Two-wavelength sinusoidal phase/modulating laser-diode interferometer insensitive to external disturbances

Osami Sasaki, Hiroyuki Sasazaki, and Takamasa Suzuki

We describe a two-wavelength laser-diode interferometer that is insensitive to external disturbances such as fluctuations in the wavelength of the laser diode and mechanical vibrations of the optical components. In sinusoidal phase-modulating interferometry this insensitivity is easily obtained by controlling the injection current of the laser diode with a feedback control system. Using an equivalent wavelength of 152 μm provided by two single-frequency laser diodes, we can measure the distance, rotation angle, and surface profile measurements with great accuracy.

I. Introduction

Two-wavelength interferometry is used to measure displacements larger than a wavelength of light.[1-4] Laser diodes are able to provide different optical wavelengths more inexpensively than any other laser. Two-wavelength interferometers using laser diodes are reported in Refs. 3 and 4. In Ref. 3 a laser diode produces two different wavelengths by switching the injection current between two levels, and the two wavelengths are time multiplexed. In Ref. 4 the two wavelengths are assigned to the two single-frequency laser diodes, and the directions of polarization of the two laser beams are perpendicular to each other, so that the two interference signals are distinguished. To obtain the phase of an interference signal in real time with electronic hardware, sinusoidal phase modulation is introduced by using a vibrating mirror.

We also present a two-wavelength interferometer using two single-frequency laser diodes. However, sinusoidal phase-modulating (SPM) interferometry[5,6] and feedback control of injection currents of the two laser diodes for eliminating external disturbances[7] are used. Sinusoidal phase modulation is created by modulating the injection current of the laser diode with a sinusoidal wave. Two different frequencies of sinusoidal phase modulation are assigned to the two laser diodes so that the two interference signals can be distinguished. The wavelength of laser diodes changes with temperature. The change in the wavelength and vibrations of optical components cause the fluctuations in the phase of the interference signal. Shearing interferometry is useful for eliminating these fluctuations. Shearing interferometry, however, cannot be applied to laser-diode interferometers where phase modulation is generated by modulating the injection current of a laser diode, since laser-diode interferometers need an optical path difference of more than a few centimeters. The fluctuations caused by external disturbances are reduced by controlling the injection current with a simple feedback control system in a laser-diode SPM interferometer. This feedback control enables us to measure displacements with great accuracy without suffering from external disturbances.

In Section II SPM interferometry using a laser diode is explained. In Section III the principle of the feedback control system for eliminating external disturbances is described, and an SPM interferometer with this feedback control system is described. A two-wavelength interferometer using the techniques described in Sections II and III is described in Section IV. The measurement results of distances, rotation angles, and surface profiles are described in Section V.

II. Sinusoidal Phase Modulation in a Laser-Diode Interferometer

A. Interference Signal

Let us consider a Twyman–Green-type interferometer that results from removing the feedback control-

The authors are with Niigata University, Faculty of Engineering, 8050 Ikarashi 2, Niigata-shi, Japan.
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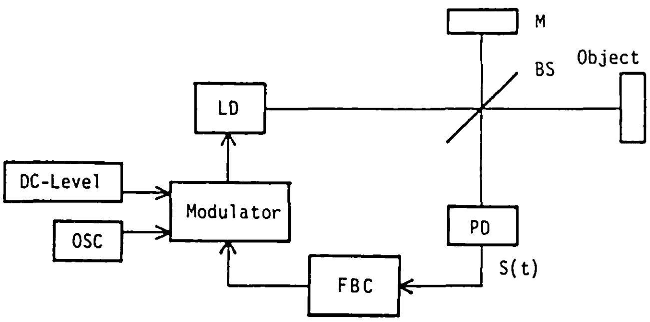

Fig. 1. Setup of the feedback control of an injection current in a laser-diode interferometer.

ler (FBC) from Fig. 1. The injection current of the laser diode (LD) consists of a dc component i_0 and a time-varying component $\Delta i_m(t)$ produced with an oscillator (OSC). The dc component determines the central wavelength of the light λ_0, and the Δi_m causes a small change in the wavelength of the laser diode:

$$\Delta\lambda(t) = \beta\Delta i_m(t), \tag{1}$$

where β is a proportional constant. Thus the wavelength of the LD is given by

$$\lambda(t) = \lambda_0 + \Delta\lambda(t). \tag{2}$$

The optical wave emitted from the laser diode is represented by

$$A(t) = \exp\left\{j2\pi c \int_0^t [1/\lambda(t)]\,dt\right\} = \exp[j\phi(t)], \tag{3}$$

where c is the velocity of light, and the amplitude is assumed to be unity. The light reflected from an object is an objective wave, and the light reflected from the mirror (M) is a reference wave. The interference pattern produced by the two waves is detected with a photodiode (PD). The optical path lengths of these waves are denoted by l_o and l_r, respectively. The time-varying component of the interference signal is given by

$$S(t) = \cos[\phi(t - \tau_o) - \phi(t - \tau_r)] = \cos\Phi(t), \tag{4}$$

where $\tau_o = l_o/c$ and $\tau_r = l_r/c$. Using the approximation

$$1/\lambda(t) \sim (1/\lambda_0)\{1 - [\Delta\lambda(t)/\lambda_0]\}, \tag{5}$$

and the definition

$$\int \Delta\lambda(t)\,dt = \Delta\Lambda(t), \tag{6}$$

the argument in Eq. (4) becomes

$$\Phi(t) = (2\pi/\lambda_0)l - (2\pi c/\lambda_0^2)[\Delta\Lambda(t - \tau_o) - \Delta\Lambda(t - \tau_r)], \tag{7}$$

where $l = l_r - l_o$. In the conditions of $\tau_o \ll 1$ and $\tau_r \ll 1$, we have the approximation

$$\Delta\Lambda(t - \tau_o) - \Delta\Lambda(t - \tau_r) \sim (l/c)\Delta\lambda(t), \tag{8}$$

and Eq. (4) becomes

$$S(t) = \cos[\alpha - (2\pi/\lambda_0^2)l\Delta\lambda(t)], \tag{9}$$

where $\alpha = (2\pi/\lambda_0)l$.

For sinusoidal phase modulation,

$$\Delta i_m(t) = a\cos(\omega_c t + \theta), \tag{10}$$

we obtain the interference signal

$$S(t) = \cos[z\cos(\omega_c t + \theta) + \alpha], \tag{11}$$

where $z = -(2\pi/\lambda_0^2)\beta a l$.

B. Signal Processing

We reported on the method used to obtain the phase α from the interference signal in Ref. 5. Here we review briefly a method for detecting the interference signal by a photodiode. Equation (11) is rewritten as

$$\begin{aligned} S(t) = {} & (\cos\alpha)[J_0(z) - 2J_2(z)\cos(2\omega_c t + 2\theta) + \cdots] \\ & - (\sin\alpha)[2J_1(z)\cos(\omega_c t + \theta) \\ & - 2J_3(z)\cos(3\omega_c t + 3\theta) + \cdots], \end{aligned} \tag{12}$$

where J_n is the nth-order Bessel function. The Fourier transform of Eq. (12) is represented by $F(\omega)$. The amplitude of the phase modulation z is obtained from the ratio $|F(\omega_c)/F(3\omega_c)|$. The phase θ is obtained from the argument $F(\omega_c)$. Using the values of z and θ, we can calculate the phase α from $F(\omega_c)$ and $F(2\omega_c)$. These computations are carried out on a computer.

III. Elimination of External Disturbances with Feedback Control

A. Principle

The wavelength of the laser diode changes by $\Delta\lambda_T$ with temperature. Optical components in the interferometer vibrate in response to external mechanical vibrations. This causes the change Δl in the optical path difference l between the object and reference waves. Both $\Delta\lambda_T$ and Δl cause a fluctuation in the phase of the interference signal. The fluctuation is compensated for by controlling the injection current to produce the change $\Delta\lambda_I$ in the wavelength of the laser diode. To consider the changes $\Delta\lambda_T$, Δl, and $\Delta\lambda_I$ in Eq. (9), l and $\Delta\lambda$ are replaced with $l + \Delta l$ and $\Delta\lambda + \Delta\lambda_T + \Delta\lambda_I$, respectively. By neglecting the term $(\Delta\lambda + \Delta\lambda_T + \Delta\lambda_I)\Delta l$, the interference signal for sinusoidal phase modulation is written as

$$S(t) = \cos[z\cos(\omega_c t + \theta) + \alpha + \delta(t)], \tag{13}$$

where

$$\delta(t) = (2\pi/\lambda_0)\Delta l - (2\pi l/\lambda_0^2)(\Delta\lambda_T + \Delta\lambda_I). \tag{14}$$

We try to reduce the phase $\delta(t)$ to zero by controlling the $\Delta\lambda_I$, which is produced by a change $\Delta i_c(t)$ in the injection current.

Let us explain how to generate the feedback signal for controlling the injection current $\Delta i_c(t)$ of the laser diode. The expansion of Eq. (13) is obtained by replacing α with $\alpha + \delta(t)$ in Eq. (12). By producing the signal $S(t)\cos(\omega_c t + \theta)$ and passing this signal through a low-pass filter, we obtain the feedback signal

$$S_F(t) = J_1(z)\sin[\alpha + \delta(t)]. \tag{15}$$

This feedback signal is available in the region of $z = 0.5$–3.5, where the value of the $J_1(z)$ is not so small.

We can keep the phases α and $\delta(t)$ at zero with a proportional and integral feedback control using the feedback signal given by Eq. (15). The feedback signal is generated in the feedback controller shown in Fig. 1, and it is fed to a proportional amplifier and an integrator. The sum of these outputs is the injection current $\Delta i_c(t)$. The proportional feedback control reduces the phase $\delta(t)$ to zero. The integral feedback control stabilizes the phase α at $2n\pi$, where n is an integer. When α is equal to $(2n + 1)\pi$, the feedback control is unstable. For the sake of simplicity, we use the expression where the phase α is kept at zero.

B. SPM Interferometer

Figure 2 shows an SPM interferometer with a feedback control system used for movement measurements. The central wavelength of laser diode 1 (LD1) is λ_{01}, and the frequency of the sinusoidal phase modulation is ω_{c1}. The change in temperature causes a change $\Delta\lambda_{T1}$ in the wavelength of LD1, and the injection current for the feedback control causes a change $\Delta\lambda_{I1}$. The light emitted from the laser diode is collimated with lens 1 (L1). The light reflected from mirror 1 (M1) is a reference wave. The light passing through the beam splitter (BS) is an object wave. A portion of the object wave is illuminated onto the object. The reflected light from the object and the reference light is superimposed on photodiode 1 (PD1). The optical path difference is l, and it changes by Δl due to mechanical vibrations of the optical components. The interference signal detected with PD1 is written as

$$S_1(t) = \cos[z_1\cos(\omega_{c1}t + \theta_1) + \alpha_1 + \delta_1(t)], \tag{16}$$

where

$$\delta_1(t) = (2\pi/\lambda_{01})\Delta l - (2\pi l/\lambda_{01}^2)(\Delta\lambda_{T1} + \Delta\lambda_{I1}),$$
$$\alpha_1 = (2\pi/\lambda_{01})l. \tag{17}$$

On the other hand, the remainder of the object wave is illuminated onto mirror 2 (M2). The light reflected by M2 and the reference light reach photodiode 2 (PD2). The optical path difference is l_f, and its change is represented by Δl_f. The interference signal detected with PD2 is written as

$$S_{1f}(t) = \cos[z_{1f}\cos(\omega_{c1}t + \theta_1) + \alpha_{1f} + \delta_{1f}(t)], \tag{18}$$

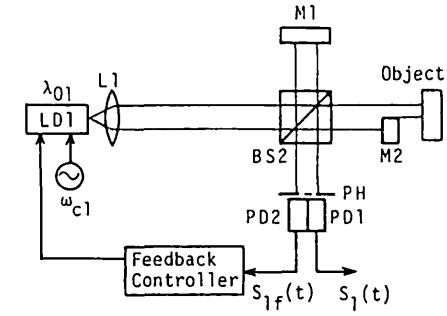

Fig. 2. SPM laser-diode interferometer with a feedback control system to eliminate external disturbances.

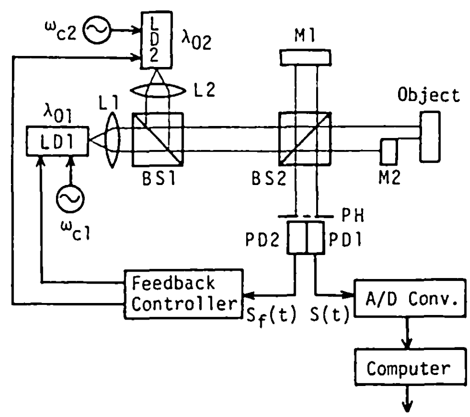

Fig. 3. Two-wavelength SPM laser-diode interferometer with a feedback control system to eliminate external disturbances.

where

$$\delta_{1f}(t) = (2\pi/\lambda_{01})\Delta l_f - (2\pi l_f/\lambda_{01}^2)(\Delta\lambda_{T1} + \Delta\lambda_{I1}),$$
$$\alpha_{1f} = (2\pi/\lambda_{01})l_f. \tag{19}$$

The feedback signal is generated from this interference signal in the feedback controller. When feedback control is operating well, phases $\delta_{1f}(t)$ and α_{1f} are kept at zero. Then the phase α_1 is represented as

$$\alpha_1 = (2\pi/\lambda_{01})(l - l_f). \tag{20}$$

This equation indicates that the phase α_1 shows the distance between the object and M2. Using the condition $\delta_{1f}(t) = 0$, Eq. (17) becomes

$$\delta_1(t) = (2\pi/\lambda_{01})[\Delta l - (l/l_f)\Delta l_f]. \tag{21}$$

The phase $\delta_1(t)$ is not always zero, because the optical path difference l and its change Δl in the signal $S(t)$ are not equal to l_f and Δl_f in the signal $S_f(t)$.

IV. Two-Wavelength Interferometer

We add another interferometer to the interferometer shown in Fig. 2, which produces the two-wavelength interferometer shown in Fig. 3. The central wavelength of the second interferometer is λ_{02}, and the frequency of the sinusoidal phase modulation is $\omega_{c2}/2\pi$. The symbols used for the second interferometer have a 2 subscript, which is to be distinguished from those used for the first interferometer. By replacing λ_{01}, $\Delta\lambda_{T1}$, and $\Delta\lambda_{I1}$ with λ_{02}, $\Delta\lambda_{T2}$, and $\Delta\lambda_{I2}$, respectively, in Eqs. (17) and (19), we define the phases α_2, $\delta_2(t)$, α_{2f}, and $\delta_{2f}(t)$ for the second interferometer.

The signals $S(t)$ and $S_f(t)$ detected with PD1 and PD2, respectively, are the sum of the two interference signals corresponding to the two wavelengths λ_{01} and λ_{02} as follows:

$$S(t) = \cos[z_1 \cos(\omega_{c1}t + \theta_1) + \alpha_1 + \delta_1(t)] + \cos[z_2 \cos(\omega_{c2}t + \theta_2) + \alpha_2 + \delta_2(t)], \tag{22}$$

$$S_f(t) = \cos[z_{1f} \cos(\omega_{c1}t + \theta_1) + \alpha_{1f} + \delta_{1f}(t)] + \cos[z_{2f} \cos(\omega_{c2}t + \theta_2) + \alpha_{2f} + \delta_{2f}(t)]. \tag{23}$$

The feedback signals for controlling the injection currents of LD1 and LD2 are obtained from the interference signal $S_f(t)$ in the feedback controller. Producing the signals $S_f(t)\cos(\omega_{ci}t + \theta_i)$ $(i = 1, 2)$ and passing these signals with low-pass filters, we obtain the feedback signals $J_1(z_{if})\sin[\alpha_{if} + \delta_{if}(t)]$ $(i = 1, 2)$, respectively. When the two feedback controls for LD1 and LD2 work well, conditions $\alpha_{if} = 0$ and $\delta_{if}(t) = 0$ $(i = 1, 2)$ hold. Thus the phases α_1 and $\delta_1(t)$ are given by Eqs. (20) and (21), and the phases α_2 and $\delta_2(t)$ are given by the equations that are produced by replacing λ_{01} with λ_{02} in Eqs. (20) and (21).

We obtain the phase $\alpha_i + \bar{\delta}_i$ $(i = 1, 2)$ from the Fourier transform $F(\omega)$ of the interference signal $S(t)$ using the method described in Subsection II.B. In the Fourier domain we can separate the two interference signals corresponding to the two wavelengths λ_{01} and λ_{02}. The phase $\bar{\delta}_i$ indicates the average value of $\delta_i(t)$ during the detecting interval or the length of the signal to be processed on a computer. Before we calculate the difference α between $\alpha_1 + \bar{\delta}_1$ and $\alpha_2 + \bar{\delta}_2$, we define the equivalent wavelength λ_e produced from wavelengths λ_{01} and λ_{02} as

$$\lambda_e = (\lambda_{01}\lambda_{02})/(\lambda_{01} - \lambda_{02}). \tag{24}$$

Then phase α is given by

$$\alpha = (2\pi/\lambda_e)(l - l_f) + (2\pi/\lambda_e)[\overline{\Delta l} - (l/l_f)\overline{\Delta l_f}], \tag{25}$$

where $\overline{\Delta l}$ and $\overline{\Delta l_f}$ are the average values of $\Delta l(t)$ and $\Delta l_f(t)$ during the detecting interval. Thus we measure the distance $l - l_f$ with the equivalent wavelength λ_e, eliminating external disturbances.

V. Experiments

A. Construction of the Interferometer

We constructed the two-wavelength interferometer shown in Fig. 3. We used GaAlAs laser diodes whose wavelengths are ~780 nm. The difference between the two wavelengths was ~4 nm. The two frequencies of the sinusoidal phase modulations $\omega_{c1}/2\pi$ and $\omega_{c2}/2\pi$ were 1 and 8 kHz, respectively. The optical path differences l and l_f were ~80 and ~60 mm, respectively. The distance between the two pinholes on PD1 and PD2 was 2 mm. The interference signal $S(t)$ was sampled with an analog-to-digital converter to be stored on a microcomputer. The sampling frequency was 64 kHz, and the length of the sampled data was 32 ms. We obtained phase α through the fast Fourier transform of the sampled data with the computer.

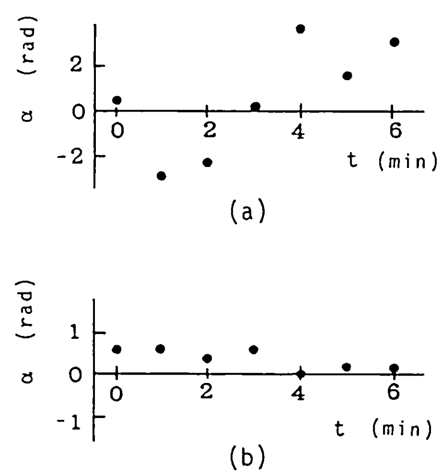

Fig. 4. Effects of eliminating external disturbances: (a) feedback control not working; (b) feedback control working well.

B. Effect of Eliminating External Disturbances

In the interferometer shown in Fig. 3, we used an aluminum plate as the object, and the measuring point on the object was fixed. When the feedback control for eliminating external disturbances did not work, measurements of phase α were repeated at intervals of 2 min. The measured phases are shown in Fig. 4(a). The maximum fluctuation in phase α is ~6 rad. The phases measured when the feedback control worked well are shown in Fig. 4(b). The feedback control reduces the fluctuation in the phase α by approximately one tenth compared with that shown in Fig. 4(a).

C. Distance Measurements

In the interferometer shown in Fig. 3, we moved the object parallel to the optical axis at intervals of 20 μm. Figure 5 shows the phases measured at several distances d. In this figure, phases α_i $(i = 1, 2)$ indicate phases $\alpha_i + \bar{\delta}_i$ $(i = 1, 2)$, and they are denoted by circles and rectangles, respectively. The phases α are denoted by double circles. The measured phase α is proportional to the distance d, and this proportional constant gives $\lambda_e = 152$ μm. The maximum deviation from the proportional relation in the phase α is 0.5 rad, and this deviation causes the measurement error of ~6 μm.

D. Rotation Angle Measurement

To measure the rotation angles of the object, we changed partially the constitution of the interferometer as shown in Fig. 6. Lens 3 makes an image of the object with unity magnification. In this constitution the interference signal to be used for generating the feedback signal is produced from the light reflected

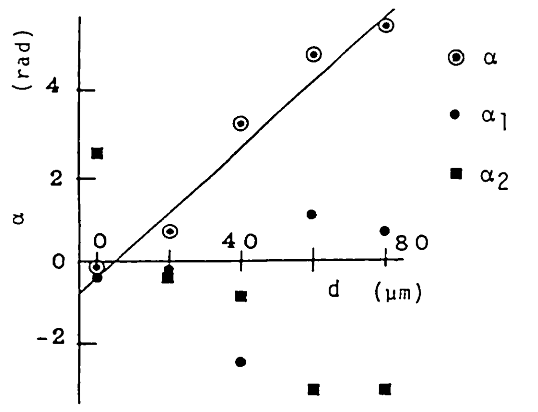

Fig. 5. Measurement results of the moving distances of an aluminum plate.

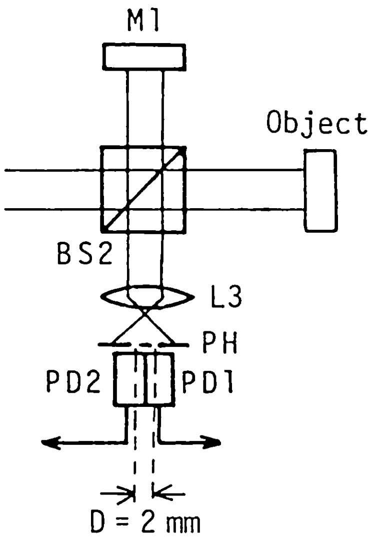

Fig. 6. Optical system used to create an image of an object in a rotation angle and surface profile measurements.

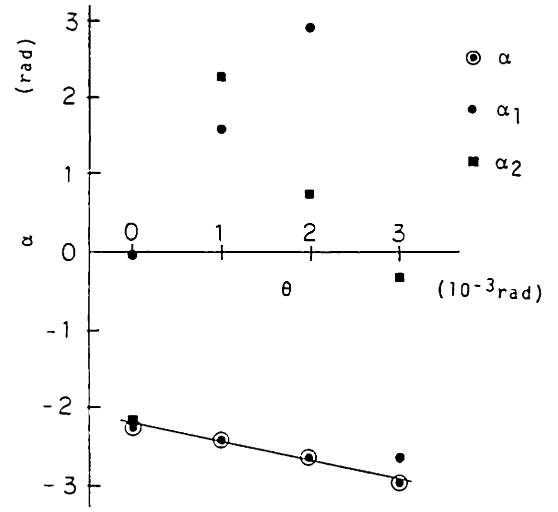

Fig. 7. Measurement results of the rotation angles of the mirror.

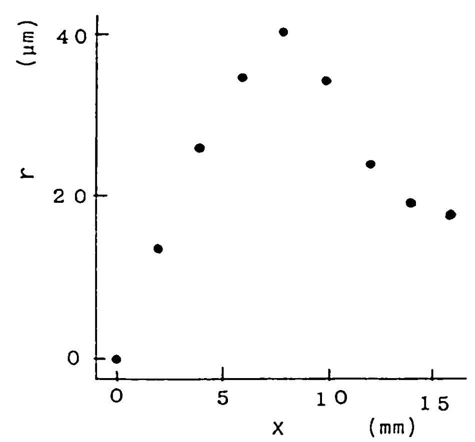

Fig. 8. Measurement results of the surface profile of an aluminum plate.

from the object. Since the conditions $l = l_f$ and $\Delta l = \Delta l_f$ are satisfied, the fluctuations in the phase α are reduced to within ± 0.015 rad. We used a mirror as an object and rotated it at intervals of $\sim 1.0 \times 10^{-3}$ rad. At each rotation angle we measured a phase α as shown in Fig. 7. The phases α_1 and α_2 change by more than 2π rad, but the phase α changes in proportion to the rotation angle. Deviations from the proportional relation in the phase α are also within ~ 0.015 rad.

E. Surface Profile Measurement

We measured surface profiles of diffuse objects with the same interferometer used for measurements of rotation angles. The object was an aluminum plate whose surface was coated with white paint. An image of the object was made on the photodiodes with unity magnification. The object was translated normal to the optical axis at intervals that are equal in distance to that between the two pinholes on the photodiodes. If the phase detected at the ith measuring point is represented by α^i ($i = 0, 1, 2, \cdots$), the surface profile at the measuring point is given by

$$r^i = (\lambda_e/4\pi)\,(\alpha^i + \alpha^{i-1} + \cdots + \alpha^1 + \alpha^0),$$

where $\lambda_e = 152$ μm. The phase α^0 corresponds to the phase of the interference signal detected with PD2 when the first measuring point is located at PD1, so that the phases α^0 and r^0 are equal to zero. We measured the phase α_i ($i = 1$–8). The measured surface profile is shown in Fig. 8. In this measurement the fluctuations in the phase α were also within ± 0.015 rad, which results in a measurement accuracy of 0.18 μm.

VI. Conclusions

We have described the two-wavelength SPM laser-diode interferometer with a feedback control system that eliminates external disturbances. Fluctuations in the wavelength of the laser diode were completely eliminated with the feedback control system. In distance measurements the fluctuations in the phase of the interference signal caused by mechanical vibrations were reduced to one tenth. By a rotation angle and surface profile measurements, the fluctuations were reduced to within ± 0.015 rad. Thus accurate measurements were made with no effects from external disturbances. In SPM interferometry we can utilize effectively the property that the light wavelength of the laser diode can be modulated by the injection current; thus we can construct two-wavelength interferometers that are insensitive to external disturbances with simple optical and electrical components.

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