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PAPER

Efficient Prefiltering for FIR Digital Filters

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SUMMARY This paper presents three types of prefiltering for FIR digital filters to decrease the number of multipliers required. The first type is based on cyclotomic polynomials. It can be applied to any types of band-selective filters. The second is a mirror-image quadratic polynomial to make a passband shaping. Both types of the prefilters are used with the interpolation technique, and this improves each primitive characteristic in terms of the sharp transition. In the prefilter-equalizer design approach, these prefilterings bring about the reduction of the number of multipliers required in hardware implementation. The prefiltering efficiency is demonstrated by a few examples.

1. Introduction

Recently there have been reported a number of methods for designing FIR digital filters efficiently[(1)-(8)]. Adams and Willson have presented the prefilter-equalizer design approach[(1),(2)]. The overall filter consists of the multiplierless prefilter cascaded with an amplitude equalizer. The efficiency of this scheme depends on not only the chracteristics of prefilters but also the matching between given specifications and the prefilters used. Owing to this fact, many kinds of prefilters have been proposed.

A prototype prefilter has the Recursive Running Sum (RRS) structure. It is very efficient because it requires only two adders, regardless of its order. In addition, the modified Kaiser-Hamming prefiltering is presented to improve the stopband attenuation. All of these types of prefilters are very efficient to lowpass and highpass filters, but it is impossible to apply to bandpass filters.

An extension to design bandpass filters has been done, based on the concept of matched filters[(3)]. Since the design concepts differ in the cases of lowpass/highpass filters and bandpass filters, it is difficult to get a unified design approach.

Vaidyanathan and Beitman have also proposed the efficient prefilter based on Dolph-Chebyshev functions[(4)]. Its amplitude response shows the equiripple attenuation and the prefilter alleviates the approximating burden on an equalizer. Yet it seems to give rather complicated implementations, because the prefilter consists of many substructures involving a multiplier.

The interpolated FIR filter presented by Neuvo, Dong, and Mitra[(5)] has the completely different origin from the prefilter-equalizer design. The difference leads to the distinct design method and the resulting properties will be also dissimilar. Nevertheless it can contribute to increase the variety of the class of prefilter-equalizer structures.

In this paper we are primarily concerned with three types of prefiltering within the prefilter-equalizer design. The first type is based on the cyclotomic polynomials. It essentially requires no multipliers. To improve the amplitude response around the passband edge, the other is led from the investigation of zeros of an optimal FIR filter. Finally the interpolation concept is combined with those prefilters. By resort to the prefilterings it is possible to increase the potential of the prefilter-equalizer design.

2. Preliminaries

In tradition designing digital filters involves two steps: approximation for a transfer function and realization of the transfer function. On one hand the approximation minimizes the approximation error. On the other hand the realization tends to minimize the hardware complexity or the computational complexity. Nevertheless, it is of course better to implement the digital filter so that it minimizes the computational complexity as long as the relevant approximation error stays within a tolerance limit given in advance.

The fact that these two aspects relates to each other in a finite wordlength implementation has been pointed out by several authors[(1),(7)-(11)]. Adams and Willson have pointed out as follows: The conventional approach[(12)] to designing FIR digital filters minimizes the length of the impulse response, but dose not necessarily minimize the computational complexity. Thus we have a question which is a better way to implement a digital filter. There is not yet a unified approach to solve the question. Instead, at present, the problem in the prefilter-equalizer design approach is to find a variety of efficient prefilters.

In the prefilter-equalizer design for FIR digital

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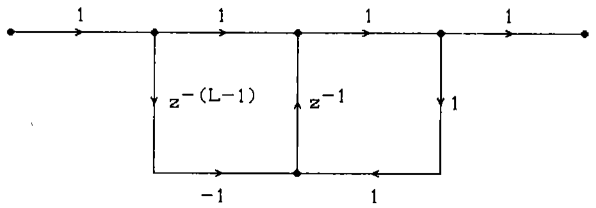

Fig. 1 The RRS structure with length L.

filters, a structure of the prefilters is very simple and usually has no multipliers. The entire FIR filter consists of the prefilter cascaded with an equalizer that equalizes the amplitude response of the prefilter to be matched with the desired response. It has been demonstrated that this approach can provide benefits in three areas: reduced computational complexity, reduced sensitivity to coefficient quantization, and reduced roundoff noise[1].

The prefilter structure in the original design[1] is of the Recursive Running Sum (RRS) as shwon in Fig.1. The RRS is very simple and it requires only two adders, regardless of its length. The frequency response of an RRS with length L is given by

$$P(e^{j2\pi f})=\frac{\sin(\pi fL)}{\sin(\pi f)}e^{-j\pi f(L-1)}, \qquad (1)$$

and it looks like a lowpass filter. The first null occurs at the frequency $1/L$.

Given the stopband edge f_s of a lowpass filter, the first step of the design procedure starts with the possible choices of the lengh as an integer slightly less than $1/f_s$. The second step is to design an amplitude equalizer with minimum length for each prefilter candidate so that the product of the prefilter and equalizer frequency response will meet the overall specifications.

In addition to the basic RRS structure, the cascaded RRS structure and modified Kaiser-Hamming structure have been also presented on the basis of the filter sharpening concept[2]. This approach is very efficient to narrow band lowpass/highpass filters. However it is difficult to apply those prefilters to bandpass/bandstop filters, because of the lack of suitable prefilter candidates.

3. Prefilters Based on Cyclotomic Polynomials

The RRS with length L has its roots at all of the equi-spaced points on the unit circle but unity. It is thus possible to factor the RRS into a product of the cyclotomic polynomials. To improve the facility of the RRS and to keep the efficiency on the level as it is, the RRS structure is extended, based on the cyclotomic polynomials.

The cyclotomic polynomial $C_k(z)$ arises from a factorization for the polynomial z^K-1 as a product of irreducible polynomials with rational coefficients[13].

$$z^K-1=\prod_{k|K}C_k(z), \qquad (2)$$

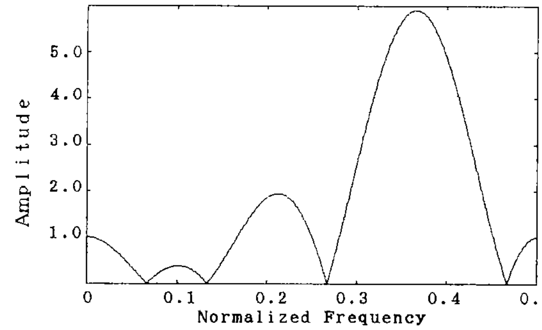

Fig. 2 Amplitude response of $C_{15}(z)$.

where $k|K$ denotes that k is a divisor of K.

There is one $C_k(z)$ for each divisor k of K, including $k=1$ and $k=K$. The roots of $C_k(z)$ are the primitive k th roots of unity. The number of such roots is given by the Euler's function $\varphi(k)$. $\varphi(k)$ is equal to the number of positive integers smaller than k which are prime to k. Therefore the degree of $C_k(z)$ is $\varphi(k)$. $C_k(z)$ is defined by

$$C_k(z)=\prod_{(i,k)=1}(z-e^{-j2\pi i/k}), \qquad (3)$$

where $(i,k)=1$ denotes that i and k are mutually prime.

The cyclotomic polynomials $C_k(z)$ have a salient property: If k has no more than two distinct odd prime factors, $C_k(z)$ has coefficients from the set $\{0, 1, -1\}$. The smallest integer k with three odd prime factors is $k=105$. Remember that even if k is greater than 105, there are infinitely a great number of cyclotomic polynomials of which coefficients pertain to 0, 1, and -1. For example, see the case for $k=128$. As long as k is a composite number without more than two odd prime factors, the coefficients of the cyclotomic polynomials are 0, 1, and -1.

From Eq.(2), the mathematical form of the RRS with length L, $P(z;L)$, is now factored as

$$P(z;L)=\prod_{\substack{k|L\\k>1}}C_k(z)z^{-\varphi(k)}. \qquad (4)$$

This equivalent decomoposition also requires no multiplications. Moreover, this factorization suggests that a product of appropriate cyclotomic polynomials can be a prefilter for every types of band-selective filters, if the appropriate set of cyclotomic polynomials is obtained by removing unwanted zeros of equi-spaced zeros on the unit circle.

Unlike the RRS, the amplitude responses of the cyclotomic polynomials have their peaks at various frequencies in addition to the special frequencies, 0 and 0.5. For example, Fig. 2 shows the amplitude response of $C_{15}(z)$. Since the spectra of cyclotomic polynomials exhibit a variety of shapes, how to use those polynomials is not as easy as the case for the RRS.

Let us now review the design with the cyclotomic resonators[7]. In the design, the basic structure consists

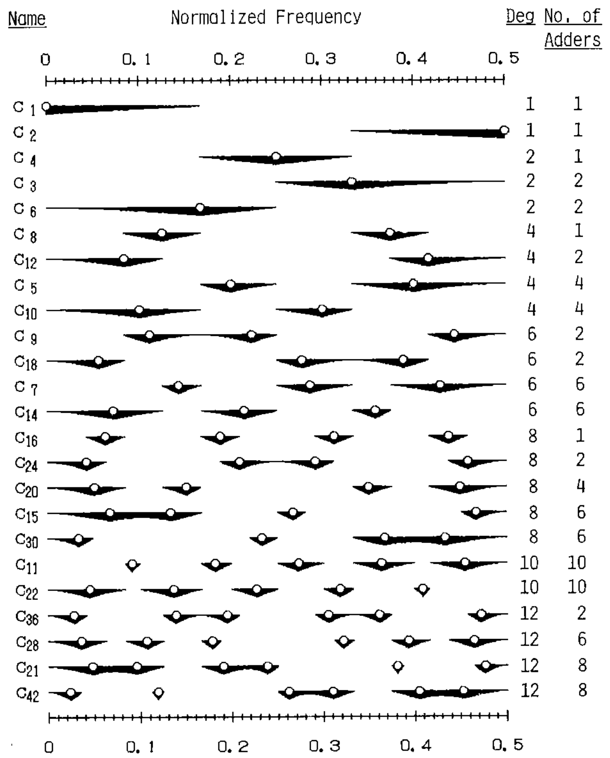

Fig. 3 Frequency ranges smaller than unity and root locations of 24 cyclotomic polynomials.

of an FIR part cascaded with cyclotomic resonators. The design is conducted so that the resulting spectrum area of the FIR part may be reduced by successive multiplication of cyclotomic polynomials to the desired frequency response. In other words, the cyclotomic resonator works in place of a principal part of the desired response. The resonator corresponds to a prefilter in the prefilter-equalizer scheme.

In addition, the design completes when a number of almost the same sizes of spectral peaks have come up at almost everywhere over the response of the FIR part[14]. Such a spectrum equivalently means a periodic sequence of a white noise with a finite interval in the time domain.

For this fact, while the degree of the FIR part slightly increases, the actual range of coefficients in the FIR part will decrease. This leads to an easier implementation in practical uses. Namely, as the response of a cyclotomic resonator approaches to the desired response, the burden on an FIR part will be relaxed.

This discussion with emphasis on a spectrum shows the truth of the following observation: If the spectrum of a prefilter resembles the desired one as close as possible, the burden of approximating an equalizer is most reduced.

By resort to this observation as a guideline, it is possible to proceed the design using a new family of prefilters based on the cyclotomic polynomials. If a cyclotomic polynomial has its roots at the stopband region of interest rather than the passband region, it is a candidate for a suitable prefilter. Selection among the candidates is performed with the aids of computer graphics and the selecting chart shown in Fig. 3. The number of cyclotomic polynomials in the figure is limited in only 24, but every other polynomial is still a possible candidate for a prefilter. The possibility depends on individual specifications.

Heavy segments in Fig. 3 represent the frequency ranges on which the amplitude response of each cyclotomic polynomial is smaller than unity. The light dots indicate the root locations. The roots of the polynomial z^k-1 lie on the equi-spaced points on the unit circle, and its frequency response is as same as a comb filter. A cyclotomic polynomial is formed from those roots by removing some of them. Thus the frequency response of a cyclotomic polynomial rises higher than unity at those removed zeros. The highest peak takes place at the frequency which is far from the adjacent zeros pertaining to the polynomial.

A product of appropriate cyclotomic polynomials will contain the cascaded RRS structure[2], and will provide a useful family of prefilters that is applicable to all types of band-selective filters.

4. Passband-Shaping Prefilters

Based on the cyclotomic polynomials, the basic RRS has been extended to the prefilters for bandpass/bandstop filters. The frequency response of such a prefilter contributes to the desired prefiltering at a certain peak. The shape of the peak is simple and single. Its roll-off rate is basically controlled over by a single vacancy of a equi-spaced zero on the unit circle. Hence it is difficult to make a sharp cut-off.

An optimal linear-phase FIR filter designed by the computer program[12] has some zeros off the unit circle but within the passband region. If a prefilter has such a zero, it will improve the cut-off characteristic around the passband edge. As a bonus, it will relax tight requirements for the amplitude equalizer from making a sharp transition from the passband to the stopband.

The simplest candidate for such a passband shaping prefilter is a real quadratic polynomial. Its two roots lie on the real axis with the mirror-image symmetry. When the root is on the positive real axis, the amplitude response of the associated polynomial will display the rising and then falling behavior that is different from a monotonic decreasing fashion. The rising part will compensate the droop of the RRS and the prefilters based on the cyclotomic polynomials.

Of course, the quadratic polynomial must not have any actual multipliers. Otherwise, the efficiency will be disappeared. The coefficients of the polynomial have been thus restricted to a sum of two powers of 2. The quadratic polynomial for the passband shaping must be of the form

$$P_2(z)=1-(2+2^M)z^{-1}+z^{-2}, \qquad \text{for } M\leqq 1. \qquad (5)$$

Empirically, the lower bound for an integer M enough to give an effective shaping is -5. When one uses this to a highpass filter, switch over the negative sign of the second term.

To get more effective shaping prefilters, one can consider a mirror-image quartic polynomial. Such a polynomial, however, cannot have the desired symmetry within rational coefficients. Hence, it is impossible to implement the real circuit based on it. A very exceptional case happens, when the four roots of the polynomial are on the imaginary axis. The root configuration for this case is obtained by folding a pair of the roots of Eq.(5) with respect to the imaginary axis and then rotating them by $\pi/2$ radian. The corresponding polynomial can be simply obtained by replacing z in Eq.(5) with $-z^2$.

5. Interpolated Prefilters

The primitive prefilters presented in the preceding sections is efficient to the design of FIR digital filters except for too wide passband applications. In the prefilters based on the cyclotomic polynomials, a single vacancy among the equi-spaced zeros on the unit circle controls over the transitional characteristic from a passband to a stopband. If a multipe use of the same prefilters or a cascaded structure of some different prefilters are employed, the transitional behavior around the passband edge tends to be like a straight line. Thus these schemes are effective to the applications with narrow passbands but not too narrow transition bands.

The interpolation technique with zero-valued samples can be used to get a prefilter with more rapid transitional characteristic. The technique will convert a primitive prefilter into the interpolated prefilters. The original idea has come from the interpolated FIR (IFIR) filters presented by Neuvo et al[(5),(6)].

An IFIR digital filter consists of a model filter interpolated by replacing each delay with several delays, cascaded with an interpolator to suppress the unwanted replicas of the model filter. The main reason for the efficiency arises from the two facts as follows. First, a wide transition band of the model filter can be shortened by inserting zero-valued samples between the original impulse response of the model filter. On the other hand, it is well known that the impulse response duration of an FIR digital filter is inversely proportional to the transition band width. Therefore it is sufficient to design the model filter with a shorter length of impulse response duration.

The same idea is useful to our primitive prefilters. If a 1 to N interpolation with $N-1$ zero-valued samples each is applied to the primitive prefilter based on the cyclotomic polynomials, the primitive transition band width is decreased by a factor N. The interpolated prefilter is simply obtained by replacing each delay with N delays.

The interpolation concept can provide a further scope to make a great deal of prefilters from some basic cyclotomic polynomials. The simplest cyclotomic polynomials are $C_2(z)$ and $C_3(z)$. The common property among them is that each has only a single zero over the real frequencies. Although $C_4(z)$ has the same property as those, it is readily obtainable by the 1 to 2 interpolation for $C_2(z)$. $C_1(z)$ and $C_6(z)$ are complementary to $C_2(z)$ and $C_3(z)$, respectively. The former behaves like a highpass filter, and the latter like a lowpass filter. They are convertible by replacing z with $-z$, neglecting a trivial sign.

Suppose each of $C_2(z)$ and $C_3(z)$ as a model filter. To yield an effective prefilter, one replica of the interpolated model filter must resemble the desired filter in terms of the band width and the position. The band width criterion specifies the maximum interpolation factor $N_{\max}$ in such a way that the interval from the dc frequency to the null of the model filter can be mapped into the band width between the center frequency and the stopband edge. In general, the replicas occur at the frequencies of integral multiples of $1/N_{\max}$. One of the replicas should be placed around the center frequency of the desired filter as close as possible. By using a particular integer K, if $K/N_{\max}$ can approximate the center frequency of interest, $N_{\max}$ turns out to be the desired interpolation factor. If K and $N_{\max}$ have a greatest common divisor, one can obtain the smaller interpolation factor as the quotient factored away from $N_{\max}$.

If $K/N_{\max}$ cannot approach to the desired center frequency for any K, choosing an integer N smaller than $N_{\max}$ and checking with this integer may be done as before. The above procedure to get interpolated prefilters is used with any primitive wide band prefilters.

Remember that we can achieve the same effect without noticing such an idea, if we understand the properties of the cyclotomic polynomials themselves. In fact we have had done so at first. The alternative method is described below, because it allows us to find such a primitive prefilter set simultaneously without using interactive steps.

The essence for the method can be found from Eqs. (2) and (3), by paying attention to which numbers specify the divisors of K and the integers prime to the index k. That is as follows.

At first, let us select a cyclotomic polynomial $C_K(z)$ that has a root close to the center frequency of a desired filter. K specifies a set $\{k|K\}$ of which members are the divisors of K.

K and $N_{\max}$ then allow us to choose an integer N that is a multiple of K and smaller than $N_{\max}$. Of course N is preferable to be highly composite. N determines a set $\{n|N\}$ that consists of the divisors of N. The cyclotomic polynomials specified by this set have in total

N equi-spaced zeros on the unit circle. The roots of $C_K(z)$ are duplicate for some of those roots, because K is a divisor of N.

Therefore, if one removes the former set from the latter, the remaining members will specify a primitive set of cyclotomic polynomials suitable for attenuating unwanted replicas. When K is a prime number or its double, K is suitable for generating a primitive prefilter set. Otherwise, undesired peaks in the amplitude response must be attenuated by other polynomials. It is as same as the case when a single set of cyclotomic polynomials cannot exceed a sufficient attenuation. Another set may be found by either methods as described before.

It should be noted that there is the possibility to simplify the primitive set of cyclotomic polynomials as the interpolator $G(z)$, based on the properties of the cyclotomic polynomials. One of them summarized by McClellan and Rader[13] is cited as follows: If p is prime, and if p dose not divide m,

$$C_{pm}(z)C_m(z)=C_m(z^p). \tag{6}$$

Since N is a highly composite number, this property often finds useful applications in practice.

There are several ways to provide interpolated prefilters, but how to get the best is beyond our scope at present.

As for the quadratic shaping prefilters, the behavior caused by interpolation is as same as the case for cyclotomic polynomial based prefilters. An interpolation by a factor N causes the original spectrum to be packed N times in a back-to-back manner. The interpolation brings about an N times reduction of the frequency axis. Hence, if a 1 to N interpolation by replacing each delay with N delays is applied to the primitive shaping polynomial as it follows that

$$P_2(z^N)=1-(2+2^M)z^{-N}+z^{-2N}, \text{ for } -5\leqq M\leqq 1 \tag{7}$$

then it is possible to enhance the changing rate of the primitive spectrum by a factor N.

6. Design Examples

Two design examples are compared to the conventional optimal FIR digital filters [12]. The amplitude equalizers have been designed by the computer program[12] with slight modifications. The design problem is formulated in the same way as found in Ref.(1).

The first example is a lowpass filter, and the specifications are listed in Table 1, where the frequency is normalized by a sampling frequency. To the specifications, the first type of the prefilters has been made of a set of the cyclotomic polynomials $C_2(z)$, $C_3(z)$, $C_4(z)$, $C_8(z)$, $C_9(z)$, $C_{13}(z)$. The second prefilter has been specified by $M=-4$.

Table 1 Specifications for the examples.

Type		Bands	Approximation Error P+E	Approximation Error Conv.
Ex.1	LPF	pass(0.00, 0.021)	0.09 dB	0.10 dB
		stop(0.07, 0.500)	-46.2 dB	-46.8 dB
Ex.2	BPF	stop(0.000, 0.168)	-63.0 dB	-59.0 dB
		pass(0.189, 0.211)	0.32 dB	0.41 dB
		stop(0.232, 0.500)	-62.1 dB	-59.0 dB

"P+E" and "Conv." denote the prefilter-equalizer design, and the conventional optimal design, respectively.

Figure 4(a) shows the frequency response of the prefilter. An equiripple design weighted by the prefilter has produced the amplitude equalizer with length 35, and the amplitude response is found in Fig.4(b). As a result, the overall filter with the prefilter-equalizer design has the response as shown in Fig.4(c). The final part of the Fig. 4(d) illustrates the coefficient quantization effects with 4-bit fixed point binary representation. Our design keeps the attenuation level under -40dB. By contrast, the same quantization causes the significant damage for the conventional digital filter, as found in Fig.5(b).

Table 2 summarizes the computational complexity for this design to compare with the conventional optimum FIR digital filter. In the table, prefilter-1 is based on the cyclotomic polynomials, and prefilter-2 is the mirror-image quadratic shaping prefilter. The number of multiplications required in our design is 18. and it amounts to 2/3 in the conventional design.

In turn, let us take a bandpass filter as the second example. The specifications listed in Table 1 are as same as those in Ref.(3). Figure 6 illustrates each part of our prefilter-equalizer design in parallel to the example 1. The computational complexity needed to this case are also found in Table 2.

The prefilter-1 doubly uses two sets of the cyclotomic polynomials. One is the cyclotomic polynomials specified by the divisors of 30 except for one and five. It should be noted that a special number 30 is the possible upper bound for the interpolation factor to this example. The other is those specified by the divisors of 20 also except for one and five. As the original forms factored with those polynomials require many additions, they have been converted to the alternative decimated versions

$$C_3(z)C_2(z^3) \tag{8 a}$$

and

$$C_2(z)C_2(z^2) \tag{8 b}$$

by using the properties of the cyclotomic polynomials, before these have been interpolated by replacing each

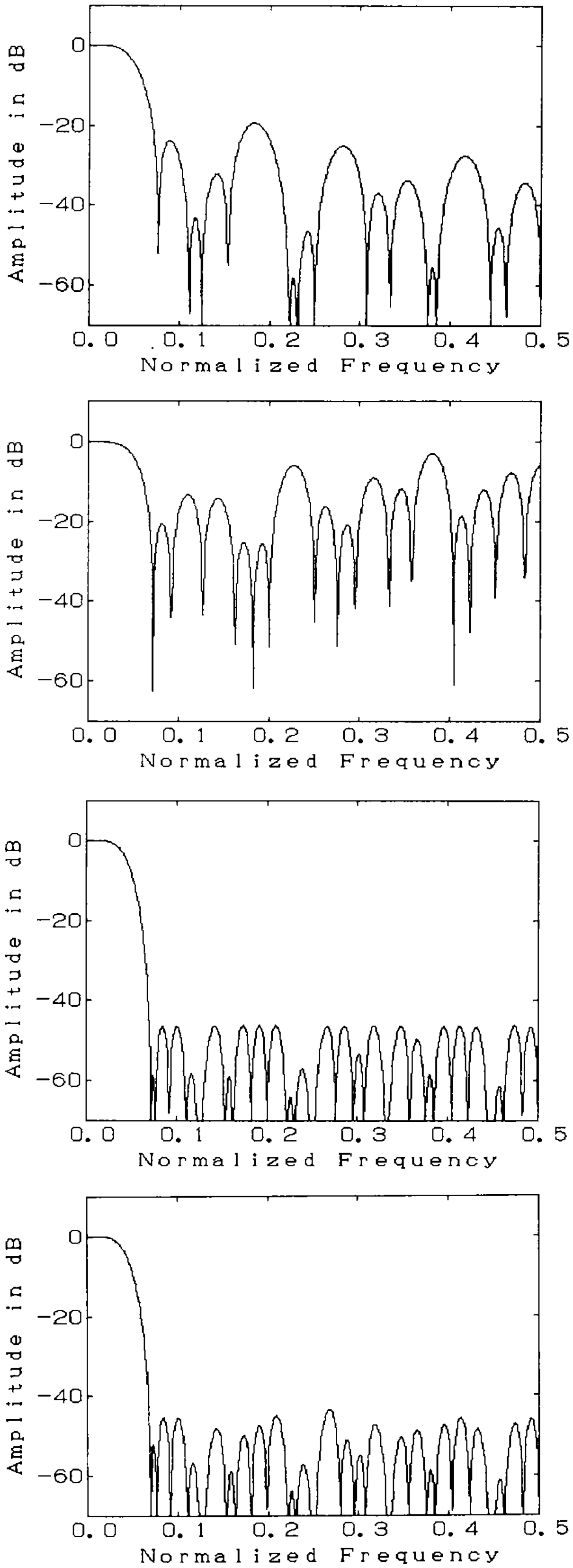

Fig. 4 Example 1. Amplitude responses of (a) the prefilter, (b) the equalizer, (c) the ideal overall filter, (d) the degraded filter with coefficients quantized to 4-bits.

Table 2 Comparison on computational complexity.

		P+E Design			Conv. Design		
		M	A	D	M	A	D
Ex.1	PREFILTER-1	0	19	27			
	PREFILTER-2	0	3	2			
	EQUALIZER	18	34	34			
	TOTAL	18	56	63	27	52	52
Ex.2A	PREFILTER-1	0	10	80			
	PREFILTER-2	0	3	10			
	EQUALIZER	40	79	79			
	TOTAL	40	92	169	61	120	120
Ex.2B	PREFILTER-1	0	19	93			
	PREFILTER-2	0	3	10			
	EQUALIZER	31	60	60			
	TOTAL	31	82	163	61	120	120

M, A, and D stand for the numbers of multiplications, additions, and delays, respectively, per sample.

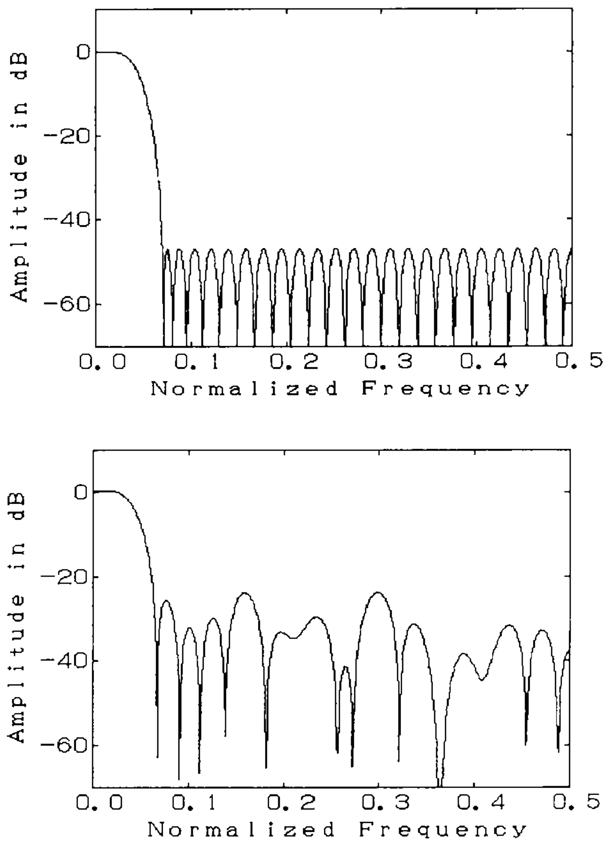

Fig. 5 Amplitude responses of the conventional lowpass filter. (a) ideal, (b) degraded by 4-bit coefficient quantization.

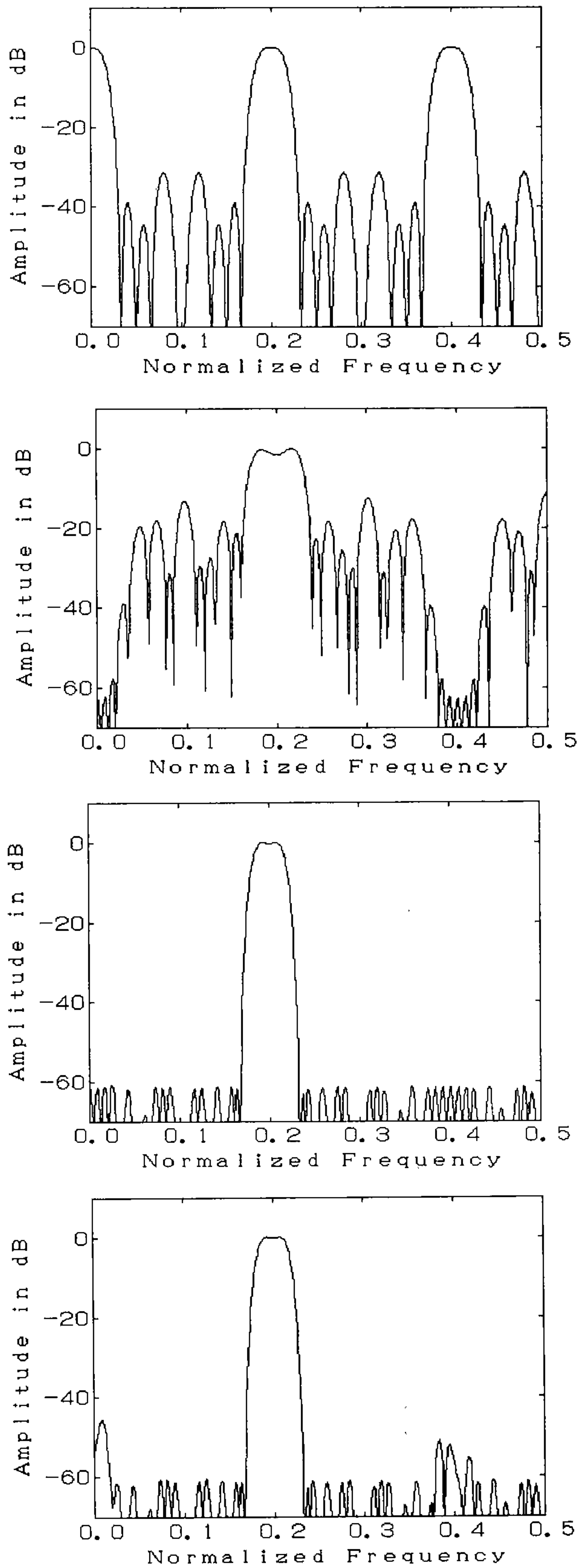

Fig. 6 Example 2a. Amplitude responses of (a) the prefilter, (b) the equalizer, (c) the ideal overall filter, (d) the overall filter degraded by 8-bit coefficient quantization.

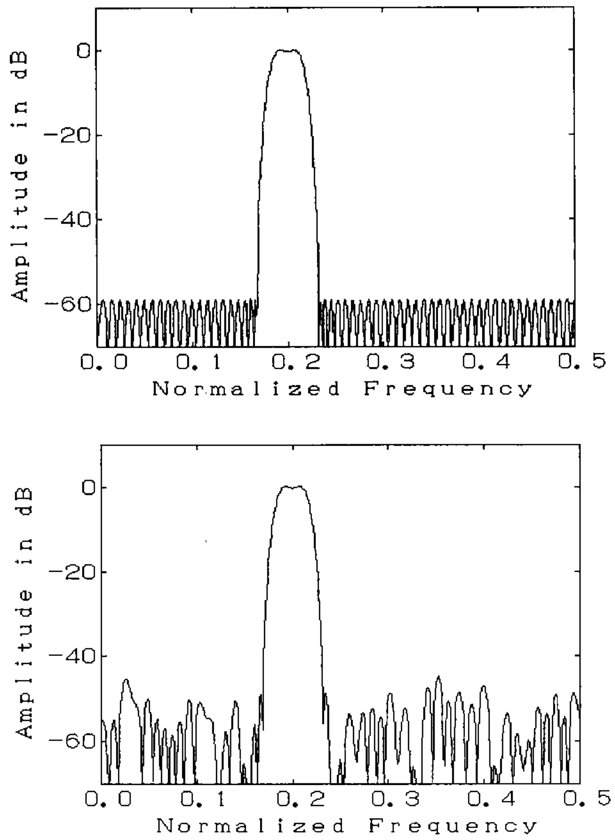

Fig. 7 Amplitude responses of the conventional bandpass filter. (a) ideal, (b) degraded by 8-bit coefficient quantization.

delay with five delays. These two sets of the prefilters require only five additions in total. The prefilter-2 with $M=-3$ is interpolated by a factor 5. Figure 6 shows the amplitude response of the individual parts and the overall bandpass filter. This implementation is referred to as the example 2a.

Table 2 shows that the number of multiplication required in the example 2a is 40 and amounts to 2/3 for the conventional design. This is one of the improvements gained by the novel design in this paper. Regarding finite wordlengh effects, the comparison between Fig.6(d) and Fig.7(b) reveals that it is yet hard to find admissible improvements. The reason why is that a prefilter derived from interpolation accompanies unwanted replicas of a desired passband. Those replicas cause the production of sharp attenuation, and this makes the equalizer design be difficult.

As can be seen from the Fig.6(a), the response has two unwanted passbands around the frequencies 0 and 0.4. In order to suppress the unwanted replicas, in the example 2b, a minor part of the prefilter-1 composed of $C_1(z)$ and $C_{42}(z)$ is added to the prefilter used in the example 2a. The total prefilter response is displayed in Fig.8(a).

The number of multiplications required in the

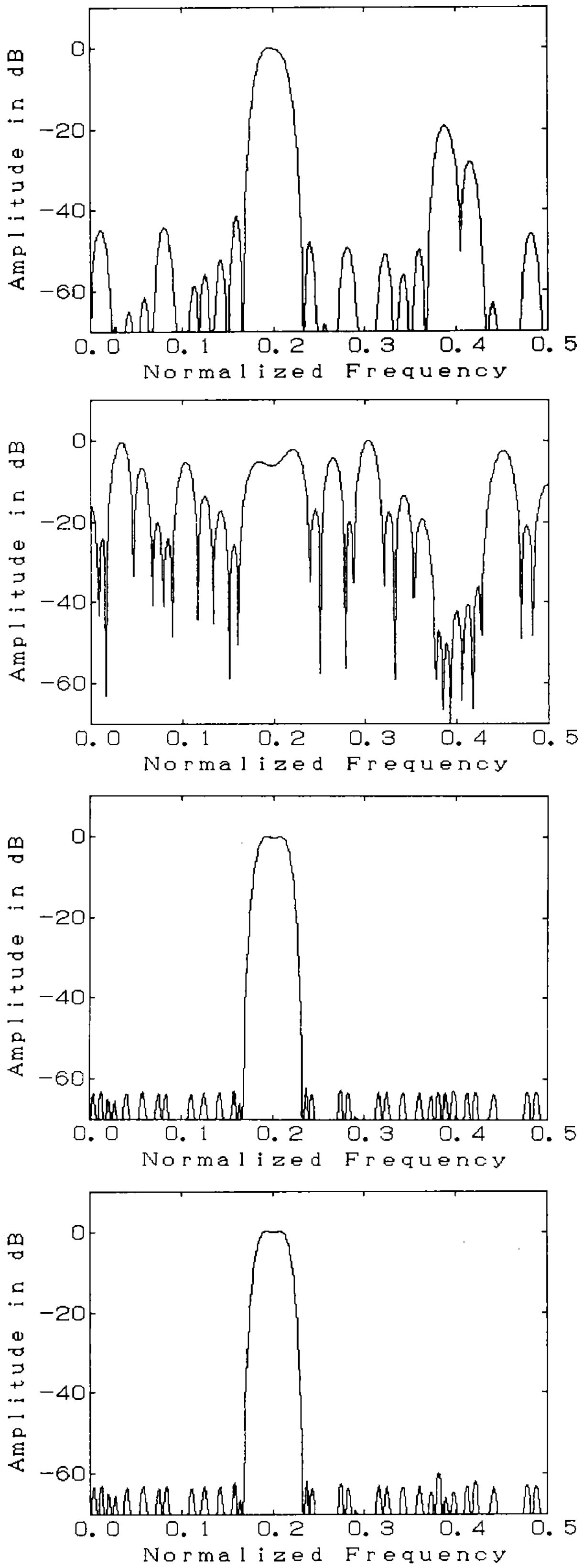

Fig. 8 Example 2b. Amplitude responses of (a) the improved prefilter, (b) the equalizer, (c) the ideal overall filter, (d) the overall filter with coefficients quantized to 8-bits.

example 2b is 31. It amounts to a half required for the conventional design. Furthermore, the required number of additions is decreased by 30% to 82. Figure 8(d) shows that the degradation by 8-bit coefficient quatization is only 2dB over the stopband. No passband degradation is admissible. On the other hand the same quantization has degraded by 14dB the stopband response of the conventionally designed filter, as shown in Fig.7(b).

7. Concluding Remarks

This paper has presented three types of the prefiltering for FIR digital filters to decrease the number of multipliers requried. The first type is based on the cyclotomic polynomials to give a sufficient attenuation desired in the stopband. The prefilters on the basis of the Recursive Running Sum are mathematically equivalent to the new prefilters. The latter can be applied to every type of band-selective filters. Thus the new class of prefilters extends the facility of the prefilter-equalizer structure.

The second type of the prefilters is the mirror-image quadratic polynomial with a slightly complicated coefficient. Yet it dose not require any actual multiplications. The prefilters contribute the passband shaping around the passband edge.

Both types of prefilters can be used in conjunction with the interpolation technique. A 1 to N interpolation causes the frequency axis to be reduced by a factor N. Hence, on one hand this increases the primitive roll-off rate of the first type of the prefilter from a passband to a stopband. On the other hand, it can afford to improve the primitive shaping effects of the second type of the prefilters.

Since the three kinds of prefiltering alleviate the burden of approximating an amplitude equalizer, the equalizer requires lower degrees. This fact has reduced the number of multipliers required in hardware implementations, when compared to conventional implementations.

Though omitted in this paper, it is possible to implement the cyclotomic polynomial based prefilter as the RRS structure. Such an implementation will more decrease the required number of additions. This fact is obvious because of the inherent property of cyclotomic polynomials. Rather, those variations are suitable for discussing an architectural problem in LSI implementations.

One of the reasons is the possibility of oscillation in an RRS structure by floating point implementation. Another issue in architectural problems lies in the pipelinability. In a recursive implementation, the simplest pipelinability with the signaling clock rate is disappeared. The pipelined processing demands providing an additional internal clock that is twice the original signaling rate. Note that both problems dose not arise in the nonrecursive structures described in this paper.

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Appendix

The explicit formulas of the 24 cyclotomic polynomials are listed for a practical use.

$C_1(z)=z-1$
$C_2(z)=z+1$
$C_4(z)=z^2+1$
$C_3(z)=z^2+z+1$
$C_6(z)=z^2-z+1$
$C_8(z)=z^4+1$
$C_{12}(z)=z^4-z^2+1$
$C_5(z)=z^4+z^3+z^2+z+1$
$C_{10}(z)=z^4-z^3+z^2-z+1$
$C_9(z)=z^6+z^3+1$
$C_{18}(z)=z^6-z^3+1$
$C_7(z)=z^6+z^5+z^4+z^3+z^2+z+1$
$C_{14}(z)=z^6-z^5+z^4-z^3+z^2-z+1$
$C_{16}(z)=z^8+1$
$C_{24}(z)=z^8-z^4+1$
$C_{20}(z)=z^8-z^6+z^4-z^2+1$
$C_{15}(z)=z^8-z^7+z^5-z^4+z^3-z+1$
$C_{30}(z)=z^8+z^7-z^5-z^4-z^3+z+1$
$C_{11}(z)=z^{10}+z^9+z^8+z^7+z^6+z^5+z^4+z^3+z^2+z+1$
$C_{22}(z)=z^{10}-z^9+z^8-z^7+z^6-z^5+z^4-z^3+z^2-z+1$
$C_{36}(z)=z^{12}-z^6+1$
$C_{28}(z)=z^{12}-z^{10}+z^8-z^6+z^4-z^2+1$
$C_{21}(z)=z^{12}-z^{11}+z^9-z^8+z^6-z^4+z^3-z+1$
$C_{42}(z)=z^{12}+z^{11}-z^9-z^8+z^6-z^4-z^3+z+1$

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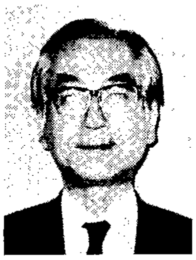

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