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PAPER

Pipelinable Low-Sensitivity Digital Filters Based on the Localization of Transmission Zeros

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SUMMARY A direct procedure to realize pipelinable low-sensitivity digital filters is developed only in the z-domain. It is possible to realize arbitrary digital transfer functions using this procedure. The key concept for the low-sensitivity property lies both in the matching concept in doubly-terminated lossless networks and in the localization of transmission zeros. The synthesis procedure is based on successive extractions of transmission zeros by means of lossless but not always reciprocal transfer scattering matrices. Since a transfer scattering matrix involves the transmission zeros as its poles, such a matrix is suitable for their localization. Furthermore a universal first/second-degree section is derived explicitly.

1. Introduction

Wave digital filters (WDFs) originally presented by Fettweis[(1)-(4)] enjoy the excellent properties. The main reason is that they are derived from doubly-terminated analog filters by means of digital imitation in terms of voltage waves. Owing to the imitation, on the one hand the synthesis procedure for a WDF needs an analog reference filter. On the other hand it is difficult to find a WDF realization for some class of transfer functions which are arbitrarily specified in the digital domain.

Indeed, WDF theory introduced the entire body of classical network structures into linear shift-invariant digital filters. Yet many classical networks have been derived by intended disciplines in the context of analog component technology. For instance, see the case of a lowpass mid-shunt lossless ladder by Fujisawa[(5)]. WDF theory does not simply answer to what is a suitable reference filter for a response specified in the digital domain, and to how a reference filter suitable for digital implementations should be synthesized. A solution to these questions will need the sophisticated and comprehensive knowledge in classical network theory and digital technology together with WDF theory.

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The main purpose of this paper is to develop a direct procedure to realize low-sensitivity digital filters. The direct procedure is described in terms of z-domain transfer scattering matrices rather than the s-domain counterparts. Thus, the synthesis procedure excludes the bilinear transformation that lays the bridge between the analog and digital domains. Hence, although the realized filters may fall into a class of WDFs, one of the differences lies in the fact that the synthesis procedure is given only in the z-domain.

Being led from a similar motivation, Vaidyanathan and Mitra have developed a theory based on the lossless bounded real (LBR) concept[(6)-(8)]. According to the theory, a transfer function is realized as a cascade of lower-degree digital two-ports by successive extraction of lower-degree LBR sections. An LBR section is basically extracted from an input function as shown in Fig. 1. The method can also allow us to realize the transfer function as a cross transfer function, but this realization is restricted to the case for the transmission zeros on the unit circle. Although a remedy for arbitrary transmission zeros is also described, the price paid for this costs the disturbance of the structurally-LBR property.

As to the input transfer function realization, another synthesis has been recently described by Watanabe with conventional two-port adaptors[(9)].

The method described in this paper yields a cascade realization as a so-called cross transfer function by means of more regular first / second-degree sections[(10),(11)].

There are two essential points for low-sensitivity

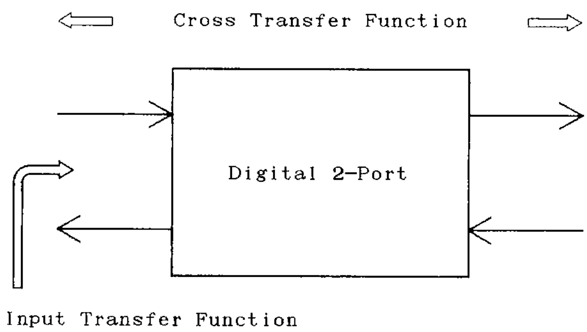

Fig. 1 Cross and input transfer functions.

structures. First, the low-sensitivity property in a passband is a consequence of matching in doubly-terminated reactance networks in which the termination is frequency-independent. Thus losslessness is crucial, but reciprocity that is conserved in other literature is not of importance.

Second, each transmission zero is realized by a series or shunt arm in a reactance ladder network. The independent or local realization of transmission zeros leads to the stopband immunity of the ladder against reactive element-value variations. This fact implies that the localization of transmission zeros is useful for the low-sensitivity in a stopband. Thus the methods based on the factorization of a scattering matrix[(12),(13)] cannot attain lower stopband sensitivity.

It should be mentioned that the topology of the universal section derived in this paper was firstly yet independently found by Deprettere and Dewilde[(14)]. In the literature the topology is used for a first-degree section with complex multiplier coefficients. Their method is highly mathematical, and the realization matrices are characterized by orthogonal matrices, hence the name 'orthogonal filters'[(14)-(16)]. By contrast, the universal section in this paper is applied to any first/second-degree section with real coefficients. Moreover the described method is straight-forward, and the orthogonality is not imposed.

Another point of this paper is to provide pipelinability for the implementation of a given transfer function without any significant modification to the given response. By using some arbitrariness in the transfer scattering matrix formulation, it is possible to accomplish pipelining. The described method is a direct procedure to realize an arbitrary digital transfer function including FIR filters.

2. Transfer Scattering Matrix

A linear time-invariant digital two-port shown in Fig. 2 can be characterized by a transfer scattering matrix $\boldsymbol{T}(z)$ defined as

$$\begin{bmatrix} X_1(z) \\ Y_1(z) \end{bmatrix} = \boldsymbol{T}(z) \begin{bmatrix} Y_2(z) \\ X_2(z) \end{bmatrix} \tag{1}$$

where

$$\boldsymbol{T}(z) = \begin{bmatrix} t_{11}(z) & t_{12}(z) \\ t_{21}(z) & t_{22}(z) \end{bmatrix}. \tag{2}$$

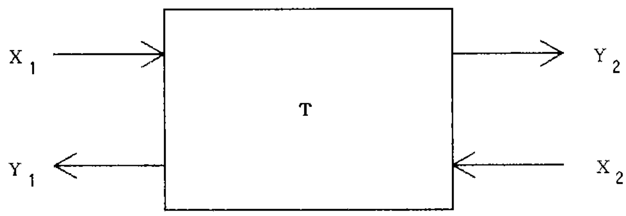

Fig. 2 Digital two-port.

For simplicity, the term 'scattering' will be henceforth suppressed for referring to a transfer scattering matrix[(17)].

A two-port is defined to be lossless, if

$$X_1(1/z)X_1(z) + X_2(1/z)X_2(z) = Y_1(1/z)Y_1(z) + Y_2(1/z)Y_2(z) \tag{3}$$

holds[(8),(10),(11),(17)]. This relation involves no port resistances and is different from the definition in terms of pseudopower by Fettweis[(2)-(4)]. A port resistance plays a key role in the wave digital filter theory. However, it is possible to derive a kind of WDFs without port resistances, and furthermore it is simpler and natural to characterize digital signals. Hence the above definition is employed[(10),(11)].

With a notation defined by $\boldsymbol{J} = \mathrm{diag}\{1, -1\}$[(14)-(17)], the losslessness imposed by Eq. (3) is expressed by the transfer matrix as

$$\boldsymbol{T}_*(z)\boldsymbol{J}\boldsymbol{T}(z) = \boldsymbol{J} \tag{4}$$

where the subscript $*$ denotes paraconjugation:

$$\boldsymbol{T}_*(z) = \boldsymbol{T}^*(1/z^*),$$

where the superscript * denotes transpose conjugation for matrices and complex conjugation for scalars, respectively. The property expressed by Eq. (4) is referred to as J-losslessness after Deprettere and Dewilde.

If $|\boldsymbol{T}(z)| = 1$, the two-port will be called reciprocal. Nevertheless, note that all of the transfer matrices in this paper are lossless but not always reciprocal.

To formulate a synthesis problem of low-sensitivity digital filters, a proper transfer matrix must be derived from a given transfer function. A rational digital filter transfer function $H(z)$ is described by

$$H(z) = F(z)/G(z) \tag{5}$$

where without loss of generality $F(z)$ and $G(z)$ are assumed Nth-degree polynomials with respect to z^{-1}, and $G(z)$ is monic, i. e. the leading coefficient is equal to unity. Actual degrees of $F(z)$ and $G(z)$ may be different to each other. For instance, if the transfer function is subject to an all-pole filter or an FIR filter, the numerator or the denominator is a constant, respectively.

The transfer matrix $\boldsymbol{T}(z)$ for $H(z)$ is constructed by

$$\boldsymbol{T}(z) = \frac{1}{F(z)} \begin{bmatrix} G(z) & \pm K_*(z)z^{-N} \\ K(z) & \pm G_*(z)z^{-N} \end{bmatrix} \tag{6}$$

where $K(z)$ is determined by

$$G_*(z)G(z) = F_*(z)F(z) + K_*(z)K(z). \tag{7}$$

$K(z)$ is thus an Nth or less degree polynomial with respect to z^{-1}. The losslessness of $\boldsymbol{T}(z)$ is easily verified by Eq. (4).

An additional assumption that the magnitude of

$H(z)$ on the unit circle is equal to or less than unity is required. This is the condition so that $K(z)$ is obtained as a real coefficient polynomial. In practice in order to give the low-sensitivity property, a transfer function should be normalized such that the maximum of the magnitude of $H(z)$ is equal to unity on the unit circle.

The sign option in Eq. (6) may be selected arbitrarily with the same occurrence. Particularly, if either the upper or the lower sign is chosen depending on whether $F(z)$ is a mirror image or an anti-mirror image polynomial, $\boldsymbol{T}(z)$ is reciprocal.

When the transfer matrix is realized as a digital two-port, the transfer function appears as

$$H(z)=1/t_{11}=(Y_2/X_1)|_{X_2=0}. \tag{8}$$

Thus the input and the output of the digital filter are X_1 and Y_2, respectively.

It should be noted that the integer N in Eq. (6) may be arbitrarily increased under preserving the input-output relation of Eq. (8) unchanged, as long as Eq. (7) is fulfilled. A specific figure of the integer is independent of the lossless property. Yet, it is not only required for realizability (causality) but also of implicit concern to invisible transmission zeros in single-input single-output digital filter applications such as represented by Eq. (8). This fact is exploited for pipeline processing, and the topic must be deferred until Sect. 6.

3. Cascade Synthesis

The realization problem for the high-degree matrix of Eq. (6) can be solved by a cascade synthesis of lossless digital two-ports. In this paper, 'cascade' means the localization of multiplicative factors of a certain quantity. Namely, each section in a cascade structure realizes a single factor for the entire body of that quantity; any global structure produced by the cascade interconnection does not affect the local realization of the factors. If a single-input single-output transfer function is considered as the above certain quantity, the term 'cascade' merely has a usual sense. It means the cascade realization with biquadratic cells.

In parallel to the case of reactance ladder filters, the localization of transmission zeros will contribute the stopband immunity against parameter variations. On the other hand the inspection of Eq. (6) proves that the numerator of a transfer function, which is the aggregation of transmission zeros, appears as a common denominator of a transfer matrix. Hence the factorization of a transfer matrix with respect to its denominator can lead to the localization of transmission zeros. Therefore the cascade synthesis based on the transfer matrix factorization is one of the best approaches for band-selective filter applications.

Let us assume that the transfer matrix $\boldsymbol{T}$ derived from a given transfer function is factored into a product of $\boldsymbol{T}_a$ and $\boldsymbol{T}_b$ as

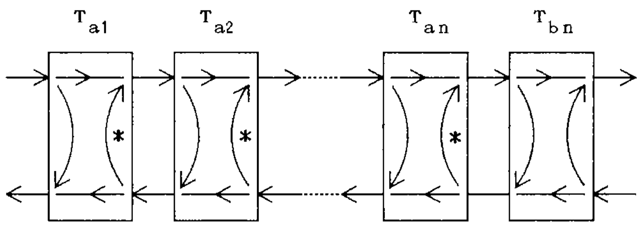

Fig. 3 Cascade realization.

$$\boldsymbol{T}=\boldsymbol{T}_a\boldsymbol{T}_b. \tag{9}$$

The common denominator of $\boldsymbol{T}$ is the numerator of a transfer function. Specifying a single real factor of the numerator, $\boldsymbol{T}_a$ is defined as a lossless transfer matrix. Then $\boldsymbol{T}_b$ having the remaining factors of the numerator described by

$$\boldsymbol{T}_b=\boldsymbol{T}_a^{-1}\boldsymbol{T} \tag{10}$$

will necessarily become a lossless transfer matrix as well as $\boldsymbol{T}$ and $\boldsymbol{T}_a$. Hence, if the degree of $\boldsymbol{T}_b$ becomes lower than that of $\boldsymbol{T}$ by means of $\boldsymbol{T}_a$ whose degree is one or two, such a decomposition can be repeated until a constant matrix $\boldsymbol{T}_{bn}$ appears. The termination of the procedure results in

$$\boldsymbol{T}=\boldsymbol{T}_{a1}\boldsymbol{T}_{a2}\cdots\boldsymbol{T}_{an}\boldsymbol{T}_{bn}. \tag{11}$$

If one can find the realization network for each transfer matrix factor, a cascade realization is obtained as shown in Fig. 3.

By the way, the computability of a digital network requires a special criterion that does not arise in analog networks. A digital network must have no delay-free loops. Paying our attention to the feedback loops in Fig. 3, it is found that if every path depicted by an asterisk has a delay, the cascade realization is computable. It is thus adequate to incorporate a delay in (1, 2) entry of each $\boldsymbol{T}_a$.

4. Factorization of Transfer Scattering Matrices

To factor the transfer matrix $\boldsymbol{T}$ of Eq. (6) and to bring about the degree reduction in the remainder matrix $\boldsymbol{T}_b$, this section describes how to extract $\boldsymbol{T}_a$ from $\boldsymbol{T}$. The extraction is based on the localization of transmission zeros.

4.1 Extraction of Real Transmission Zeros

For a real transmission zero located at $z=\alpha$, one can write $\boldsymbol{T}_a$ and $\boldsymbol{T}_b$ as

$$\boldsymbol{T}_a=\frac{1}{f(1-\alpha z^{-1})}\begin{bmatrix}1+g_1z^{-1} & \pm k_0z^{-1}\\ k_0 & \pm(g_1+z^{-1})\end{bmatrix} \tag{12}$$

$$\boldsymbol{T}_b=\frac{1}{F_b(z)}\begin{bmatrix}G_b(z) & \pm K_{b*}(z)z^{-N+1}\\ K_b(z) & \pm G_{b*}(z)^{-N+1}\end{bmatrix} \tag{13}$$

where f, g_1, and k_0 are unknown real parameters. The upper or the lower signs in $\boldsymbol{T}$ and $\boldsymbol{T}_a$ can be specified independently, respectively. The sign in $\boldsymbol{T}_b$ is automatically determined by the combination. Although the form of Eq. (12) does not represent the case for the transmission zero at infinity, this causes no problems, because there is only an exceptional class of transfer functions with a trivial factor of negative powers of z.

The losslessness induced by Eq. (7) claims that $\boldsymbol{T}_a$ satisfies

$$1+g_1^2=k_0^2+f^2(1+a^2) \tag{14}$$

$$g_1=-f^2a. \tag{15}$$

The requirement for the degree of $\boldsymbol{T}_b$ to reduce to $N-1$ imposes an additional condition on the relationship between $\boldsymbol{T}$ and $\boldsymbol{T}_a$. Since $F(z)$ has a root at $z=a$, the function $F_b(z)$ defined by

$$F_b(z)=F(z)/f(1-az^{-1}) \tag{16}$$

is a polynomial. At the same time $G_b(z)$ and $K_b(z)$ which are computed from Eq. (10) as

$$\begin{bmatrix} G_b(z) \\ K_b(z) \end{bmatrix}=\frac{1}{f^2(1-az^{-1})(z^{-1}-a)}\begin{bmatrix} G_c(z) \\ K_c(z) \end{bmatrix} \tag{17}$$

where

$$G_c(z)=(g_1+z^{-1})G(z)-k_0z^{-1}K(z) \tag{18 a}$$

$$K_c(z)=\mp k_0G(z)\pm(1+g_1z^{-1})K(z) \tag{18 b}$$

must also be polynomials. Hence the condition for the first-degree reduction becomes

$$G_c(a)=G_{c*}(a)=0 \tag{19 a}$$

$$K_c(a)=K_{c*}(a)=0 \tag{19 b}$$

for $a^2\neq1$. The condition in the case of $a^2=1$ is

$$G_c(a)=G_c'(a)=0 \tag{20 a}$$

$$K_c(a)=K_c'(a)=0 \tag{20 b}$$

where the prime stands for the derivative with respect to z^{-1}.

As a matter of fact, there are three unknown parameters to be determined, while there are six constraining equations. Let us define a function $P(z)$ as

$$P(z)=K(z)/G(z) \tag{21}$$

then, by inspection of Eq. (7), this function is complementary to the transfer function $H(z)$, and a useful relationship

$$1-P_*(a)P(a)=0 \tag{22}$$

is proved at the transmission zero. By exploiting this, one can select a set of independent constraints to determine the unknown parameters.

As an example, for $a^2\neq1$ it follows that Eqs. (19) yield

$$g_1+a^{-1}-k_0a^{-1}P(a)=0$$

$$g_1+a-k_0aP_*(a)=0$$

$$k_0-(1+g_1a^{-1})P(a)=0$$

$$k_0-(1+g_1a)P_*(a)=0$$

Taking Eq. (22) into account proves that the first and second equations are dependent and that the last two ones are also dependent. Furthermore Eq. (14) can be obtained by a simple calculation with the last two equations and Eqs. (22), (15). Thus one gets an independent system of three equations that consists of Eq. (15) and the above first and third equations. Since the system is linear with respect to f^2, k_0, and g_1, it can be easily solved.

The solutions to both cases for $a^2\neq1$ and $a^2=1$ are represented as

$$f^2=\begin{cases} \dfrac{P^2(a)-1}{P^2(a)-a^2} & \text{for} \quad a^2\neq1 \\ \dfrac{P'(a)/P(a)}{P'(a)/P(a)+a} & \text{for} \quad a^2=1 \end{cases} \tag{23}$$

$$k_0=(1-f^2)P(a). \tag{24}$$

The solution for g_1 is Eq. (15) itself. Note that this is a direct consequence of the losslessness produced by Eq. (7) and the computability requirement which implies the lack of the leading coefficient of $K(z)$ in $\boldsymbol{T}_a$.

4.2 Extraction of Complex Transmission Zeros

In the parallel way to the case of real transmission zeros, a second-degree section $\boldsymbol{T}_a$ for localizing a complex conjugate pair of transmission zeros located at $z=z_0$ and $z_0{}^*$ is represented by

$$\boldsymbol{T}_a=\frac{1}{f(1-z_0z^{-1})(1-z_0{}^*z^{-1})} \cdot\begin{bmatrix} 1+g_1z^{-1}+f^2|z_0|^2z^{-2} & \pm(k_1z^{-1}+k_0z^{-2}) \\ k_0+k_1z^{-1} & \pm(f^2|z_0|^2+g_1z^{-1}+z^{-2}) \end{bmatrix} \tag{25}$$

where the unknown parameters are f, g_1, k_0, and k_1. Unlike Eq. (12), the leading coefficient in the numerator of (1, 1) entry has been already specified by taking account of the fact in the last paragraph in Subsec. 4.1. Hence the remaining conditions for losslessness are of the forms

$$1+g_1^2+f^4|z_0|^4=k_0^2+k_1^2+f^2[1+(z_0+z_0{}^*)^2+|z_0|^4] \tag{26 a}$$

$$g_1(1+f^2|z_0|^2)=k_0k_1-f^2(z_0+z_0{}^*)(1+|z_0|^2). \tag{26 b}$$

As for the second-degree reduction, let us define

$$F_a(z)=f(1-z_0z^{-1})(1-z_0{}^*z^{-1}). \tag{27}$$

If the extraction of $\boldsymbol{T}_a$ from $\boldsymbol{T}$ causes the desired degree reduction, the associated functions F_b, G_b, and K_b in $\boldsymbol{T}_b$ computed by Eq. (10) must be polynomials of z^{-1}. In effect,

$$F_b(z)=F(z)/F_a(z) \tag{28}$$

is a polynomial, since $F(z)$ has zeros at z_0 and z_0^*. The other functions are of the forms

$$G_b(z)=G_c/F_aF_{a*}z^{-2} \tag{29 a}$$

$$K_b(z)=K_c/F_aF_{a*}z^{-2} \tag{29 b}$$

where

$$G_c(z)=(f^2|z_0|^2+g_1z^{-1}+z^{-2})G(z) -(k_1z^{-1}+k_0z^{-2})K(z) \tag{30 a}$$

$$K_c(z)=\pm[-(k_0+k_1z^{-1})G(z) +(1+g_1z^{-1}+f^2|z_0|^2z^{-2})K(z)]. \tag{30 b}$$

If a factor $(1-z_0z^{-1})$ divides G_c and K_c, its complex conjugate does the same, since these polynomials are real. Hence the condition for the second-degree reduction can be derived as

$$G_c(z_0)=G_{c*}(z_0)=0 \tag{31 a}$$

$$K_c(z_0)=K_{c*}(z_0)=0 \tag{31 b}$$

in the case of $|z_0|\neq 1$. The condition in the other case becomes

$$G_c(z_0)=G_{c*}'(z_0)=0 \tag{32 a}$$

$$K_c(z_0)=K_{c*}'(z_0)=0. \tag{32 b}$$

One can thus solve the simultaneous system of Eqs. (26) and either Eq. (31) or Eq. (32) like the previous subsection after tedious calculation. However, the solution, which is outlined in Appendix, brings about a realization problem: The solution suggests the need of much more hardware complexity than canonic realizations, thereby resulting the loss of the structurally-insensitive property to multiplier coefficient variations.

Nevertheless, provided that the complementary function $P(z)$ is real at $z=z_0$, a simple solution can be obtained. Eqs. (A・4) and (A・7) in Appendix then reduce to

$$f^2=\begin{cases}\dfrac{P^2(z_0)-1}{P^2(z_0)-|z_0|^4} & \text{for } |z_0|\neq 1\\[2ex] \dfrac{P'(z_0)/P(z_0)}{P'(z_0)/P(z_0)+2z_0/|z_0|} & \text{for } |z_0|=1\end{cases} \tag{33}$$

$$g_1=-(1+f^2|z_0|^2)(z_0+z_0^*)/(1+|z_0|^2) \tag{34}$$

$$k_0=(1-f^2)P(z_0) \tag{35}$$

$$k_1=-k_0(z_0+z_0^*)/(1+|z_0|^2). \tag{36}$$

Therefore, the realization problem caused by complex $P(z_0)$ can be removed by introducing a preamble so that the operation makes a new complementary function be real. When $P(z_0)$ in issue is complex, the extraction procedure is divided into the following two steps: At first extract a preamble section to eliminate the imaginary part of the resulting complementary function at the transmission zero of interest. Then extract the second-degree section described in this subsection to localize the transmission zero.

4.3 Preamble for Imaginary Part Elimination

Regarding a pair of transmission zeros z_0 and z_0^*, consider the following operation such that it decomposes $\boldsymbol{T}$ with complex $P(z_0)$ into a product of a preamble section $\boldsymbol{T}_a$ and the residual $\boldsymbol{T}_b$ with real $P_b(z_0)$. The operation will be referred to as the preamble for imaginary part elimination in a complementary function, and is illustrated in Fig. 4.

In formal description, the factorization by Eq. (10) with $\boldsymbol{T}_a$ makes the degree of $\boldsymbol{T}_b$ higher than that of $\boldsymbol{T}$ by that of $\boldsymbol{T}_a$. The degree of a preamble section is preferred as low as possible. By inspection of Eq. (12) and noting that Eqs. (14), (15) give $k_0^2=(1-f^2)(1-f^2a^2)$, let the section be of the form

$$\boldsymbol{T}_a=\frac{1}{f(1-az^{-1})}\begin{bmatrix}1-f^2az^{-1} & \pm k(1-f^2)z^{-1}\\ k(1-f^2) & \pm(-f^2a+z^{-1})\end{bmatrix} \tag{37}$$

where

$$a^2=1 \tag{38}$$

$$k^2=1. \tag{39}$$

The selection of the signs in a and k is independently arbitrary.

After the same calculation as in Eqs. (17), (18), one can formally obtain

$$\begin{bmatrix}G_b(z)\\ K_b(z)\end{bmatrix}=\frac{F_b(z)}{F(z)f(-a+z^{-1})}\begin{bmatrix}G_c(z)\\ K_c(z)\end{bmatrix} \tag{40}$$

where

$$G_c(z)=(-f^2a+z^{-1})G(z)-(1-f^2)kz^{-1}K(z) \tag{41 a}$$

$$K_c(z)=\pm[-(1-f^2)kG(z)+(1-f^2az^{-1})K(z)] \tag{41 b}$$

hence

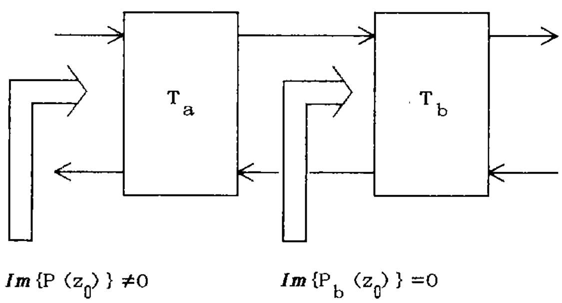

Fig. 4 Preamble for imaginary part elimination.

$$P_b(z_0)=\pm\frac{-(1-f^2)k+(1-f^2 a z_0^{-1})P(z_0)}{-f^2 a+z_0^{-1}-(1-f^2)kz_0^{-1}P(z_0)} \tag{42}$$

is obtained. In order for the imaginary part of $P_b(z_0)$ to vanish,

$$\mathrm{Im}\{[-(1-f^2)k+(1-f^2 a z_0^{-1})P(z_0)] \cdot[-f^2 a+z_0^{-1}-(1-f^2)kz_0^{-1}P(z_0)]^*\}=0 \tag{43}$$

is necessary. Hence f^2 is determined as

$$f^2=\left[1-\frac{(P(z_0)-P(z_0)^*)[(1+|z_0|^2)a-(z_0+z_0^*)]}{(z_0-z_0^*)[(1+|P(z_0)|^2)k-(P(z_0)+P(z_0)^*)]}\right]^{-1} \tag{44}$$

The preamble procedure is thus completed, thereby resulting $\boldsymbol{T}_b$ which consists of a triplet of the following polynomials

$$F_b(z)=f(-a+z^{-1})F(z) \tag{45 a}$$

$$G_b(z)=G_c(z) \tag{45 b}$$

$$K_b(z)=K_c(z) \tag{45 c}$$

The degree of $\boldsymbol{T}_b$ is formally higher by one than that of $\boldsymbol{T}$. However the preamble is followed by an extraction of a pair of complex transmission zeros. Thus the successive decomposition terminates necessarily.

When $\boldsymbol{T}$ has a transmission zero at $z=1$ or -1, the preamble is performed without changing the degree of $\boldsymbol{T}_b$. Let α_0 be such a zero, then $P^2(\alpha_0)=1$ holds from Eq. (22). Hence it is possible to set

$$a=\alpha_0, \quad k=P(\alpha_0) \tag{46}$$

and substituting this into Eq. (41) one can verify

$$G_c(\alpha_0)=G_{c*}(\alpha_0)=0 \tag{47 a}$$

$$K_c(\alpha_0)=K_{c*}(\alpha_0)=0 \tag{47 b}$$

Therefore it is found that $(-\alpha_0+z^{-1})$ divides all of the polynomials G_c, $G_{c*}z^{-(N+1)}$, K_c, and $K_{c*}z^{-(N+1)}$. This fact allows us to define

$$F_b(z)=F(z) \tag{48}$$

so that

$$G_b(z)=G_c(z)/f(-\alpha_0+z^{-1}) \tag{49 a}$$

$$K_b(z)=K_c(z)/f(-\alpha_0+z^{-1}) \tag{49 b}$$

become polynomials of at most N th-degree. This fact means even though the preamble is needed several times in a synthesis procedure, it is sufficient to increase the degree by one once for all, unlike in Ref. (8).

4.4 Constant Lossless Section

At the end of the successive extraction described in the preceding subsections, a given transfer matrix is factored as in Eq. (11). The termination of the extraction results in a constant lossless transfer matrix

$$\boldsymbol{T}_{bn}=\frac{1}{f_b}\begin{bmatrix}1 & \pm k_b\\ k_b & \pm 1\end{bmatrix} \tag{50 a}$$

where

$$f_b^2+k_b^2=1. \tag{50 b}$$

5. Transfer Scattering Matrix Realization

This section describes a universal realization of the transfer matrices obtained in the last section and the special alternatives.

5.1 Universal Section

Summarizing the results in the last section, one can obtain a unified representation

$$\boldsymbol{T}_a=\frac{1}{f(1-aW(z))}\cdot\begin{bmatrix}1-f^2aW(z) & \pm(1-f^2a^2)W(z)/k\\ (1-f^2)k & \pm(W(z)-f^2a)\end{bmatrix} \tag{51}$$

for all of the transfer matrix factors which are the first or second degree section to localize transmission zeros, and the preamble section as well as the constant section. The individual parameters that characterize each section, except f^2, are summarized as follows.

The first-degree section to extract a real transmission zero at $z=a$ is characterized by

$$k=P(a) \tag{52 a}$$

$$W(z)=z^{-1}. \tag{52 b}$$

The second-degree section for a pair of complex transmission zero at $z=z_0$ and the conjugate is characterized by

$$k=P(z_0) \tag{53 a}$$

$$a=-|z_0|^2 \tag{53 b}$$

$$W(z)=z^{-1}(\beta+z^{-1})/(1+\beta z^{-1}) \tag{53 c}$$

$$\beta=-(z_0+z_0^*)/(1+|z_0|^2). \tag{53 d}$$

It may be worth noting that Eq. (53 c) is a special case of the lowpass-bandpass frequency transformation[18].

For the preamble section,

$$a=\alpha_0 \tag{54 a}$$

$$k=P(\alpha_0) \tag{54 b}$$

$$W(z)=z^{-1} \tag{54 c}$$

where α_0 is either naturally specified by a transmission zero at 1 or -1, or arbitrarily selected as 1 or -1. In the former case no degree increment happens, unlike the latter. For both cases the magnitude of both a and k is unity.

The constant lossless section is automatically appears as the result of the preceding factorizations. Thus it contains no parameters to be specified. The parameterization

$$W(z)=1, \quad a=0, \quad k=1/k_b, \quad f=f_b \tag{55}$$

is merely the convenience to fit the unified representation.

This preliminary review paves the way to the realization of a transfer matrix in Eq. (51). In the notation used in this paper, as is shown in Fig. 2, a transfer matrix is lack of the direct correspondence between its entry and the actual signal flow. By contrast, a scattering matrix defined by

$$\begin{bmatrix} Y_1(z) \\ Y_2(z) \end{bmatrix} = \boldsymbol{S}(z) \begin{bmatrix} X_1(z) \\ X_2(z) \end{bmatrix} \tag{56}$$

where

$$\boldsymbol{S}(z) = \begin{bmatrix} s_{11}(z) & s_{12}(z) \\ s_{21}(z) & s_{22}(z) \end{bmatrix} \tag{57}$$

is capable of representing the actual signal flow. Hence the scattering matrix representation is convenient to find the realization of Eq. (51).

Through the algebraic formula

$$\boldsymbol{S} = \begin{bmatrix} t_{21}/t_{11} & |\boldsymbol{T}|/t_{11} \\ 1/t_{11} & -t_{12}/t_{11} \end{bmatrix} \tag{58}$$

the transfer matrix $\boldsymbol{T}_a$ is converted into the corresponding scattering matrix $\boldsymbol{S}_a$ as

$$\boldsymbol{S}_a = \frac{1}{1-f^2 aW(z)} \cdot \begin{bmatrix} (1-f^2)k & \pm f(-a+W(z)) \\ f(1-aW(z)) & \mp(1-f^2a^2)W(z)/k \end{bmatrix}. \tag{59}$$

By careful inspection in conjunction with step-by-step decompositions, $\boldsymbol{S}_a$ can be factored as

$$\boldsymbol{S}_a = \begin{bmatrix} 1/f & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{1-f^2 aW} \begin{bmatrix} 1 & -f^2 \\ -aW & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & f^2 a \\ 1 & W \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \mp 1/k \end{bmatrix} \begin{bmatrix} f & 0 \\ 0 & 1 \end{bmatrix}. \tag{60}$$

Substituting this into Eq. (56) and rearranging the result give

$$\begin{bmatrix} 1 & f^2 \\ aW & 1 \end{bmatrix} \begin{bmatrix} fk^{-1}Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & f^2 a \\ 1 & W \end{bmatrix} \begin{bmatrix} fX_1 \\ \mp k^{-1}X_2 \end{bmatrix}. \tag{61}$$

This relationship leads to the final expression

$$(fk^{-1}Y_1) = (fX_1) - f^2[a(\pm k^{-1}X_2) + Y_2] \tag{62 a}$$

$$Y_2 = (fX_1) - W[(\pm k^{-1}X_2) + a(fk^{-1}Y_1)]. \tag{62 b}$$

Figure 5 is drawn with this expression. It is a unified realization of the transfer matrix in Eq. (51).

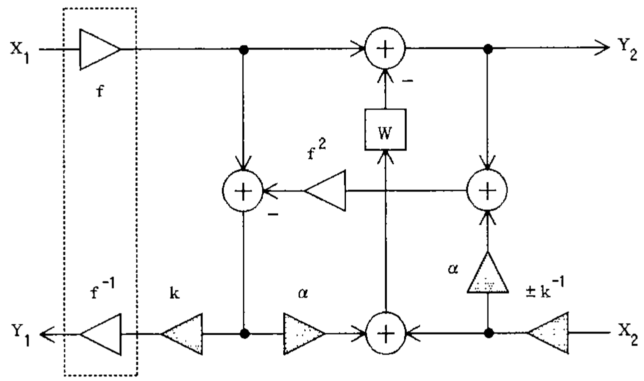

Fig. 5 Unified realization of basic transfer matrices.

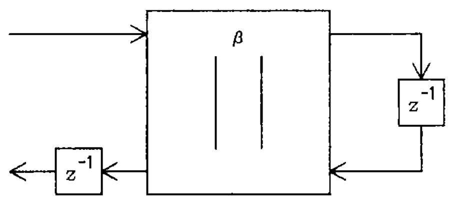

Fig. 6 Realization of the second-degree delay function block by a two-port adaptor.

Two multipliers inside the dashed box in the figure can be removed by the cutset scaling technique(19). This is because the scaling is equivalent to convert Eq. (11) into

$$\boldsymbol{T} = (f_1 \boldsymbol{T}_{a1}) \cdots (f_n \boldsymbol{T}_{an}) \boldsymbol{T}_{bn} \prod_{i=1}^{n} (1/f_i) \tag{63}$$

where each f_i is the parameter f associated with $\boldsymbol{T}_{ai}$.

The topology of the signal flow graph outside the dashed box is presented in Ref. (14). However, the realization principles in the literature and in the paper are different. Furthermore, the topology is used for a complex first-degree section in the literature, but Fig. 5 is applied to any first/second-degree section with real multiplier coefficients.

In the cascade realization as shown in Fig. 3, two multipliers $\pm 1/k$ and k in both sides on the bottom edge may be combined between successive two sections, thereby reducing to a single multiplier.

In practical applications of digital filters most of the transmission zeros lie on the unit circle. In this case both a and k become either 1 or -1, and the four multipliers shaded in Fig. 5 thus reduce to simple connecting wires which may be accompanied with an inverter. The general form of reciprocal lossless bounded-real sections(7),(8) falls into this special class.

The delay function block $W(z)$ in the second-degree section may be realized with one multiplier and two adders, as was done in Refs. (10), (11). For less sensitivity, however the best realization of this block is to rely on the two-port adaptor(20) shown in Fig. 6. The two-port adaptor consists of one multiplier and three adders.

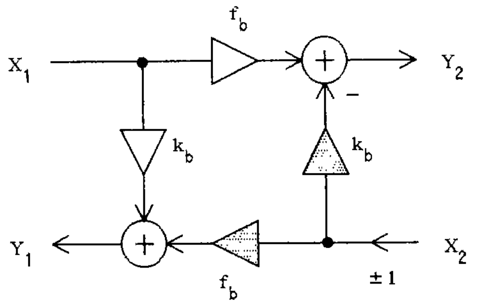

Fig. 7 Simplified realization of the constant lossless section.

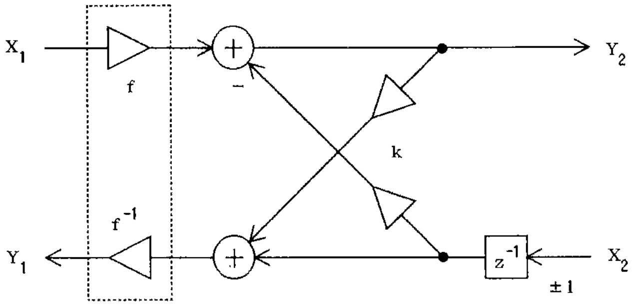

Fig. 8 Lattice realization of the first-degree section for the transmission zero at $z=0$.

The detailed structures with six variations are found in Ref. (20).

5.2 Three Special Cases

At first, the constant lossless section described by Eq. (50) is redrawn as Fig. 7, based on its scattering matrix representation. In digital filter applications, since $X_2=0$, the two multipliers are of no need, hence two adders reduce to interconnection wires.

The effect of the cutset scaling as represented by Eq. (63) appears in f_b in the upper edge. As a result the effective f_b denoted by f_{bs} takes the form

$$f_{bs}=f_b/\prod_{i=1}^{n} f_i=H(\infty) \tag{64}$$

where the last relation is verified by the construction of the original transfer matrix and by the factorization procedure itself. This is useful to ascertain the accuracy in numerical computation in practice.

Secondly, an alternative realization of the first-degree section for the transmission zero at $z=0$ is of quite importance. Setting $\alpha=0$ in Eqs. (22), (23) and (52 a) gives

$$f^2=1-P_*^2(0) \tag{65 a}$$

$$k=1/P_*(0). \tag{65 b}$$

Exploiting these in Eq. (62), the lattice realization is obtained as shown in Fig. 8. The network outside the dashed box is identical to the Gray-Markel lattice structure[21] for all-pole digital filters.

Thirdly, a pair of mirror image transmission zeros on the real axis can be extracted at once in a similar way as for the complex transmission zeros. Denoting the real pair as α_0 and $1/\alpha_0$, if $P^2(\alpha_0)=1$, the parameters for the desired extraction are summarized as

$$f^2=\frac{P'(\alpha_0)/P(\alpha_0)}{P'(\alpha_0)/P(\alpha_0)+2\alpha_0} \tag{66 a}$$

$$\alpha=-1 \tag{66 b}$$

$$k=P(\alpha_0) \tag{66 c}$$

$$W(z)=z^{-1}(\beta+z^{-1})/(1+\beta z^{-1}) \tag{66 d}$$

$$\beta=-(\alpha_0+1/\alpha_0)/2. \tag{66 e}$$

When $P^2(\alpha_0)\neq 1$, another preamble specified by

$$f^2=(\alpha_0+\alpha)(P(\alpha_0)-k)/2(\alpha P(\alpha_0)-\alpha_0 k) \tag{67 a}$$

$$\alpha^2=1, \quad k^2=1, \quad W(z^{-1})=z^{-1} \tag{67 b}$$

is required to transform the magnitude into unity, before extracting the second-degree section parameterized by Eq. (66).

6. Pipeline Processing and Other Topics

6.1 Pipelinable Cascade Realization

Pipelinability is a desirable feature in modern digital signal processing[15],[22]. A high throughput rate is achieved by pipeline processing in which consecutive operations in a proper precedence relation are concurrently processed. Hence pipeline processing requires hardware elements for concurrent processing, buffers to block the direct propagation of signals, and the pipelinability in computation algorithms.

The cascade realizations described in the preceding sections are not pipelinable. As can be seen from Fig. 3, the global structure posesses a critical path which traverses all the constituent sections.

The critical path can be shortened, if the filter has transmission zeros at $z=0$. Because such a zero makes the parameter α in Fig. 5 be zero, and this eliminates the direct backward-signal propagation. Although this is merely a rare case, the fact suggests a way to introduce the pipelinability.

According to the last paragraph in Sect. 2, a given transfer function can be embedded into the lossless transfer matrix in conjunction with an integer N. N is arbitrary, in so far as it exceeds or equals the lower bound imposed by causality. Even though the integer is increased, the transmission from the input X_1 to the output Y_2 remains unchanged. The increase in N produces a multiple pole at $z=0$ in the transfer matrix. The pole is a multiple transmission zero of the corresponding two-port, but is invisible in the transfer function of interest.

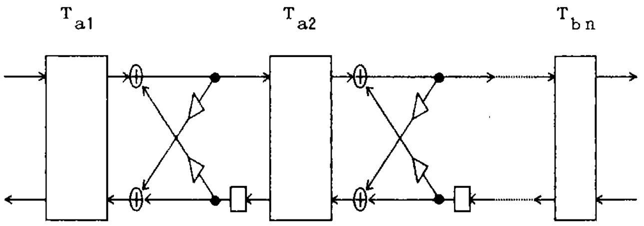

Fig. 9 Reflection-delayed pipelinable realization.

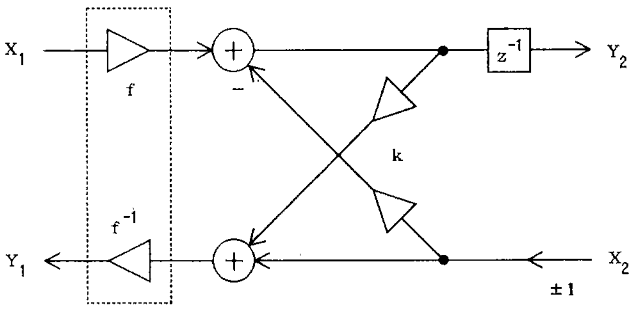

Fig. 10 Lattice realization of the first-degree section for the infinite transmission zero.

The above discussion leads to the way for pipelining as follows:

1) On the transfer function embedding into a transfer matrix, N is increased by the number of real coefficient factors in the numerator $F(z)$.

2) The synthesis procedure is conducted by the alternate localization rule in such a way that every section to localize a set of original transmission zeros off the origin is followed by a pipeline section which localizes an invisible transmission zero at $z=0$.

The pipeline section has the structure shown in Fig. 8. The resulting pipelinable cascade realization is schematically depicted in Fig. 9. One can see that every reflection signal at the junction between a section and the following section is buffered by a delay element. The absence of the direct propagation of those reflection signals means a shorter critical path. This scheme will be referred to as reflection-delayed pipelining.

Similarly, the other pipelining called as transmission-delayed pipelining can be introduced on the basis of localizing a multiple transmission zeros at infinity. This pipeline brings about constant delay in the input-output response, and the added transmission zeros explicitly appears in the transfer function.

The pipeline section based on the infinite transmission zero is described by the transfer matrix

$$T_a = \frac{1}{fz^{-1}} \begin{bmatrix} 1 & kz^{-1} \\ k & z^{-1} \end{bmatrix} \tag{68}$$

where two parameters f and k are specified by Eq. (65), which pertains to the case for the transmission zero at $z=0$. The realization structure is drawn in Fig. 10.

6.2 Coefficient Sensitivity

The digital filters designed by the described procedure display good coefficient sensitivity properties, as was intended. At present the discussion on the coefficient sensitivity in passband is well summarized in Ref. (4). It is thus omitted in this paper except mentioning the stopband sensitivity.

As for the accuracy in positioning transmission zeros, the transfer matrix based realization is approximately as same as the cascade realization with biquadratic cells. The former is superior than scattering matrix based realizations[(12),(13)]. This is due to the localization of transmission zeros. Such an interpretation is a lesson of experience in the filter theory and is also stated in Refs. (15), (23).

6.3 FIR Digital Filters

There are no significant differences between FIR and IIR digital filters with respect to the realizability requirements. A slight difference lies in the fact that an FIR filter has a multiple pole at $z=0$. Therefore there are no obstacles to prevent from synthesizing FIR filters by the proposed method.

The resulting strucures, however, posesses no practical advantages: they dissipate much more multiplications and additions than those required for the realizations with FIR lattice sections[(4),(13),(24)].

It is worth noting that the cascade realization with the nonrecursive lattice sections is obtained by the method based on scattering matrix factorization as a special case. This corresponds with the special case for the transfer matrix counterpart described in subsect. 5.2, which is particulary useful for all-pole digital filters.

7. Concluding Remarks

A direct method to synthesize a low-sensitivity digital filter from an arbitrary digital transfer function has been described only in the z-domain. The procedure is based on the localization of transmission zeros. Pipeline processing is derived by utilizing a fact that the localization concept permits to incorporate additional but insignificant transmission zeros into the two-port transmission scheme.

After a mathematical expression is found for realizing a transfer function, the realization problem requires additional work, as was worked out in Sect. 5.

Only one design example is included in Appendix 2 because of the page limit. Others have been presented in Refs. (10), (11), (25).

While there is arbitrariness in the order of localizing transmission zeros, on the other hand the order may affect the complexity in digital filter implementations. This is left as an open problem.

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Appendix 1

The calculation of the parameters for the second-degree extraction is outlined. Since a complex transmission zero $z=z_0$ is a root of $F(z)=0$,

$$P_*(z_0)P(z_0)=1 \tag{A·1}$$

is obtained from Eqs. (7), (21).

In particular, in the case of $|z_0|=1$, differentiating Eq. (7) with respect to z^{-1} and evaluating it at $z=z_0$ as well as noting $z_0=1/z_0^*$ result in

$$P'(z_0)/z_0P(z_0)=(P'(z_0)/z_0P(z_0))^*. \tag{A·2}$$

Hence defining

$$\mu=P'(z_0)/z_0P(z_0), \tag{A·3}$$

μ is real.

For the complex pair which is off the unit circle, Eq. (31) is rewritten as

$$f^2|z_0|^2+g_1z_0^{-1}+z_0^{-2}=(k_1z_0^{-1}+k_0z_0^{-2})P(z_0)$$

$$f^2|z_0|^2+g_1z_0+z_0^2=(k_1z_0+k_0z_0^2)P_*(z_0)$$

$$k_0+k_1z_0^{-1}=(1+g_1z_0^{-1}+f^2|z_0|^2z_0^{-2})P(z_0)$$

$$k_0+k_1z_0=(1+g_1z_0+f^2|z_0|^2z_0^2)P_*(z_0).$$

By using Eq. (A·1), one can select the first and second equations as independent expressions. Thus splitting them into real and imaginary parts and denoting $P(z_0)=\sigma e^{j\phi}$ yield a linear system of four equations. The solution is expressed by

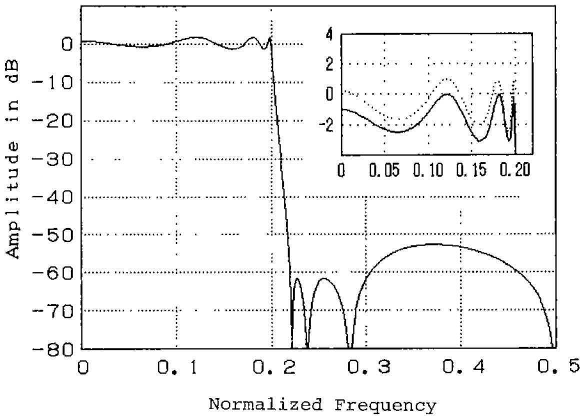


Fig. A•1 Amplitude response of a 7th-order lowpass filter with stepwise ripples.

$$f^2=[r^2(\sigma^2-1)^2\sin^2\lambda-\sigma^2(r^2-1)^2\sin^2\phi]/\Delta$$
$$g_1=r[r^2(\sigma^2-1)(r^2-\sigma^2)\sin\lambda\sin 2\lambda+\sigma^2(r^2-1)(r^4-1)\sin\phi\sin(\phi-\lambda)]/\Delta$$
$$k_1=r\sigma(r^2-1)\sin\lambda[2r^2(1-\sigma^2)\cos\lambda\sin(\phi-\lambda)-(r^2+1)(r^2-\sigma^2)\sin\phi]/\Delta$$
$$k_0=r^2\sigma(r^2-1)\sin\lambda[(r^2-1)(\sigma^2+1)\cos\lambda\sin\phi-(r^2+1)(\sigma^2-1)\sin\lambda\cos\phi]/\Delta \tag{A•4}$$

where

$$\Delta=r^2[(r^2-\sigma^2)^2\sin^2\lambda-\sigma^2(r^2-1)^2\sin^2(\phi-\lambda)] \tag{A•5}$$
$$z_0=re^{j\lambda}. \tag{A•6}$$

In a similar way, Eq. (32) can be solved as

$$f^2=(\mu^2\sin^2\lambda-\sin^2\phi)/\Delta$$
$$g_1=2[\sin\phi\sin(\phi-\lambda)-\mu(\mu+1)\sin^2\lambda\cos\lambda]/\Delta$$
$$k_1=-2\sin\lambda[\mu\cos(\phi-\lambda)\sin\lambda+\sin\phi]/\Delta$$
$$k_0=2\sin\lambda(\mu\cos\phi\sin\lambda+\sin\phi\cos\lambda)/\Delta \tag{A•7}$$

where

$$\Delta=(\mu+1)^2\sin^2\lambda-\sin^2(\phi-\lambda) \tag{A•8}$$

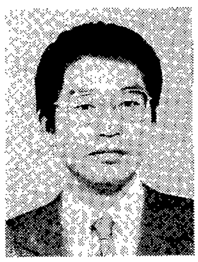
with an additional attention such that μ in Eq. (A•3) is real.

Appendix 2

A 7th-order lowpass filter with stepwise ripples[25] has been synthesized by the described procedure. The transfer function was designed by the algorithm given in Ref. (26), and its amplitude response is shown by the solid lines in Fig. A•1. The dotted lines show the amplitude response degraded by the coefficient quantization in floating binary with 4-bit mantissa. The corresponding parameters are listed in Table A•1. Note that, under the same condition, the conventional biquadratic cascade realization resulted in oscillation, thereby demonstrating the effectiveness of the procedure described in this paper.

Table A•1 Parameters for the design example.

Section	Alpha	Beta	f_0^2	k_0	$1/k_0$
1	-1		.171875	1	1
2	-1	.203125	.28125	-1	-1
3	-1		.234375	1	1
4	-1	-.171875	.46875	-1	-1
5	-1		.375	1	1
6	-1	-.078125	.40625	-1	-1
7	-1		.34375	1	1
8	k_b=-.625		f_b=.0146484375		



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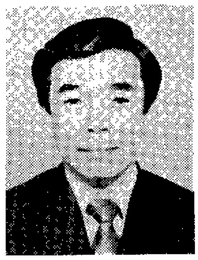

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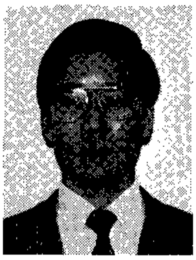

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