\boldsymbol{b}(\boldsymbol{\theta}, \boldsymbol{\rho})s_1(t) + \boldsymbol{n}(t), \tag{5}$$

where $\boldsymbol{b}(\boldsymbol{\theta}, \boldsymbol{\rho})$ is the mode vector of the coherent multipath wave defined by

$$\boldsymbol{b}(\boldsymbol{\theta}, \boldsymbol{\rho}) = \rho_1\boldsymbol{a}(\theta_1) + \cdots + \rho_K\boldsymbol{a}(\theta_K) = \boldsymbol{A}\boldsymbol{\rho}, \tag{6a}$$
$$\rho_j = s_j(t)/s_1(t). \tag{6b}$$

$\boldsymbol{\rho}$ is a K-dimensional column vector whose coefficient ρ_j denotes relative complex amplitude of the j-th wave ($s_j(t)$) to the 1-st wave ($s_1(t)$) as a reference. Clearly, ρ_1 is equal to 1 in this case.

As shown in (5) and (6a), the coherent mode vector $\boldsymbol{b}$ becomes the function of the DOAs ($\boldsymbol{\theta}$) and the relative complex amplitudes ($\boldsymbol{\rho}$). Only if we know the exact $\boldsymbol{\theta}$ and $\boldsymbol{\rho}$, we can apply the concept of single-path calibration scheme with the exact multipath mode vector $\boldsymbol{b}(\boldsymbol{\theta}, \boldsymbol{\rho})$ as

$$\tilde{\boldsymbol{b}}(\boldsymbol{\theta}, \boldsymbol{\rho}) = \boldsymbol{C}\boldsymbol{b}(\boldsymbol{\theta}, \boldsymbol{\rho}), \tag{7}$$

where $\tilde{\boldsymbol{b}}(\boldsymbol{\theta}, \boldsymbol{\rho})$ is the error contained multipath mode vector. However, estimation of ρ_j is almost impossible in practice because it is extremely sensitive to path-difference and reflection/scattering coefficient.

3. Array Calibration Technique for Multipath Reference Waves

3.1 Proposed Calibration Technique

In the calibration, we often use the reference dataset with known DOA. The signal and the noise subspace in each calibration data can be estimated by the covariance matrix defined by

$$\boldsymbol{R} = E[\boldsymbol{r}(t)\boldsymbol{r}^H(t)], \tag{8}$$

where ^{H} denotes complex conjugate and transpose. Applying eigendecomposition of the covariance matrix, the $\boldsymbol{R}$ can be decomposed into

$$\boldsymbol{R} = \sum_{i=1}^{N} \lambda_i \tilde{\boldsymbol{e}}_i \tilde{\boldsymbol{e}}_i^H. \tag{9}$$

where $\tilde{\boldsymbol{e}}_i$ denotes the eigenvector corresponding to the eigenvalue λ_i. Here we assume that the eigenvalues are indexed in descending order, or $\lambda_1 \geq \cdots \geq \lambda_N$. The signal subspace, $\tilde{\boldsymbol{E}}_S$, and the noise subspace, $\tilde{\boldsymbol{E}}_N$, can be defined by

$$\tilde{\boldsymbol{E}}_S = \tilde{\boldsymbol{e}}_1, \tag{10a}$$
$$\tilde{\boldsymbol{E}}_N = [\tilde{\boldsymbol{e}}_2, \cdots, \tilde{\boldsymbol{e}}_N], \tag{10b}$$

Note that we obtain only one signal subspace eigenvector due to the coherent multipath propagation.

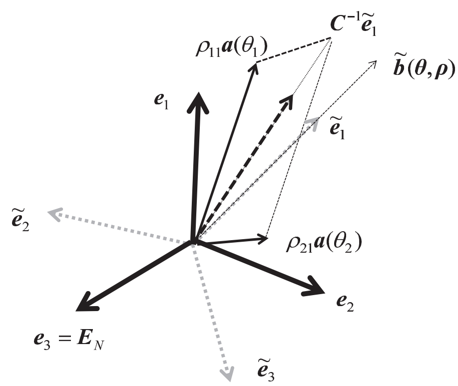

Fig. 2 Relation between signal and noise subspaces of the proposed technique for 3 elements array with coherent 2-wave incidence.

Obviously, the signal subspace eigenvector $\tilde{\boldsymbol{e}}_1$ holds the following property.

$$\tilde{\boldsymbol{e}}_1 \propto \boldsymbol{C}\boldsymbol{b}(\boldsymbol{\theta},\boldsymbol{\rho}) = \boldsymbol{C}\boldsymbol{A}\boldsymbol{\rho}. \tag{11}$$

Multiplying the matrix $\boldsymbol{C}^{-1}$ from the left in both sides of (11), we have

$$\boldsymbol{C}^{-1}\tilde{\boldsymbol{e}}_1 \propto \boldsymbol{C}^{-1}\boldsymbol{C}\boldsymbol{A}\boldsymbol{\rho} = \boldsymbol{A}\boldsymbol{\rho}. \tag{12}$$

Of course, we do not know $\boldsymbol{C}$ at this stage. This is just an algebraic manipulation to derive a calibration equation.

Here we define the ideal, or error-free, signal/noise subspace for the incoherent K waves as [10]

$$\boldsymbol{E}_S\boldsymbol{E}_S^H = \boldsymbol{A}(\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H, \tag{13a}$$
$$\boldsymbol{E}_N\boldsymbol{E}_N^H = \boldsymbol{I} - \boldsymbol{E}_S\boldsymbol{E}_S^H. \tag{13b}$$

Projection (12) onto the signal subspace in (13a), we obtain

$$\boldsymbol{E}_S\boldsymbol{E}_S^H\boldsymbol{C}^{-1}\tilde{\boldsymbol{e}}_1 \propto \boldsymbol{A}\boldsymbol{\rho}. \tag{14}$$

It is clear that the vector $\boldsymbol{C}^{-1}\tilde{\boldsymbol{e}}_1$ is unchanged because all of the *calibrated* waves are on the signal subspace spanned by $\boldsymbol{E}_S$. On the other hand, when we project (12) onto the noise subspace in (13b), we obtain

$$\boldsymbol{E}_N\boldsymbol{E}_N^H\boldsymbol{C}^{-1}\tilde{\boldsymbol{e}}_1 = \boldsymbol{0},$$

or

$$\boldsymbol{E}_N^H\boldsymbol{C}^{-1}\tilde{\boldsymbol{e}}_1 = \boldsymbol{0}. \tag{15}$$

The vector $\tilde{\boldsymbol{e}}_1$ can be obtained by the calibration measurement, and $\boldsymbol{E}_N$ can be easily calculated by the eigenanalysis of $\boldsymbol{I} - \boldsymbol{A}(\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H$. The columns of $\boldsymbol{E}_N$ are the eigenvectors of $\boldsymbol{I} - \boldsymbol{A}(\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H$ whose eigenvalues are nonzero. Therefore the matrix $\boldsymbol{C}^{-1}$ is solvable with enough number of the reference waves. Note that K should be smaller than N to derive the noise subspace defined in (13b).

Relation between the signal subspace of the reference wave ($\tilde{\boldsymbol{e}}_1$), the ideal (uncorrelated) signal and noise subspaces ($\boldsymbol{E}_S, \boldsymbol{E}_N$) is depicted in Fig. 2. This figure shows the example for a 3-element array ($N = 3$) and a multipath reference wave having two elementary waves ($K = 2$). The vectors, $\tilde{\boldsymbol{e}}_i, i = 1, 2, 3$, denote the eigenvectors derived from the covariance matrix by the measured reference data, which contains errors. On the other hand, the vectors, $\boldsymbol{e}_i, i = 1, 2, 3$, denote the eigenvectors of the covariance matrix without error defined by the two elementary waves assuming that they are uncorrelated. Note that only the 1-st eigenvector ($\tilde{\boldsymbol{e}}_1$) corresponds to the signal subspace among these three eigenvectors of $\tilde{\boldsymbol{e}}_j, j = 1, 2, 3$ for the error contained eigenvectors. On the other hand, two eigenvectors ($\boldsymbol{e}_1, \boldsymbol{e}_2$), correspond to the signal subspace for the ideal uncorrelated signal model. The multipath reference wave is a sum of elementary waves having $\boldsymbol{a}(\theta_1)$ and $\boldsymbol{a}(\theta_2)$, hence the calibrated signal subspace of the reference wave, $\boldsymbol{C}^{-1}\tilde{\boldsymbol{e}}_1$, locates on the (ideal uncorrelated) signal subspace spanned by $\boldsymbol{E}_S = [\boldsymbol{e}_1, \boldsymbol{e}_2]$. Obviously, the noise subspace, $\boldsymbol{E}_N = \boldsymbol{e}_3$, is orthogonal to the signal subspace $\boldsymbol{E}_S$, then we obtain (15).

With some algebraic manipulations, we can obtain the following simultaneous equation for $\boldsymbol{C}^{-1}$.

$$(\tilde{\boldsymbol{e}}_1^T \otimes \boldsymbol{E}_N^H)\, vec(\boldsymbol{C}^{-1}) = \boldsymbol{0}, \tag{16}$$

where $\otimes$ denotes Kronecker product and *vec* is the operator to transform a matrix into a vector by stacking the columns of the matrix: [11]

$$vec(\boldsymbol{X}) = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_N \end{bmatrix}, \tag{17a}$$
$$\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N]. \tag{17b}$$

As shown in (13a) and (13b), $\boldsymbol{E}_N$ is orthogonal to the ideal mode matrix $\boldsymbol{A}$, hence complex amplitude $\boldsymbol{s}(t)$ is not affected in this equation. Therefore, the derived equation is robust for any $s_i(t), i = 1, 2, \cdots, K$. This means that the Eq. (15) holds even when some of given elementary wave in (12) have almost power of zero. This is useful property in the practical calibration, because it can affords us to include zero-power wave(s), hence we can overestimate K when the number of effective elementary waves is hardly determined.

The proposed calibration algorithm can be summarized as follows. Assuming that we have M multipath reference data, $\boldsymbol{r}^{(m)}(t), m = 1, 2, \cdots, M$, having different DOA with each other. In addition, we suppose that each multipath reference data, $\boldsymbol{r}^{(m)}(t)$, is composed of K coherent elementary waves, and each $\boldsymbol{r}^{(m)}(t)$ has enough number of snapshots for simplicity. Then, the signal eigenvector, $\boldsymbol{e}_1^{(m)}$, for $\boldsymbol{r}^{(m)}(t)$ can be derived by the estimated $\boldsymbol{R}^{(m)}$ in (8) by using the snapshots. In this case we have M set of equations for (16). When we denote the m-th equation in (16) as

$$(\tilde{\boldsymbol{e}}_1^{T(m)} \otimes \boldsymbol{E}_N^{H(m)})\, vec(\boldsymbol{C}^{-1}) = \boldsymbol{0}, \qquad m = 1, 2, \cdots, M \tag{18}$$

the overall equation can be given by

$$\begin{bmatrix} (\tilde{\boldsymbol{e}}_1^{T(1)} \otimes \boldsymbol{E}_N^{H(1)}) \\ (\tilde{\boldsymbol{e}}_1^{T(2)} \otimes \boldsymbol{E}_N^{H(2)}) \\ \vdots \\ (\tilde{\boldsymbol{e}}_1^{T(M)} \otimes \boldsymbol{E}_N^{H(M)}) \end{bmatrix} vec(\boldsymbol{C}^{-1}) = \boldsymbol{0}. \tag{19}$$

The unknown vector, $vec(\boldsymbol{C}^{-1})$, is the N^2-dimensional vector having $N^2 - 1$ unknowns because the calibration matrix is unaffected by the constant scale factor. Also, size of the coefficient matrix in (19) is $M(N-K) \times N^2$. Therefore the following condition is required to obtain the unique solution for $\boldsymbol{C}^{-1}$:

$$M \geq \frac{N^2 - 1}{N - K}. \tag{20}$$

Note that the equation in (19) has the same form as shown in [12].

3.2 Discussion

One may think that the conventional calibration technique proposed in [7], for example, is available because we can estimate the DOA of the elementary waves. However, it is impossible because the conventional calibration technique with reference waves requires precise mode vector for each reference wave, that is $\boldsymbol{b}(\boldsymbol{\theta}, \boldsymbol{\rho})$ in (6a). Hence, $\boldsymbol{\rho}$ should be estimated before the calibration. These amplitude coefficients in $\boldsymbol{\rho}$ seems to be roughly estimated by using the known mode matrix of $\boldsymbol{A}$; however, it becomes

$$\hat{\boldsymbol{\rho}} \propto (\boldsymbol{A}^H \boldsymbol{A})^{-1} \boldsymbol{A}^H \boldsymbol{e}_1 \propto (\boldsymbol{A}^H \boldsymbol{A})^{-1} \boldsymbol{A}^H \boldsymbol{C} \boldsymbol{A} \boldsymbol{\rho}. \tag{21}$$

It is clear that $\hat{\boldsymbol{\rho}} \propto \boldsymbol{\rho}$ holds only if $\boldsymbol{C} = \boldsymbol{I}$. Therefore we can see that precise calibration will be hardly realized by the direct application of the conventional technique with estimated $\boldsymbol{b}(\boldsymbol{\theta}, \hat{\boldsymbol{\rho}})$.

Calibration performance of the proposed technique may also depend on the DOA accuracy of the reference waves. In the next section, we will evaluate calibration performance of the proposed and the conventional technique by computer simulations. The calibration technique proposed in [7] with the multipath modification discussed in this subsection is used as the conventional technique.

4. Simulation Results

In order to check validity of the proposed calibration technique with multipath reference waves, we first show a simple numerical example. In this example, a 4-element dipole array of half-wavelength inter-element spacing terminated with dummy (50 Ω) is employed as listed in Table 1. The received data affected by the mutual coupling is calculated by the method of moments. Figure 3 shows the MUSIC spectrums of one-wave incidence without calibration. The DOA in each trial is 0°, ±15°, ±30°, ±45°, ±60°, and ±75°, respectively. The calibrated spectrums by the proposed technique are shown in Fig. 4. In the calibration, we employ 8 reference datasets ($M = 8$). Each of them includes 2 coherent waves ($K = 2$) whose power ratio, $|s_1^{(m)}(t)|^2/|s_2^{(m)}(t)|^2$, is 10 dB without noise. The combinations of the DOAs in each dataset are

$$\begin{aligned}(\theta_1^{(m)}, \theta_2^{(m)}) = {} & (0°, 22°), (8°, 51°), (31°, 3°), \\ & (42°, 30°), (-23°, -42°), (33°, 10°), \\ & (60°, -60°), (-12°, -25°).\end{aligned}$$

Clearly, all of the MUSIC spectrums are correctly recovered by the proposed calibration.

Table 1 Array parameter.

Antenna element	Half-wavelength dipole
Number of elements (N)	4
Frequency	2.45 GHz
Inter-element spacing	6.1 cm (0.5λ)
Radius of the wire	0.5 mm
Terminated Load	50 Ω

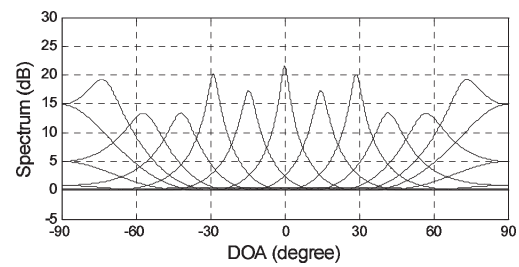

Fig. 3 MUSIC spectrums without calibration.

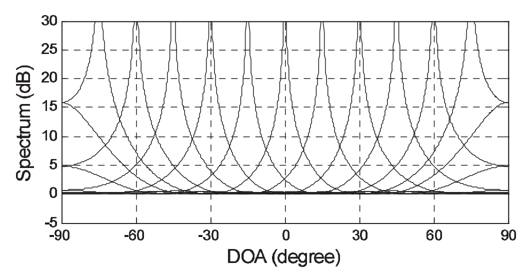

Fig. 4 MUSIC spectrums calibrated by the proposed technique.

In the above example, precise DOAs are used to verify validity of the calibration equation in (19). On practical calibration in the actual environment, although DOA of the direct wave can be given precisely by direction of the reference antenna, the DOA of the reflected waves could often contain bias. Robustness for DOA bias of the reflected waves is important to clarify availability of the proposed technique in practice. For this purpose, we evaluate RMSE (Root-Mean Square Error) of DOAs estimated by the MUSIC statistically. In this evaluation, the averaged RMSE derived by the DOA estimation results for one wave incidence in every 10° from −60° to 60°.

In calibration, we need set of multipath reference waves. The number of elementary waves and their DOAs highly depend on the propagation environment. It is difficult to evaluate statistical performance of the technique by using

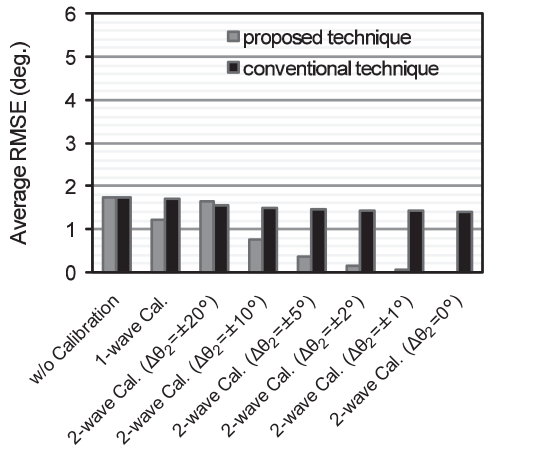

Fig. 5 Averaged RMSE of DOA estimation by MUSIC algorithm calibrated with multipath reference waves having signal #1 and signal #2 ratio of 20 dB.

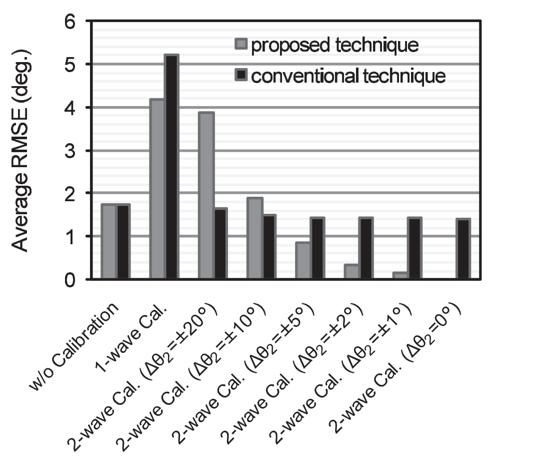

Fig. 6 Averaged RMSE of DOA estimation by MUSIC algorithm calibrated with multipath reference waves having signal #1 and signal #2 ratio of 10 dB.

such a specific environment. Therefore, we employed random combination of two elementary waves ($K = 2$) having diverse DOAs for simplicity. Nine sets of reference waves ($M = 9$) is used in each trial of the calibration. The DOAs of the direct (signal #1) and the reflected (signal #2) waves, θ_1 and θ_2, in the reference wave are selected randomly in the range $[-60°, 60°]$. In the calibration matrix derivation, we introduce the DOA estimation error ($\Delta\theta_2$) of the reflected wave. The $\Delta\theta_2$ is selected also randomly according to the Gaussian probability distribution function whose center is θ_2. Standard deviation, σ, of the function defines degree of DOA estimation error. In the simulations, we evaluate several $\Delta\theta_2$s defined by 3σ width ($\Delta\theta_2 = \pm 3\sigma$). The calibrated MUSIC spectrum is evaluated by 100 independent sets of $\Delta\theta_2$ with given θ_1. Also 100 different θ_1s are used in this evaluation; therefore, total number of the estimated $\boldsymbol{C}$ used in each trial for the RMSE derivation becomes 100×100.

The derived average RMSEs by the proposed and the conventional technique are shown in Figs. 5 and 6, respectively. In both simulations, no noise is added to show the DOA estimation error caused by the calibration itself. The RMSE of calibration results assuming that $K = 1$ (direct wave only) is also shown in these figures for reference.

Figure 5 shows the results for power ratio of 20 dB between the direct and reflected wave. In this case, the reflected waves are weak enough, hence we can slightly improve the averaged RMSE even if we apply the proposed calibration with $K = 1$ (1-wave cal.). The RMSEs of the proposed technique decrease rapidly as $\Delta\theta_2$ becomes smaller as we expected. On the other hand, the RMSEs of the conventional technique are almost the same in all cases. No improvement can be seen even when we used the exact DOAs in the calibration. This was caused by the estimation error of $\hat{\rho}$ as discussed in Sect. 3.1. Figure 6 shows the results for power ratio of 10 dB between the diirect and reflected wave. Clearly, this is the case for the reference waves having relative strong reflected wave, so the 1-wave calibration do not work properly in both techniques. The RMSEs become worse for $\Delta\theta_2 \geq 10°$; however, they becomes less than 1° when $\Delta\theta_2$ is less than 5° by the proposed technique, although no improvement can be obtained by the conventional one.

From these results, we can say that the proposed technique is available in the multipath environment when the DOA of elementary waves can be almost estimated.

5. Conclusions

In this paper, we have considered array calibration problem with multipath reference waves and proposed a new technique. We prove the robustness of the proposed technique against complex amplitude of elementary waves. This method can accurately calibrate the array without estimating the complex amplitude of elementary waves in the reference signal when the DOA of the waves are almost known. This property will be preferable for the calibration in actual multipath environment. Of course, the exact DOA of the waves is hardly estimated in practice. Hence, the robustness of the technique for DOA error of reflected waves is also evaluated by computer simulations. We show, in this paper, the simulation results only for the dipole array with the limited scenario. Quantitative analysis for the other arrays and experimental verification are the further problems. In addition, further improvement of the robustness for DOA error of elementary waves will be preferable for practical calibration. They will be done in the near future.

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References

[1] R.O. Schmidt, “Multiple emitter location and signal parameter esti-

mation," IEEE Trans. Antennas Propag., vol.AP–34, no.3, pp.276–280, March 1986.
[2] R. Roy and T. Kailath, "ESPRIT — Estimation of signal parameters via rotational invariance techniques," IEEE Trans. Acoust. Speech Signal Process., vol.37, no.7, pp.984–995, July 1989.
[3] J.C. Liberti, Jr. and T.S. Rappaport, Smart antennas for wireless communications, Prentice Hall, 1999.
[4] I.J. Gupta, J.R. Baxter, W. Ellingson, H.G. Park, H.S. Oh, and M.G. Kyeong, "An experimental study of antenna array calibration," IEEE Trans. Antennas Propag., vol.51, no.3, pp.664–667, March 2003.
[5] N. Tyler, B. Allen, and H. Aghvami, "Adaptive antennas: The calibration problem," IEEE Commun. Mag., vol.42, no.12, pp.114–122, Dec. 2004.
[6] A. Leshem and M. Wax, "Array calibration in the presence of multipath," IEEE Trans. Signal Process., vol.48, no.1, pp.53–59, Jan. 2000.
[7] C.M.S. See, "Sensor array calibration in the presence of mutual coupling and unknown sensor gains and phases," Electron. Lett., vol.30, no.5, pp.373–374, March 1994.
[8] I.J. Gupta and A.A. Ksienski, "Effect of mutual coupling on the performance of adaptive arrays," IEEE Trans. Antennas Propag., vol.31, no.5, pp.785–791, Sept. 1983.
[9] B. Friedlander and A.J. Weiss, "Direction finding in the presence of mutual coupling," IEEE Trans. Antennas Propag., vol.39, no.3, pp.273–284, March 1991.
[10] H. Krim and M. Viberg, "Two decades of array signal processing research," IEEE Signal Process. Mag., vol.13, no.4, pp.67–94, July 1996.
[11] H.L. Van Trees, Optimum Array Processing — Part IV of Detection, Estimation, and Modulation Theory, Wiley Interscience, New York, 2002.
[12] T. Naito, H. Yamada, and Y. Yamaguchi, "Array calibration technique with virtual elements for DOA estimation," IEICE Trans. Commun. (Japanese Edition), vol.J92–B, no.1, pp.216–223, Jan. 2009.

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