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The Influence of Mechanical Properties of Spherical Indenters on Hardness*

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Hardness is influenced by mechanical properties of a spherical indenter in the elastic-plastic transient indenting process.To clarify that point,we indented steel and tungsten carbide spherical indenters into several standard blocks for hardness.In order to experimentally establish the relations of the elastic-plastic transient indenting process. we related the hardness Pm (mean contact pressure) with the true profile coefficient of a permanent indentation (d/Dp), not with an apparent profile coefficient (d/D) used traditionally, and formulated as follows; $P_m=P_{up}(d/D_p)^{x_p}$ We can calculate the hardness affected by the different mechanical properties of the spherical indenter using this formula for arbitrary conditions.Further, we compared the Brinell hardness obtained by Yoshizawa with our calculated values,and found good coincidence between them.

Key Words : Material Testing,Mechanical Property,Hardness,Mean Contact Pressure Elastic-Plastic Transient Area,Profile Coefficient of Indentation Spherical Indenter,Brinell Hardness,Deformation of Indenter

1. Introduction

In Brinell hardness test, when the hardness value is between 320 and 450,it is an established practice[1] that the notation HBS and HBW must be used for the steel ball indenter and the tungsten carbide ball indenter respectively. Further, when the hardness value is 450 and 650 (maximum value in JIS),only the tungsten carbide ball indenter must be used.

This practice in Brinell hardness test is based on consideration of influence of (1) the mechanical properties [Young's modulus,etc.] of the indenters and (2) the permanent deformation of the indenters ,on hardness value.

The influence (1) appears in the elastic-plastic transient indenting process where the load of the indenter is very low or the hardness of a specimen is very high ,but it can't be completely explained by Hertz's Law and Meyer's Law of Contact.

The influence (2) appears when the strength of an indenter is not high enough in comparison with that of a specimen.

This paper discusses the above points ,mainly the influence (1).

First,the relation of the physical quantities of a specimen and a spherical indenter in the elastic-plastic transient indenting process are obtained experimentally,on the basis of the relation between the mean contact pressure and not the apparent profile coefficient used traditionally,but the true profile coefficient of a permanent indentation.

Thus , the influence of mechanical properties of spherical indenters on hardness can be analyzed.

Furthermore , the influence (2) is quantitatively considered on the basis of the fact that Brinell hardness values for hard specimens are given with both the mechanical properties and the degree of permanent deformation of the indenter.

2. Elastic Contact between Specimen and a Spherical Indenter

First , we considered the elastic contact between a specimen and a spherical indenter before the onset of the plastic deformation of a specimen. According to Meyer,hardness is defined as the mean contact pressure given in Eq.(1)[4], and the ratio of a load to square of the diameter of an indenting sphere is defined as the load ratio in Eq.(2).

Hertz's elastic contact law[2] between a specimen and an indenting sphere is summarized as Eqs. (3) to (6) and the influence of mechanical properties of the spherical indenters on hardness is expressed in Eq.(5) or Eq. (6) containing the elastic parameter f(E).

The calculated Pme-(de/D) curves from Eq.(6) for a rigid ball indenter (Rigid ball),a tungsten carbide ball (W ball) and a steel ball (S ball) are shown in Figs.2,3 and 5 respectively.

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$$P_{me}=0.102\times 4L/\{\pi d_e^2\} \quad (1)$$

$$C_{LR}=0.102L/D^2 \quad (2)$$

$$L=d_e^3/\{3Df(E)\} \quad (3)$$

$$f(E)=\frac{1-\mu_i^2}{E_i}+\frac{1-\mu_s^2}{E_s} \quad (4)$$

$$P_{me}=0.102\frac{4}{\pi}\left[\frac{9.8C_{LR}}{\{3f(E)\}^2}\right]^{1/3} \quad (5)$$

$$P_{me}=0.102\frac{4}{3\pi f(E)}\left(\frac{d_e}{D}\right) \quad (6)$$

where

L:load

de:contact diameter in the elastic contact

R,D:radius,diameter of an indenting sphere

Ei,Es,μi,μs:Young's modulus and Poisson's ratio of an indenting sphere and a specimen respectively

f(E):elastic parameter for contact between a specime and an indenting sphere[9]

C_{LR}:load ratio

Pme:hardness in the elastic contact [mean contact pressure]

3. Meyer's Law of Similarity

Next,we consider a contact phenomenon in fully plastic deformation of a specimen.

When the load is increased, the onset of the plastic deformation of the specimen under the center of the contact point takes place,and the area deformed plastically enlarges gradually, which is called the elastic-plastic transient indenting process.

Finally, the whole contact area is covered with the plastically deformed metal ,which is called the fully plastic deformation of a specimen.

Meyer's Law of Similarity[4] explains the fully plastic deformation of the specimen and it is summarized in Eqs. (8) to (11).

Here,in accordance with the former section,hardness is defined as the mean contact pressure Pm in Eq.(7).

$$P_m=0.102\times 4L/\{\pi d^2\} \quad (7)$$

$$L=ad^m \quad (8)$$

$$P_u=0.102(4a/\pi)D^{m-2} \quad (9)$$

$$P_m=P_u^{2/m}(4C_{LR}/\pi)^{(m-2)/m} \quad (10)$$

$$P_m=P_u(d/D)^{m-2} \quad (11)$$

where

a:coefficient variable with the diameter of an indenting sphere

m:constant depending on the material only[Meyer index]

d:cordal diameter of a permanent indentation

Pu:ultimate hardness by a spherical indenter

Pm:hardness[mean contact pressure]

Eqs.(10) and (11) show that in the state of fully plastic deformation of a specimen, hardness is not influenced by mechanical properties of an indenting sphere,and that if the value of (d/D) is the same,namely if the value of C_{LR} is the same in the indenting conditions, the same hardness value will be obtained although the load L and the diameter of an indenting sphere D may be changed.

The ultimate hardness[3] Pu means the hardness Pm for the value of (d/D)=1.

4. Profile and Profile Coefficient of a Permanent Indentation in the Elastic-Plastic Transient Indenting Process

In this section , we consider the profile of an indentation which is the basis of the elastic-plastic transient indenting process , showing the elastic-plastic feature,and we analyze the profile of an indentation as follows.

4.1 Profile of a permanent indentation

Two cross sections of a permanent indentation produced by a spherical indenter are shown in Fig.1 , and the profiles are broadly divided into (a) type of piling-up and (b) type of sinking-in[5].

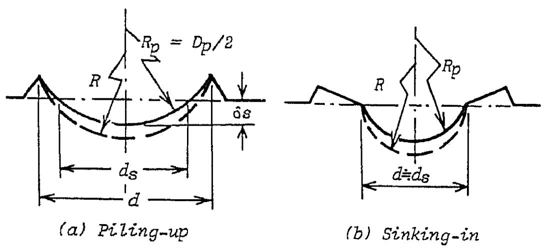

Figure 1. The cross section of a permanent indentation

In the experiment , specimens HRB33 and HRB51 of the standard blocks for hardness of brass metal behaves as type of sinking-in,and the relation of d≒ds is obtained in the same way as Kuroki's experiments[6].

ds and δs denote the cordal diameter and the depth of a permanent indentation measured from the original surface level of a specimen respectively.

Although a specimen behaves as the type of piling-up under the load,the edge of the indentation profile decreases in height and in steepness with decreases of the load,and gradually the profile changes into the type of sinking-in.

Therefore the difference between d and ds can't be clearly distinguished in the indentations produced in harder specimens under lower load.

4.2 Profile coefficient of a permanent indentation

In the elastic - plastic transient indenting process,when the load is removed ,the indentation left on the metal surface has a radius of curvature Rp(=Dp/2) which is several times as large as that of the indenter R.

This effect is known as "shallowing"[5] and has been ascribed to the elastic recovery in the direction of depth.

This shallowing effect is dependent on mechanical properties of the indenting sphere, consequently various profiles of a permanent indentation emerge corresponding to mechanical properties of the indenter.

Accordingly, in order to clearly distinguish the apparent profile of an indentation from the true one, they are traditionally treated without distinction, they are defined as follows in this paper.

(d/D): apparent profile coefficient of a permanent indentation

(d/Dp): true profile coefficient of a permanent indentation

4.3 Interrelationship between the apparent and the true profile coefficient of a permanent indentation

Tabor cofirmed[5] that if the indenter is replaced on the recovered (formed) indentation and the original load is applied, the surfaces deforme elastically, therefore the relations among a cordal diameter d, the diameter Dp of the spherical cup of a permanent indentation respectively (Fig.1), and the load L and an elastic parameter f(E) satisfy Hertz's elastic contact Equation (12) between a sphere and a spherical cup.

$$L=\frac{1-(D/D_p)}{3Df(E)}d^3 \quad \cdots\cdots(12)$$

Consequently the interrelationship of both coefficients is given as

$$\left(\frac{d}{D_p}\right)=\left(\frac{d}{D}\right)-\frac{3\times 9.8C_{LR}f(E)}{(d/D)^2} \quad \cdots\cdots(13)$$

5. Experimental Method

5.1 Indenter

Steel and tungsten carbide balls of diameters 2 mm 5 mm are cemented in the holder, and used as indenters.

The specifications of indenting spheres are shown in Table 1.

Table 1. Specification of indenting spheres

Indenter	Ball Material	D mm	Ei GPa	ν_i
S 2 S 5	Steel	2 5	201	0.29
W 2 W 5	Tungsten carbide	2 5	608	0.21

1 GPa= 102 Kgf/mm²

5.2 Specimen

The commercial standard blocks for hardness of Shore and Rockwell B scale are used as specimens for assuring the homogeneity and the standard.

The specifications of specimens are shown in Table 2.

Table 2. Specification of specimens

Standard Blocks for Hardness		Hardness HS (HRB)	HV	E_s GPa	f(E) 1/{10³GPa} S ball	W ball
Steel	SK2	101	1000	188	9.38	6.36
	SK5	92	847	191	9.30	6.28
		80	676			
		69	550	196	9.18	6.16
		61	458			
		53	382			
		41	276	201	9.07	6.05
		31	206			
	S20C	22	138	211	8.86	5.84
	SK5	(101)	260	201	9.07	6.05
Brass	C2600P	(81)	155	98	13.8	10.8
		(51)	97			
		(33)	79			

1 GPa= 102 Kgf/mm²

5.3 Load ratio

A load ranging 9.8 N to 490 N is applied with Vickers hardness tester and for the load over 490 N a small compression tester having a maximum load 4900 N is used.

The condition of an indenting test is as follows: three measurements are carried out with the same load on a specimen, the holding time of a load being 30 seconds.

In order to prevent the permanent deformation of an indenting sphere in this test, the study by Yoshizawa[8] regarding the permanent deformation of a steel and a tungsten carbide ball in Brinell hardness test is referred to, thus the maximum load ratios for hard specimens are determined as shown in Table 3.

Table 3. Maximum load ratio for harder specimens

Indenter	HS101	HS92	HS80	HS69	HS61
S 2	10	13.8	16.3	25	30
S 5	-	-	7	-	15
W 2	17.5	25	31.3	45	62.5
W 5	8.6	13	15	15	15

5.4 Measurement of the profile of a permanent indentation

The dimensions of a permanent indentation shown in Fig.1 are measured using a tool maker's microscope.

The diameter d is measured with magnification 50 or 100, the diameter ds and the depth δs are measured with magnification 400 respectively.

At the same time, the dimensions of several specimens measured using a profilometer are compared with those measured by a microscope, to confirm the validity of measurement using a microscope.

6. Experimental Results and Discussion

6.1 Permanent deformation of an indenting sphere

After the indenting test, the radius of curvature of an indenting sphere is measured using a profilometer and the occurrence of permanent deformation of an indenter is investigated.[8][10]

Among the maximum load ratios in Table 3, when a steel ball indenter (S5) is indented into the specimen HS80 with,

C_{LR} =7,a permanent deformation of about 5-6% at the tip of a sphere occurs.Therefore ,the data on this indenter are eliminated from this paper.

In other conditions , the measured radii of curvature of indenters have values close to that of a new ball used for calibration in the difference of ± 3%,so that these indenters are considered to have suffered no permanent deformations.

The relations between hardness Pm and profile coefficients of indentation (d/D), (d/Dp) obtained in experiments are as follows.

6.2 Pm-(d/D)

Hardness Pm plotted against the apparent profile coefficient of a permanent indentation (d/D) on logarithmic coordinates is shown in Figs.2 and 3 with a open circle.

Although hardness Pm is given in unit of MPa,the hardness values which have no unit are shown in the figures in accordance with Brinell and Vickers hardness test. It is seen that in soft specimens such as HS40.4 (HV276) and HRB80.6 (HV155) , the hardness value is independent of material and size of indenting spheres when the value of (d/D) is larger than about 0.15.

This is the reason why the same notations are used usually in Brinell hardness test [in JIS] even when different indenters are used.

The other experimental points for the same indenter are eliminated from figures to avoid cofusion.

Further,for the hard specimens such as HS101 (HV1000) and HS80 (HV676) , the indenting process is an elastic - plastic trnsient indenting one though the values of (d/D) are so large that the difference among the hardness values using different indenters is remarkable, i.e. the hardness values with a steel ball indenter are much lower than those with a tungsten carbide ball.

Namely , in the elastic - plastic transient indenting process based on the relation between Pm and (d/D), difference among the hardness values using different mechanical properties of spherical indenters becomes large as the hardness of a specimen increases and the load decreases,and thus the curve of Pm-(d/D) is gradually closer to the Pme-(de/D) curve calculated from Hertz's Eq.(6) as the value of (d/D) decreases.

6.3 Pm-(d/Dp)

It is difficult to formulate hardness Pm in a simple form with a profile coefficient (d/D) as Meyer's Eq.(11).

So before formulating the relation between Pm and (d/D),we focus our attention upon the relation between hardness Pm and the true profile coefficient of a permanent indentation (d/Dp),because the profile of

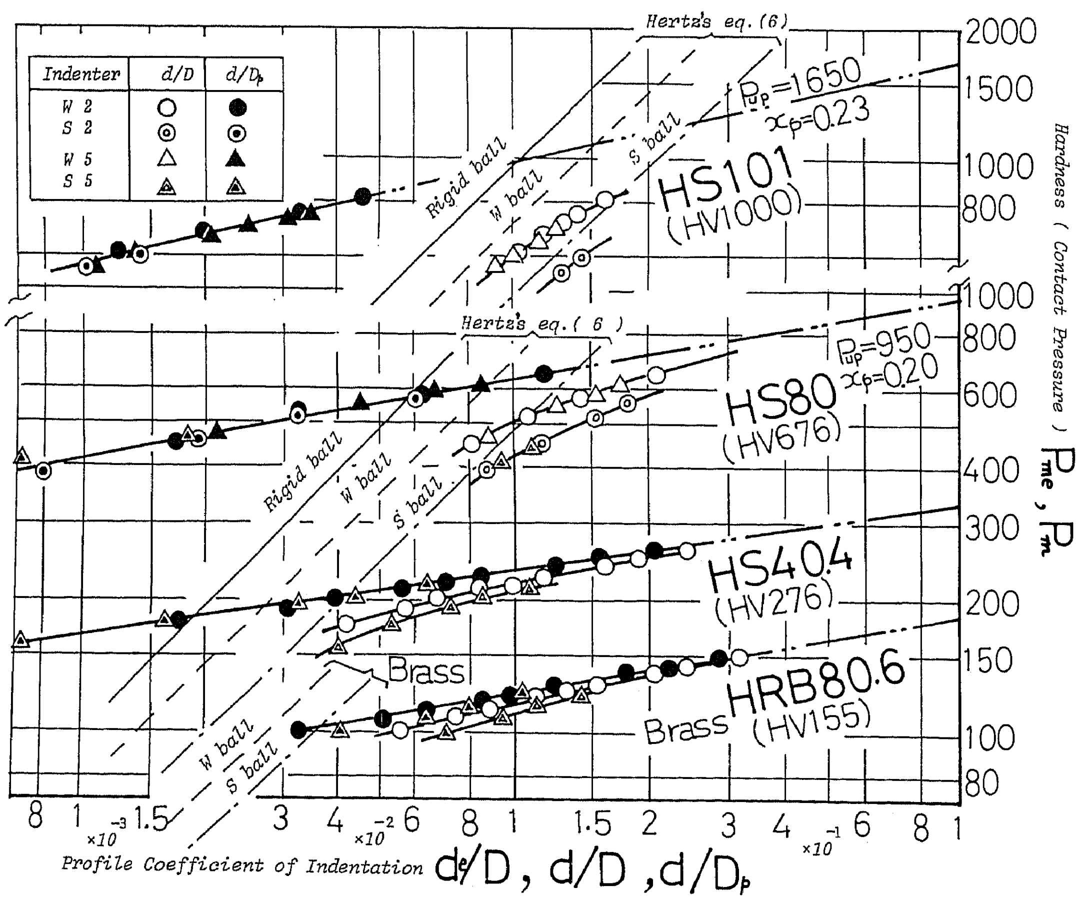

Figure 2. The relation between hardness and profile coefficient of an indentation

an indentation is markedly influenced by the mechanical property of an indenter , namely the value of Dp is not equal to that of D in the elastic - plastic transient indenting process.

Hardness Pm plotted against the true profile coefficient of a permanent indentation (d/Dp) is shown in Figs. 2 and 3 with a solid circle.

The relation of Pm-(d/Dp) is different from that of Pm-(d/D) mentioned above and the experimental data approximately fall on a straight line on logarithmic coordinates.

For the fully plastic deformation of a specimen or for a soft specimen, it is seen in Fig.3 that this line approaches that calculated by Meyer's Eq.(11).

The relation of Pm-(d/D) is considered to come from the analogy the relation of stress σ -strain ε traditionally discussed by many workers.

Here the relation between the stress σ and the total strain ε , the sum of a permanent strain εp and an elastic strain εr, is generally shown on logarithmic coordinates in Fig.4.

In many metals, the relation between a stress σ and a permanent strain εp [Eq.(15)] which is the difference obtained by subtracting an elastic strain εr [Eq.(14)] from a total strain ε is expressed by Eq.(16) [Hardenning Law of the n-th power] , as shown with a broken line in Fig.4 while the curve of σ - ε has been shown with a full line.

However in Fig.4,the ranges of elastic -plastic and fully plastic deformation are shown only for the curve of σ - ε .

$$\varepsilon_r = \sigma / E_s \quad \cdots\cdots (14)$$

$$\varepsilon_p = \varepsilon - \varepsilon_r \quad \cdots\cdots (15)$$

$$\sigma = C\varepsilon_p^n \quad \cdots\cdots (16)$$

On the basis of the above relations, first the curve of Pm-(d/Dp) in Fig.3 is compared with that of σ -εp in Fig.4.

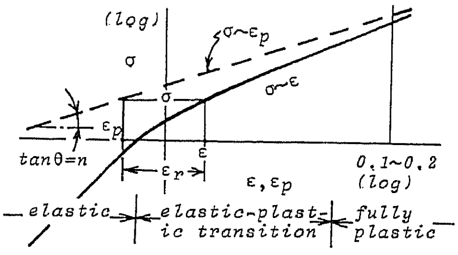

Figure 4. The stress-strain characteristic curve of general materials

Comparison between the curve of Pm-(d/D) and that of σ-ε ,it shows that the relation of Pm-(d/Dp) and σ-εp are clearly analogous.Consequently,it may be concluded that the relation of Pm-(d/Dp) is obtained corresponding to that of σ-εp.

Next,when the relation of Pm-(d/Dp) is formulated,the relation of Pm-(d/Dp) , corresponding to the curved part of Pm-(d/D) which approaches a straight line calculated using Hertz's Eq.(6), is drawn as a straight line on logarithmic coordinates.

Thus the relation of Pm-(d/Dp), referred to the Hardenning Law of n-th power for σ-εp, may be simply formulated with an exponential function as far as near point of the elastic contact.

Therefore the relation of Pm-(d/D) is expressed as Eq.(17) under the condition of (d/Dp) larger than 0 and the quantitative relationship between the physical quantities in the elastic-plastic transient indenting process are obtained as Eq.(18) and (19).

Namely,first the characteristic values of Pup, x_p of a specimen and that of f(E) of a specimen and an indenting sphere are given in Eq.(18),next by calculating the value of hardness Pm which satisfies Eq.(18) for an arbitrary value of (d/D), the relation of Pm-(d/D) may be obtained.

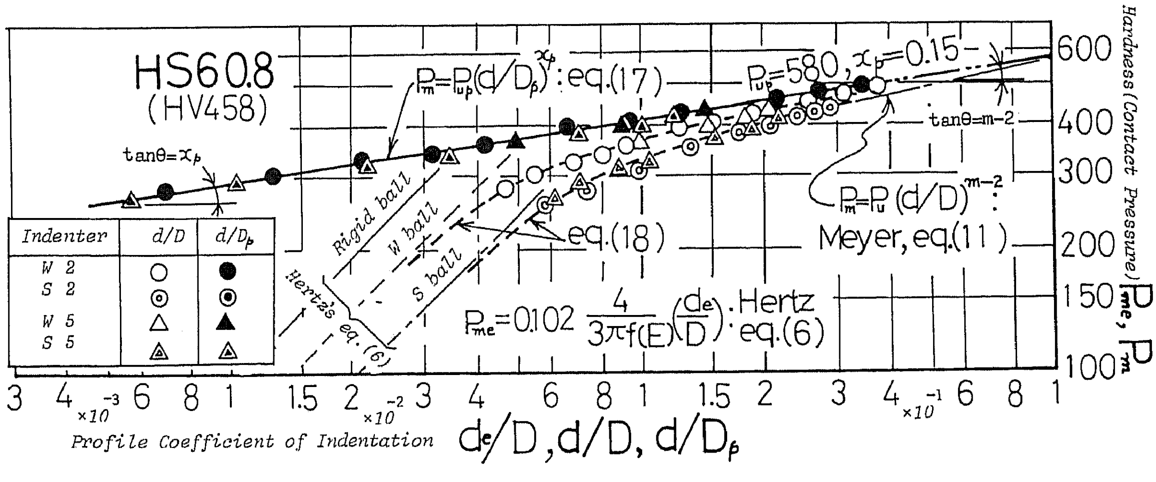

Figure 3. The interrelationship between Hertz's, Meyer's and Authors's Equation

In the same way,the relation between the load ratio C_{LR} and the profile coefficient (d/D) may be obtained from Eq.(19).

In these equations ultimate plastic hardness Pup means hardness Pm for the value of (d/Dp)=1,and the term x_p is called plastic hardenning index for a spherical indenter.

Further in the case of fully plastic deformation of a specimen, Eq.(17) will coincide with Meyer's Eq.(11) by the fact that the value of (d/Dp) is nearly equal to that of (d/D) in Eq.(13).[Pup≒Pu, x_p≒m-2]

$$P_m=P_{up}(d/D_p)^{x_p} \cdots\cdots(17)$$

$$P_m=P_{up}\left[\left(\frac{d}{D}\right)\left\{1-\frac{3\times9.8\pi f(E)}{4(d/D)}P_m\right\}\right]^{x_p} \cdots(18)$$

$$C_{LR}=\frac{\pi}{4}P_{up}\left(\frac{d}{D}\right)^2\left[\left(\frac{d}{D}\right)\left\{1-\frac{3\times9.8f(E)}{(d/D)^3}C_{LR}\right\}\right]^{x_p} \cdots\cdots(19)$$

6.4 The interrelationship between Hertz's , Meyer's and Author' contact equations

In Fig.3, the experimental results of Pm-(d/D) and Pm-(d/Dp) for the specimen HS61 (HV458) and the calculated curves using Hertz's Eq.(6), Meyer's Eq.(11) and Authors' Eqs.(17),(18) are shown in order to reveal the interrelationship between them.

Fig.3 shows that when an indenter contacts a specimen,first the contact is purely elastic so that the hardness increases along the straight line of Hertz's Eq.(6) as load becomes larger, then after the onset of plastic deformation , the hardness begins to climb and follow gradually the curved broken line of Authors' Eq.(18),and finally the hardness approaches the straight line of Meyer's Eq.(11).

6.5 The Brinell hardness value for hard specimen

Under the influence (2) mentioned in the Introduction , the actual Brinell hardness test for hard specimens produces the permanent deformation of an indenting sphere.

For that reason, the influence of mechanical properties of indenters and the permanent deformation of an indenter on the Brinell hardness value,defined as Eq.(20)[1],are studied in this paragraph.

$$HBS,\ HBW=\frac{2}{\pi}\frac{C_{LR}}{1-\sqrt{1-(d/D)^2}} \cdots\cdots(20)$$

The relations between hardness Pm, HB and the profile coefficient (d/D) may be calculated using Eqs.(18)-(20) when the indenter doesn't produce a permanent deformation. The relations of PmS,PmW,HBS, HBW-(d/D) calculated for the specimens HS 101, HS80, HS61 respectively are shown in Fig.5.

Fig.5 shows that hardness Pm and Brinell hardness HB exhibit similar values in range of the value of (d/D) less than 0.2, although Pm increases beyond this

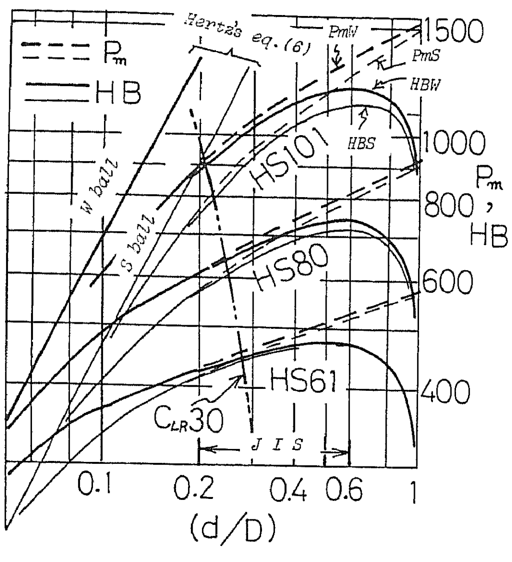

Figure 5. The relation between hardness Pm,HB and a profile coefficient (d/D)

value,the curve of HB has a maximum value and finally reaches the half value of Pm for the value of (d/D)=1.(PmS and PmW are the hardness using a steel ball indenter and a tungsten carbide ball indenter respectively.)

At the same time, the relation of HB-(d/D) calculated for the standard indenting test condition of JIS: C_{LR} =30,is shown with a two dot chain line.

Next the actual values of Brinell hardness test for hard specimens carried out by Yoshizawa[8] with the diameter of an indenter D=10 mm , corresponding to the calculated values for the standard condition C_{LR} =30,are quoted here and both values are given in Table 4.

We consider the reason why there is a difference between the experimental value of HB825 by Yoshizawa and the calculated value of HB892.The reason may be as follows.

Yoshizawa reports that the permanent deformation of an indenter is produced even when Brinell hardness test is carried out using a tungsten carbide ball indenter for the specimen HV478.Therefore the difference between them is attributed to the permanent deformation of an indenting sphere.

On the basis of the approximate method in Fig.6 for calculating the mean diameter of a deformed sphere following Yamashiro[13], the mean diameter of the tip of a deformed indenter D_Δ is calculated by Eq.(21) using

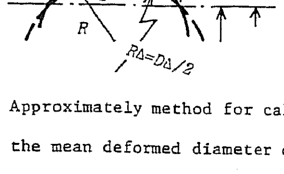

Figure 6. Approximately method for calculating the mean deformed diameter of a sphere

Yoshizawa's experimental values of Δ (the permanent deformation) and d_b , shown in Table 4.

$$D_\Delta = D / \{1 - (4\Delta D / d_b^2)\} \quad \cdots\cdots (21)$$

Table 4. Experimental and calculated values of hardness HBW, HBS

Standard Blocks for Hardness HS (HV)	Calculation Eq.(18), Eq.(19), Eq.(20)				Yoshizawa's data	
	HBW (HBWΔ)	DΔ mm	HBS (HBSΔ)	DΔ mm	HBW	HBS
101 (1000)	892 (835)	12	790 (491)	25	825	662
80 (676)	639 (634)	10.2	607 (576)	12	629	585
61 (458)	453 (453)	10	443 (440)	10.5	458	436

The suffix Δ is attached to the value of HBS and HBW calculated from Eq. (18) - (20), and they are put in parentheses and given under the values of HBS and HBW in Table 4.

Comparing the calculated values with Yoshizawa's experimental values, it is seen that they have a good coincidence.

But in the case of an indenting test for a specimen HS101 using a steel ball indenter , the calculated value is much higher than the experimental one.

The reason may be as follows; Yamashiro has reported that if the permanent deformation of an indenter is excessive , the profile of the tip of an indenting sphere is nearly flat, so that the above difference is attributable to the fact that the premise of the indenting test using a sphere is no longer present.

It is shown in Table 4 that the influence of the permanent deformation of an indenting sphere on the hardness value is relatively small with the degree of about a quarter of that deformation.

7. Conclusion

The results obtained are summarized as follows.

(1) In the elastic - plastic trnsient indenting process using a spherical indenter, hardness Pm is formulated easily with the exponential function of a true profile coefficient (d/Dp) ranging from the near point of the elastic contact to the fully plastic deformation of a specimen.

(2) On the basis of the analogy between Pm-(d/D) and σ - ε pointed out traditionally, it may be said that the relation of Pm-(d/Dp) corresponds to that of stress σ - permanent strain εp.

(3) As the relations of the physical quantities between a specimen and an indenting sphere are formulated in the elastic-plastic transient indenting process ,the influence of different mechanical properties of spherical indenters on hardness can be analyzed in an arbitrary indenting condition.

(4) The actual Brinell hardness values for hard specimens are in good coincidence with the calculated values considering the permanent deformation of an indenting sphere by Authors' Equations , consequently a consistent basis to the hardness test using a spherical indenter from the soft specimens to the hardest ones is obtained.

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